

MEASUREMENTS

1. **ALL MEASUREMENTS have SOME DEGREE of UNCERTAINTY.**
2. The last digit of a measurement is always estimated and therefore, uncertain. Always include the uncertain digit with the reported measurement.
 - a. For measurements obtained with analog scales, the estimated/uncertain digit is located at 1 decimal place beyond the smallest division of the scale.
 - b. For measurements obtained with digital scales, the estimated/uncertain digit is the last digit of the display. It is electronically estimated.
3. Significant Figures, “SFs”, are simply the figures of a measurement that were actually measured along with the single estimated digit. Here are some rules for determining whether a figure is significant or not:
 - a. All non-zero figures (1-9) are significant
 - b. Zeros that lie between non-zero figures are significant
 - c. Leading zeros are NEVER significant (they only serve as place holders)
 - d. Trailing Zeros are only significant if there is a decimal point *anywhere* in the number
4. Exact Numbers are different than measurements. There is no estimating/uncertainty involved Here are some examples of exact numbers:
 - a. Counting numbers are exact (e.g. 15 pennies)
 - b. Whole number fractions are exact (e.g. 1/3)
 - c. The symbol, π , is treated as an exact number. The number π can be written with as many SFs as one chooses (3.14, 3.14159, etc.)

- d. Definitions, such as 3ft/1 Yd., are exact numbers
5. Scientific notation is a convenient way of recording only the SFs of a measurement. An important skill is to be able to quickly recognize and count the SFs in a measurement. That is the first step in knowing how to properly round mathematical results based on sig figs.

Examples:

<u>Decimal notation</u>	<u>Scientific notation</u>	<u># of SFs</u>
523.87	5.2387×10^2	5
0.00423	4.23×10^{-3}	3
19000	1.9×10^4	2
341000.	3.41000×10^5	6
19.20	1.920×10^1	4

6. When multiplying or dividing measurements, find the *measurement* with the fewest number of SFs and round the mathematical result to that number of SFs. Do not consider exact numbers, since they are infinitely significant.

Examples:

$$123\text{m}/54\text{s} = 2.2777\text{m/s} = 2.3\text{m/s} \quad \text{rounded to 2 sig figs}$$

$$0.00234\text{cm} \times 62567\text{cm} = 146.40678\text{cm}^2 = 146\text{cm}^2 \quad \text{rounded to 3 sig figs}$$

$$6.5\text{ft} \times 1\text{Yd.}/3\text{ft} = 2.16667\text{Yd} = 2.2\text{Yd} \quad \text{rounded to 2 sig figs}$$

$$30\text{mm} \times 33\text{mm} = 990\text{mm}^2 = 1000\text{mm}^2 \quad \text{rounded to 1 sig fig}$$

7. When adding or subtracting measurements, round the result to the last significant figure of the least precise measurement (as indicated by the underlined figure in each of the following examples).

Examples:

$$543.123\text{mm} + 101.\underline{4}\text{mm} = 644.523\text{mm} = 644.5\text{mm} \quad \text{rounded to the tenths place.}$$

$$6\underline{4}00 \text{ mL} + 12 \text{ mL} = 6412 \text{ mL} = 6400 \text{ mL}$$

place

rounded to the hundreds

$$0.00235 \text{ m} + 0.00\underline{1} \text{ m} = 0.00335 \text{ m} = 0.003 \text{ mL}$$

place

rounded to the thousandths

STATISTICS

8. **Average** – A measure of where a distribution of data is centered.

$$\bar{X} = \frac{\sum X_i}{n}$$

9. **Sample Standard Deviation** – A measure of how widely the data is distributed to either side of the average.

$$s = \sqrt{\frac{\sum(\bar{X} - X_i)^2}{(n - 1)}}$$

10. **Accuracy** – The closeness of a measurement to the actual quantity being measured. An accurate measurement is characterized by a small % relative error.

$$\% \text{ Relative Error} = \left(\frac{|Average - Actual|}{Actual} \right) \times 100$$

11. **Precision** – The repeatability of a measurement. A Precise measurement is characterized by small % relative standard deviation.

$$\%RSD = \left(\frac{\text{Standard Deviation}}{\text{Average}} \right) \times 100$$

12. Systematic Errors – Errors due to procedural or instrumental factors that cause measurements to be consistently either too large or too small. This type of error impacts accuracy. Systematic errors are also called “determinate” errors because they can, in theory, be determined and minimized and possibly eliminated.

13. Random Errors – Errors that cause measurements to be intermittently too large or too small, without directional bias. This type of error impacts the precision of a measurement. Random errors are also called “indeterminate” errors because they CANNOT be determined and eliminated. The impact that random errors have on the accuracy of a measurement is minimized by taking multiple measurements and obtaining an average result.

14. Confidence Interval, λ The distance from the average in which the true mean is expected to be found with a given level of confidence.

$$\lambda = \frac{t \cdot s}{\sqrt{n}}$$

$$\text{Lower limit} = \text{average} - \lambda \quad \text{Upper limit} = \text{average} + \lambda$$

For a given confidence level (such as 95%), look up the value of t on the “Student’s t” table, below.

n-1	t _{50%}	t _{80%}	t _{90%}	t _{95%}	t _{99%}
1	1	3.078	6.314	12.706	63.657
2	0.816	1.886	2.92	4.303	9.925
3	0.765	1.638	2.353	3.182	5.841
4	0.741	1.533	2.132	2.776	4.604
5	0.727	1.476	2.015	2.571	4.032
6	0.718	1.440	1.943	2.447	3.707
7	0.711	1.415	1.895	2.365	3.500
8	0.706	1.397	1.860	2.306	3.355
9	0.703	1.383	1.833	2.256	3.250
10	0.700	1.372	1.812	2.228	3.169
15	0.691	1.341	1.753	2.131	2.947
20	0.687	1.325	1.725	2.086	2.845
∞	0.674	1.282	1.645	1.960	2.576

Example: For a given set of data, where $\bar{X} = 23.2\text{g}$, $s = 1.20\text{g}$ and $n=3$, the 95% confidence interval is calculated as:

$$\lambda = \frac{(t)(s)}{\sqrt{n}} = \frac{(4.303)(1.20)}{\sqrt{3}} = 2.98\text{g}$$

$$\text{Lower limit} = (\bar{X} - \lambda) \quad \text{Upper limit} = (\bar{X} + \lambda)$$

$$= (23.2 - 2.98) \quad = (23.2 + 2.98)$$

$$= (20.2) \quad = (26.2)$$

$$95\% \text{ Confidence Interval} = (20.2, 26.2)\text{g}$$