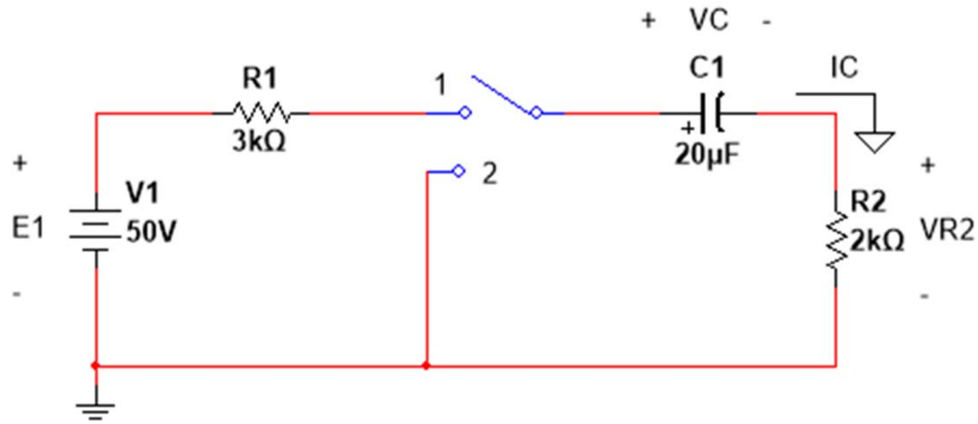
Example – Charge and Discharge



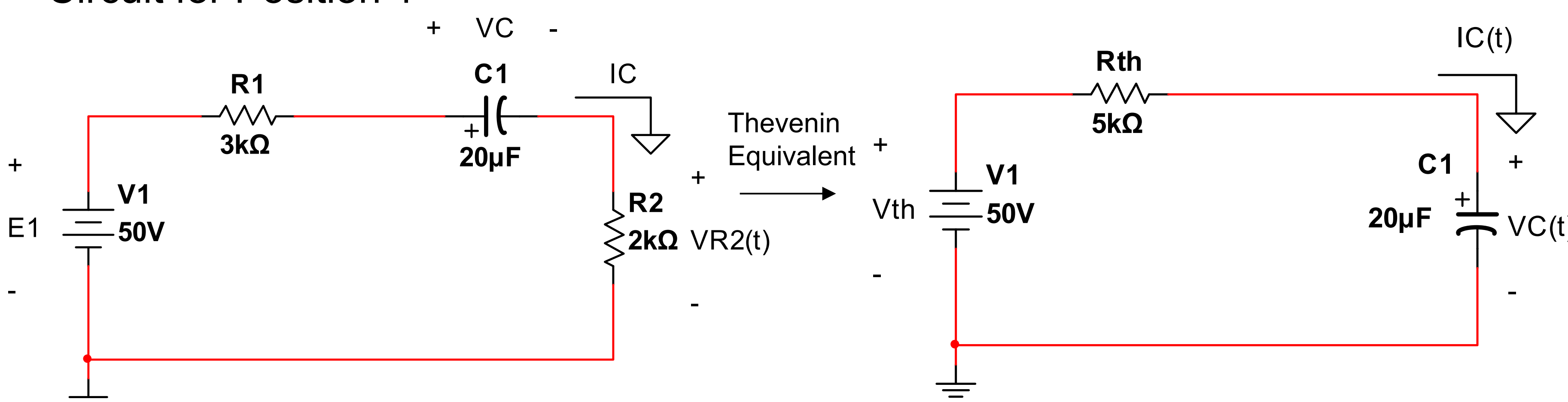
No initial charge

$V_C = 0$ V, $t < 0$

Position 1: C_1 **charges** through R1 and R2

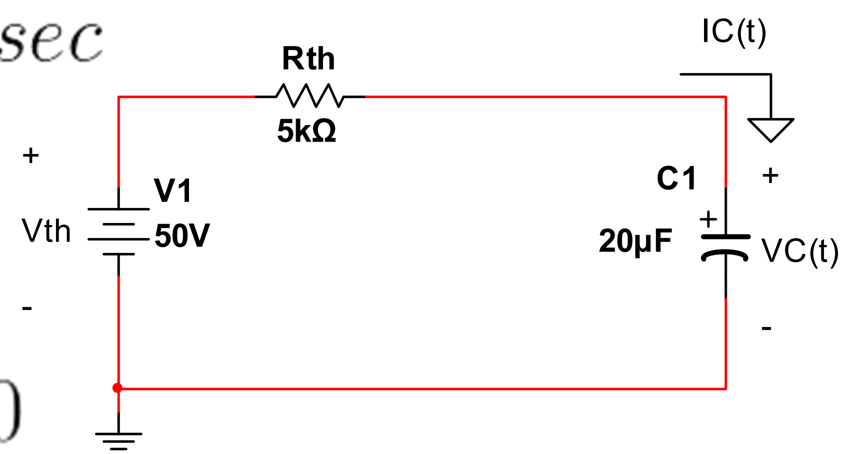
Position 2: C_1 **discharges** through R2

Circuit for Position 1



Example – Charge and Discharge

$$\tau = RC = 5k\Omega \cdot 20\mu F = 0.1sec$$

