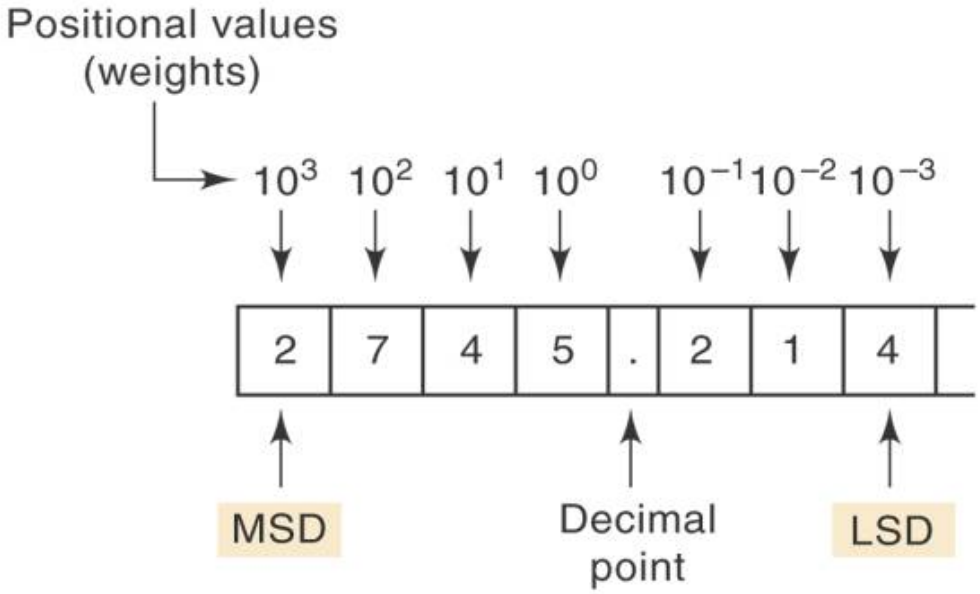
Number Systems

Number Systems

- Understanding digital systems requires an understanding of the decimal, binary, and hexadecimal numbering systems.
- Number systems differ in the amount of symbols they use
  - Decimal : Base 10;
    - 10 symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
  - Binary : Base 2;
    - 2 symbols (0, 1)
  - Hexadecimal : Base 16
    - 16 symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

Decimal Number Systems

- The Decimal (base 10) System
  - 10 symbols: 0, 1, 2, 3, 4, 5, 6 , 7, 8, 9.
  - Each number is a *digit* (from Latin for *finger*).
  - Most Significant Digit (MSD) & Least Significant Digit (LSD).



Decimal Number Systems

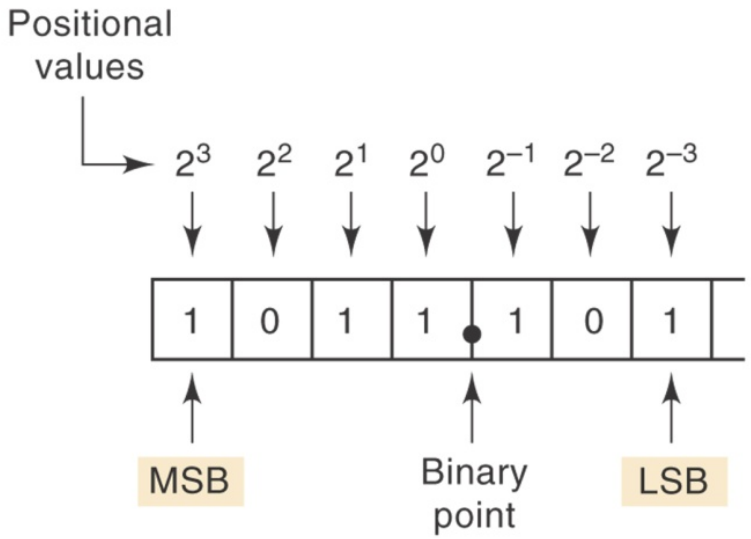
- Example : 5702_{10} ← subscript represents base

Digit	5	7	0	2
weighting	10^3	10^2	10^1	10^0
value	5x1000	7x100	0x10	2x1
	5000 + 700 + 0 + 2 = 5702			

- What would the equation look like for 15,702?

Binary Number Systems

- The Binary (base 2) System
  - 2 symbols: 0, 1
  - Each number is a *bit*
  - Most Significant Bit (MSB) & Least Significant Bit (LSB).



  - Binary lends itself to electronic circuit design since only two different voltage levels are required

Converting Binary to Decimal

- Each digit is multiplied by the corresponding power of 2
  - The next 3 years will be a lot easier if you memorize the powers of 2 NOW.