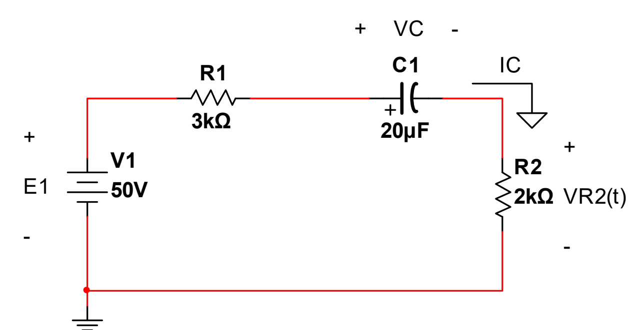


$$i_c(t) = (\frac{E}{R})e^{\frac{-t}{\tau}}\ A, t > 0$$

$$i_c(t) = (\frac{50V}{5k\Omega})e^{\frac{-t}{0.1sec}}\ A, t > 0$$

$$v_c(t) = E(1 - e^{\frac{-t}{\tau}})\ V, t \geq 0$$

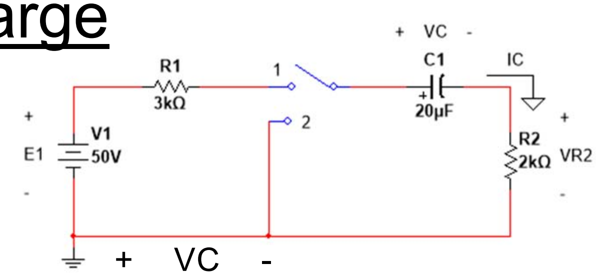
$$v_c(t) = 50(1 - e^{\frac{-t}{0.1sec}})\ V, t \geq 0$$



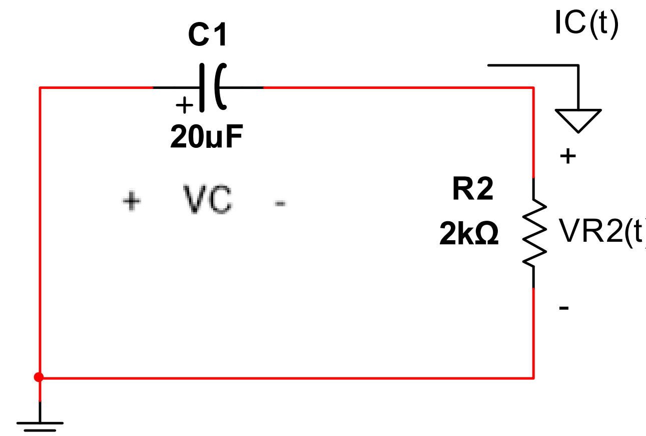
$$V_{R2}(t) = i_C(t) \cdot R_2 = 20e^{\frac{-t}{0.1}} V, t > 0$$

Example – Charge and Discharge

At t = 1 sec (1 sec > 5 τ), switch moves to position 2



- V_C (t = 1 sec) = 50V, final V_C(t) from charge phase
  - V_C cannot change instantaneously
  - V_C will decrease exponentially to 0 V
    - Depends on τ value for timing
  - The capacitor acts like a battery during discharge



$$v_c(t) = Ee^{\frac{-t}{\tau}}V$$

Initial Charge   Assumes E at t=0

$$E = 50V \qquad \tau = R \cdot C = 2k\Omega \cdot 20\mu F = 40ms$$

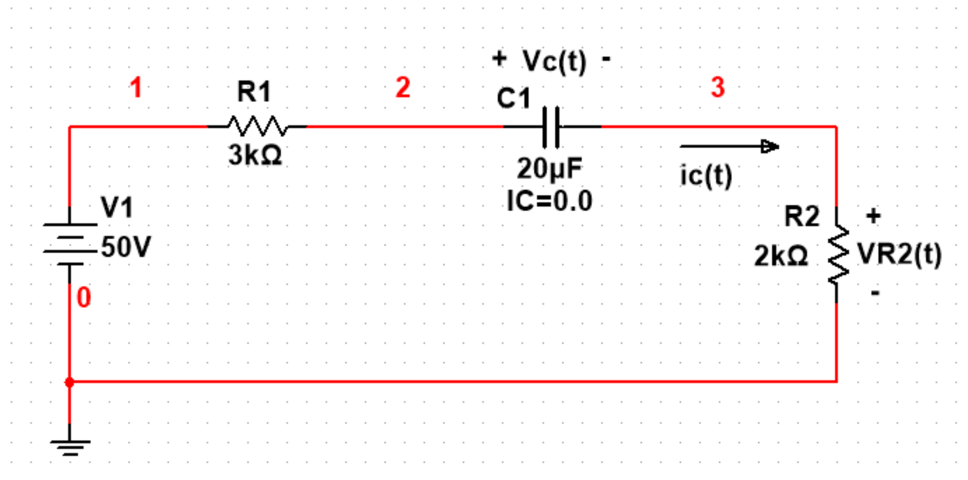
$$v_c(t) = 50e^{\frac{-(t-1)}{.04sec}}V, t \geq 1sec$$

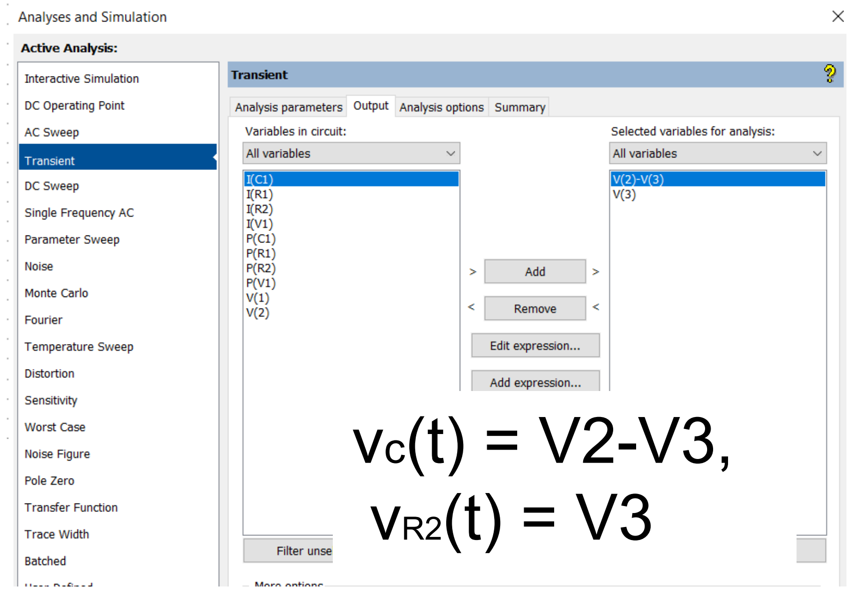
Note: $t_{initial}$ = 1 sec (not 0 sec)

$$v_{R_2}(t) = -v_c(t) = -50e^{\frac{-(t-1)}{0.04}}\ V, t > 1sec$$

$$i_c(t) = \frac{v_{R_2}(t)}{R_2} = -25 \cdot 10^{-3}e^{\frac{-(t-1)}{0.04}}A, t > 1sec$$