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
256	128	64	32	16	8	4	2	1

- Example: $1101\ 1001_2$

Digit	1	1	0	1	1	0	0	1
Weighting	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	128 +	64 +		16 +	8 +		2 +	1 =

Decimal to Binary

- Converting binary to decimal is easy (provided you memorized the powers of 2)
- You must also know the powers of 2 to convert decimal to binary
- Example: convert 358_{10} to binary

358
-256 => 2^8
102
-64 => 2^6
38
-32 => 2^5
6
-4 => 2^2
2
-2 => 2^1
0

$358_{10} = 1\ 0110\ 0110_2$

1	0	1	1	0	0	1	1	0
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Decimal to Binary Conversion

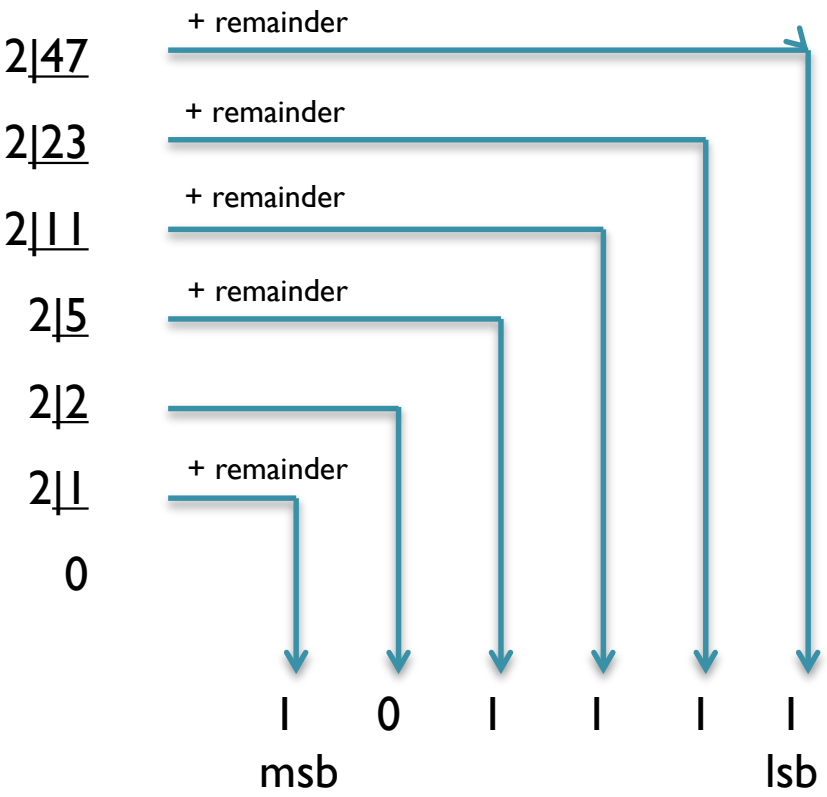
- Convert 373_{10} to binary

373
-256 => 2^8
117
-64 => 2^6
53
-32 => 2^5
21
-16 => 2^4
5
-4 => 2^2
1
-1 => 2^0
0

372 = 101110101

Decimal to Binary Method #2

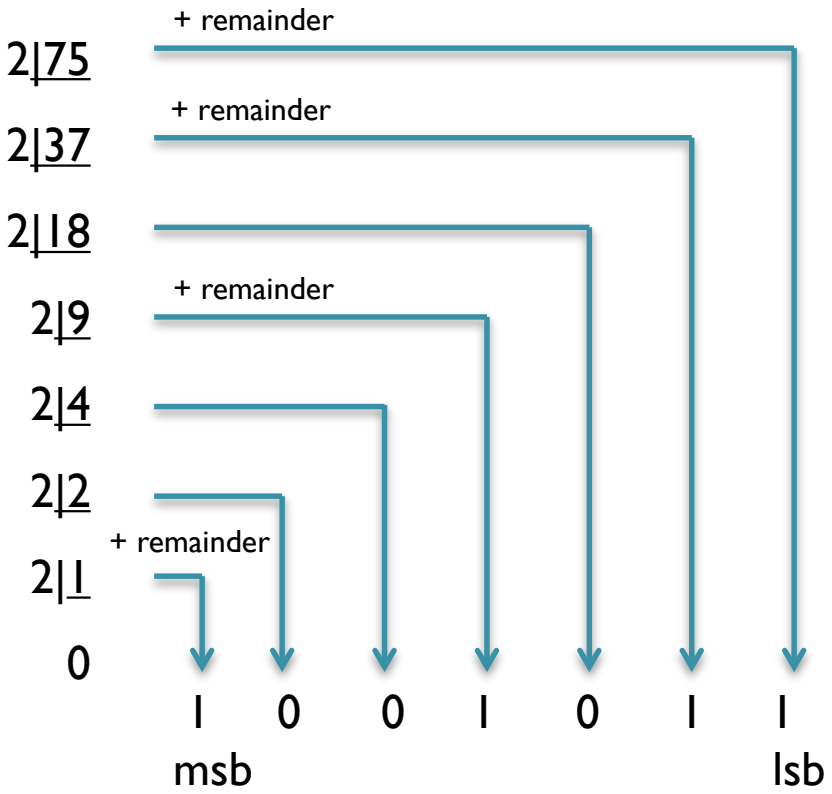
- Successively divide by 2



$47_{10} = 10\ 1111_2$

Decimal to Binary Conversion

- Convert 75 to binary using the successive division method.



$75_{10} = 1001011_2$

Hex Numbering System

- Base 16
- 16 digits
  - (0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F)
  - Hex is very common
    - ¼ the digits of binary
    - More intuitive than binary
    - Guess what you should do?

Decimal	Binary	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hex Conversions

- Binary to Hex
  - Divide binary number into groups of four bits (starting from the lsb)
  - Write the equivalent Hex digits
  - $1010001101011_2 = 1\ 0100\ 0110\ 1011_2 = 146B_{16}$
- Hex to Binary
  - Convert each hex digit to its 4-bit binary code
  - $73AB_{16} = 111\ 0011\ 1010\ 1011_2$
- Hex to decimal
  - Multiply each digit by its weighting factor (power of 16)
  - $73AB_{16} = (7x16^3) + (3x16^2) + (10x16) + 11 = 29{,}611_{10}$