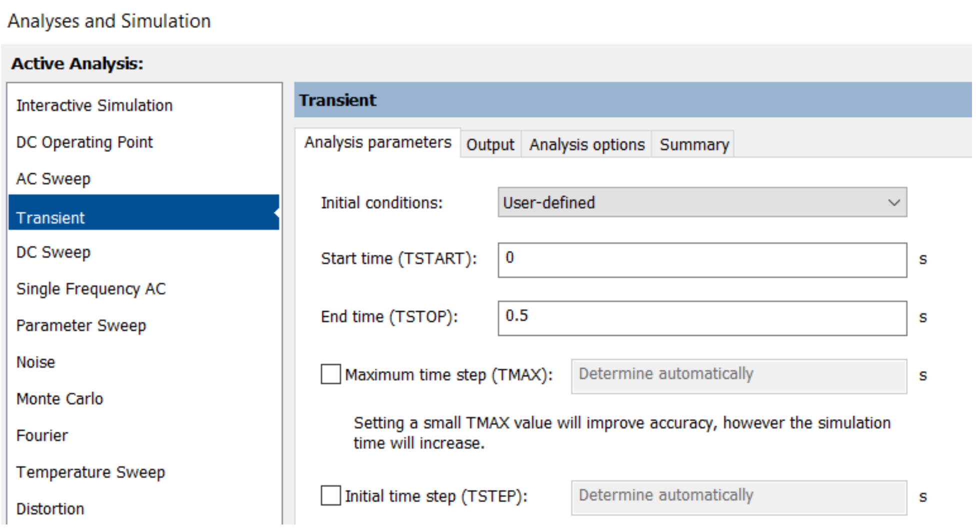
Simulation – Charge and Discharge

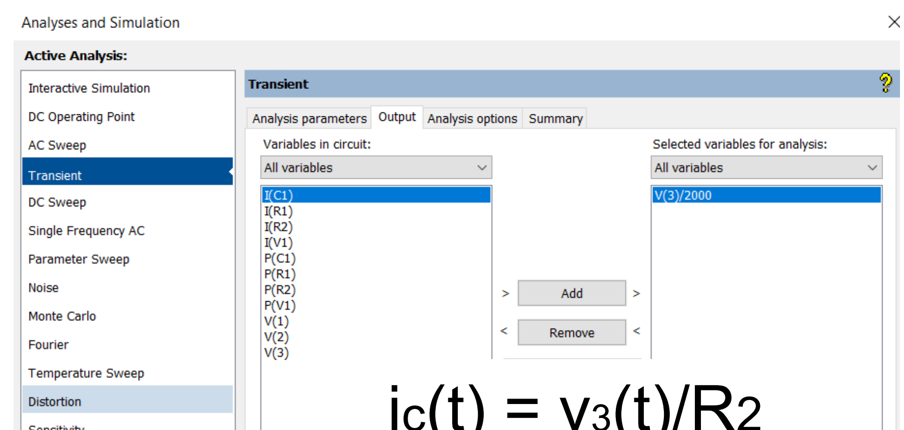




$v_C(t) = V2\text{-}V3,$

$v_{R2}(t) = V3$



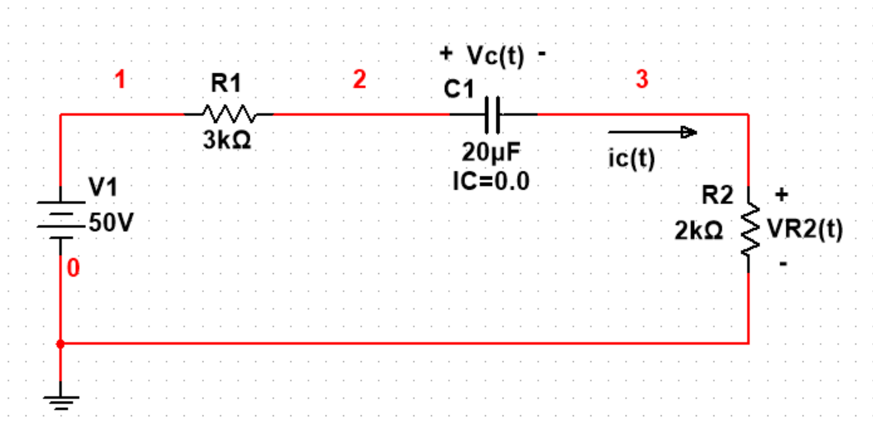


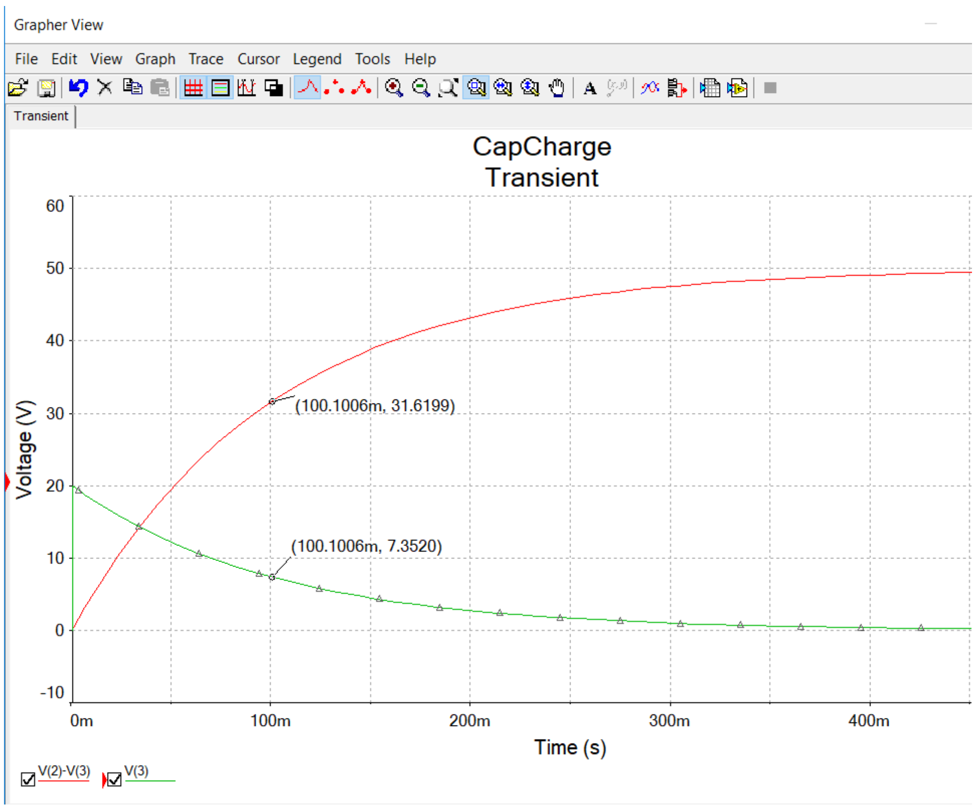
Recall: τ_{charge} = 0.1 sec

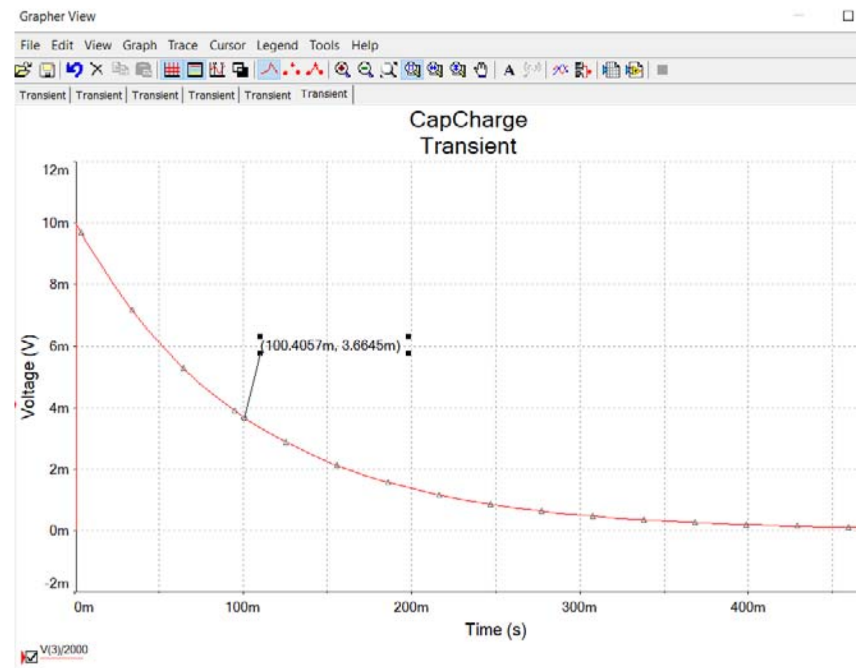
Simulate for 5τ

$i_C(t) = v_3(t)/R_2$

Simulation – Charge and Discharge







At one time constant (0.1 sec):

$v_C(0.1sec) = 31.6V$

$v_{R2}(0.1sec) = 7.35V$

$i_C(0.1sec) = 3.66mA$

Example – Charge and Discharge

Checking the charge phase results (theory and simulation):

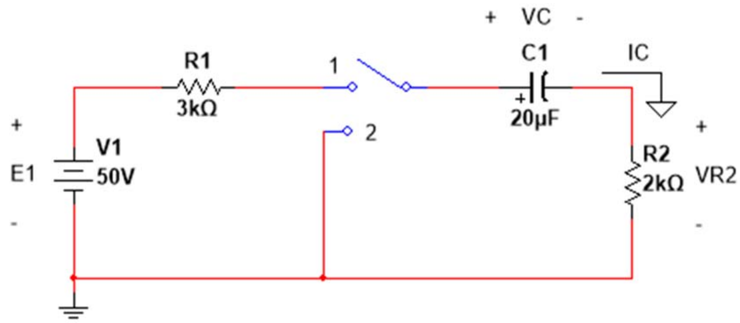
$$i_c(t) = 10 \cdot 10^{-3} \cdot e^{\frac{-t}{0.1}} A, t > 0$$

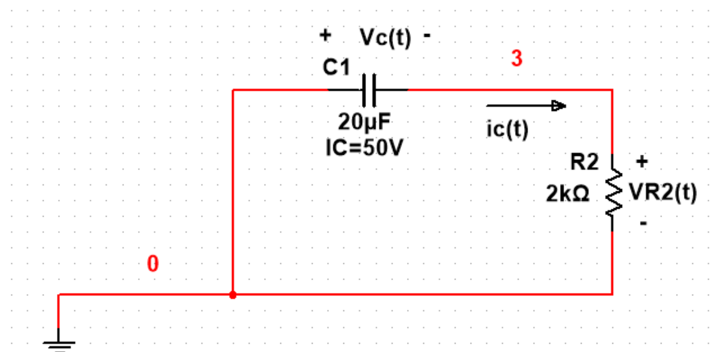
$$v_c(t) = 50(1 - e^{\frac{-t}{0.1}})V, t \geq 0$$

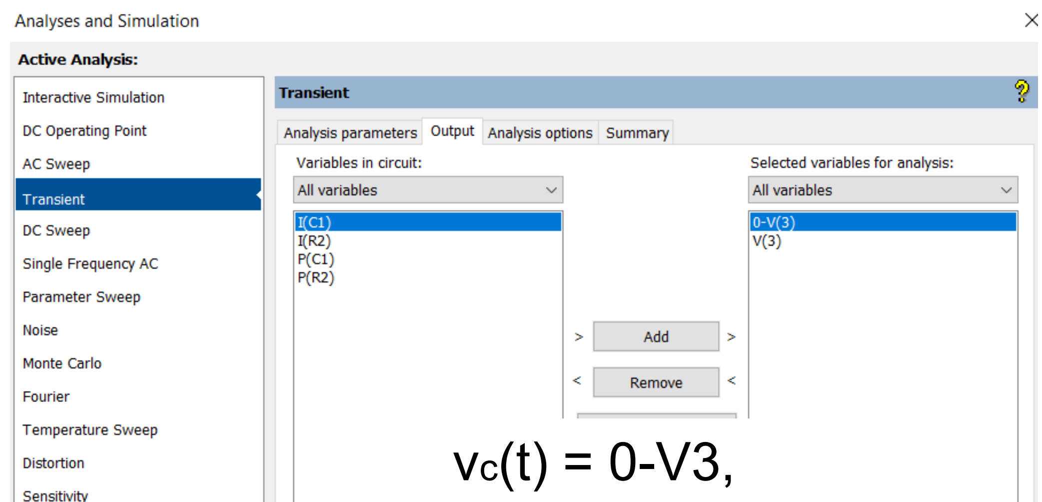
$$v_{R2}(t) = i_c(t) \cdot R_2 = 20e^{\frac{-t}{0.1}} V, t > 0$$

t (sec)	$i_C(t)$		$v_C(t)$		$v_{R2}(t)$	
	Calc	Multisim	Calc	Multisim	Calc	Multisim
0^+	10 mA	9.9 mA	0 V	0V	20 V	19.9 V
0.1 (τ)	3.68 mA	**3.66 mA**	31.6 V	**31.6 V**	7.36 V	**7.35 V**
0.25	0.82 mA	0.82 mA	45.9 V	45.8 V	1.64 V	1.63 V
0.5 (5τ)	67.4 μA	66.7 μA	49.7 V	49.6 V	0.13 V	0.13 V
1	0.45 μA	0.45 μA	49.98 V	49.85 V	0.91 mV	0 V

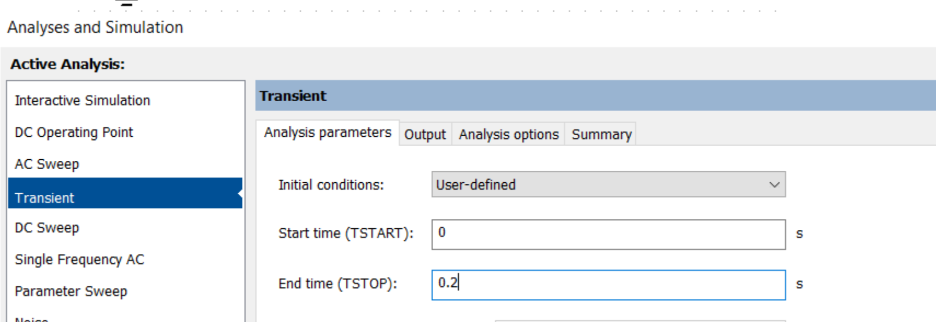
Simulation – Charge and Discharge

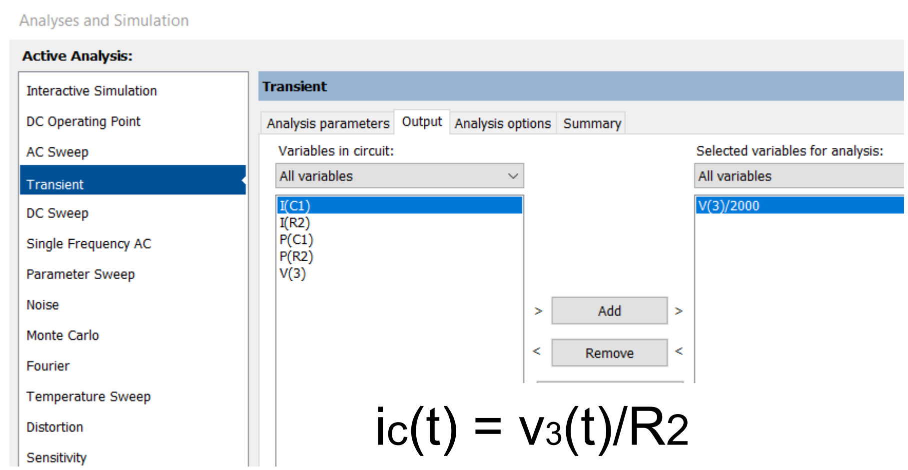






$v_C(t)$ = 0-V3,
$v_{R2}(t)$ = V3

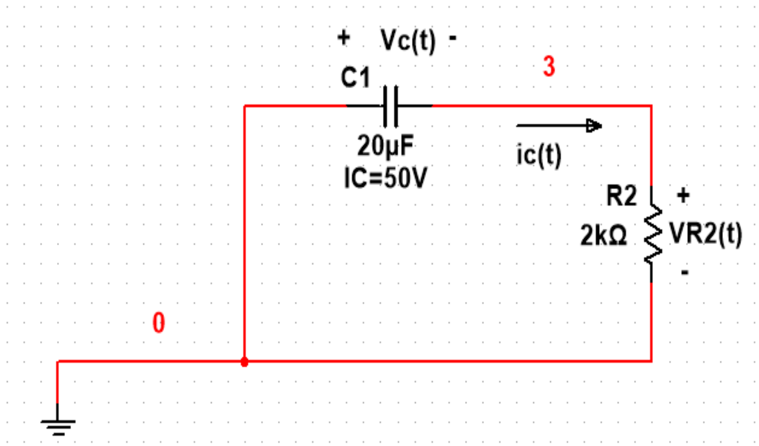


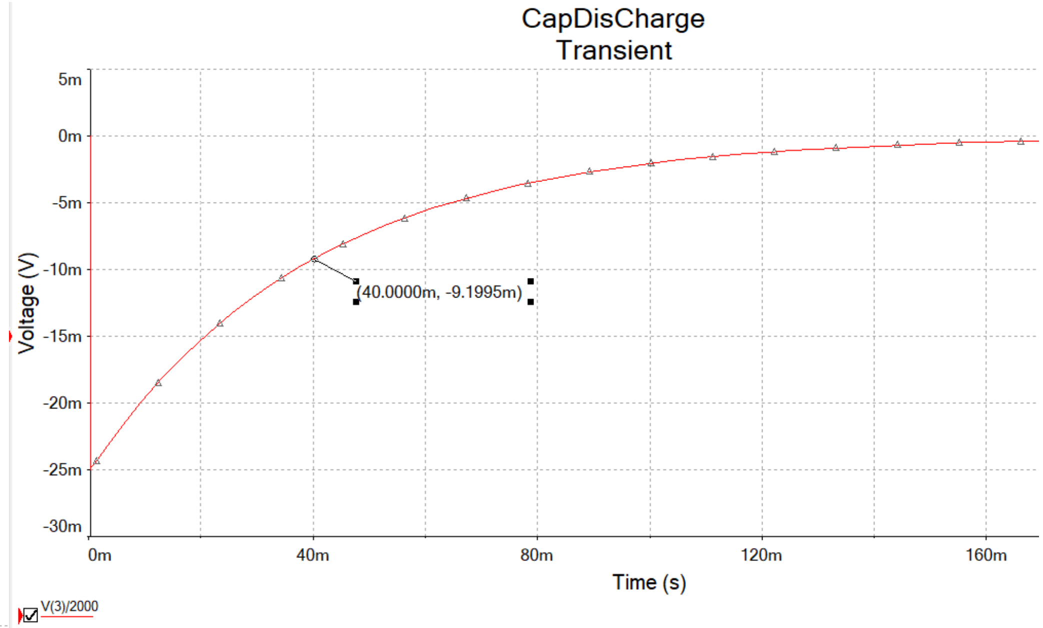


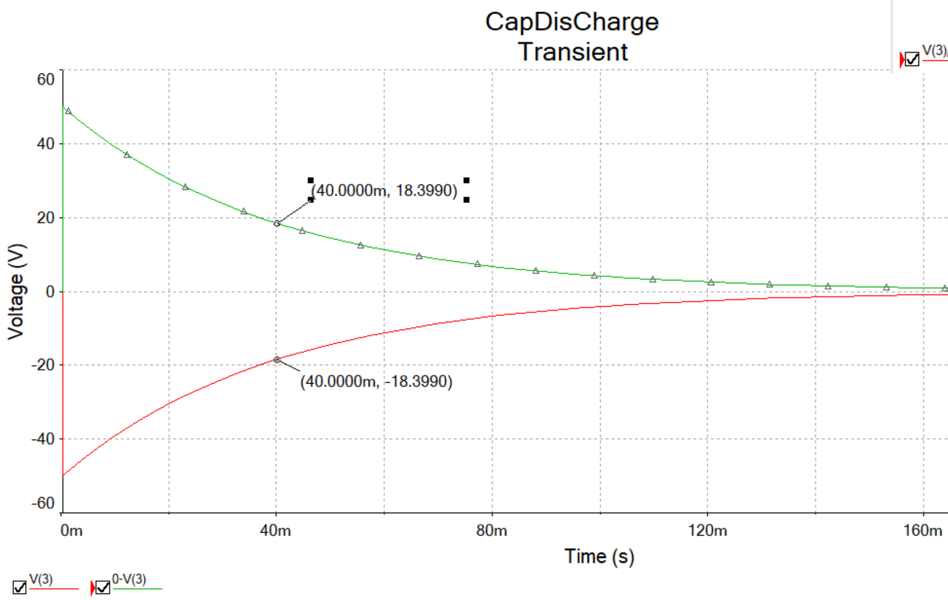
$i_C(t) = v_3(t)/R_2$

Recall: $\tau_{discharge}$ = 40msec
Simulate for 5τ (Simulation starts at t=0 sec, actual time is t=1 sec)

Simulation – Charge and Discharge







<u>At one time constant into discharge (40ms, actual problem time of 1.04 sec):</u>

$v_C(1.04sec) = 18.4V$

$v_{R2}(1.04sec) = -18.4V$

$i_C(1.04sec) = -9.2mA$

Example – Charge and Discharge

Checking the discharge phase results (theory and simulation):

$$v_C(t) = 50e^{\frac{-(t-1)}{0.04}} V, t \geq 1$$

$$v_{R_2}(t) = -50e^{\frac{-(t-1)}{0.04}} V, t > 1$$

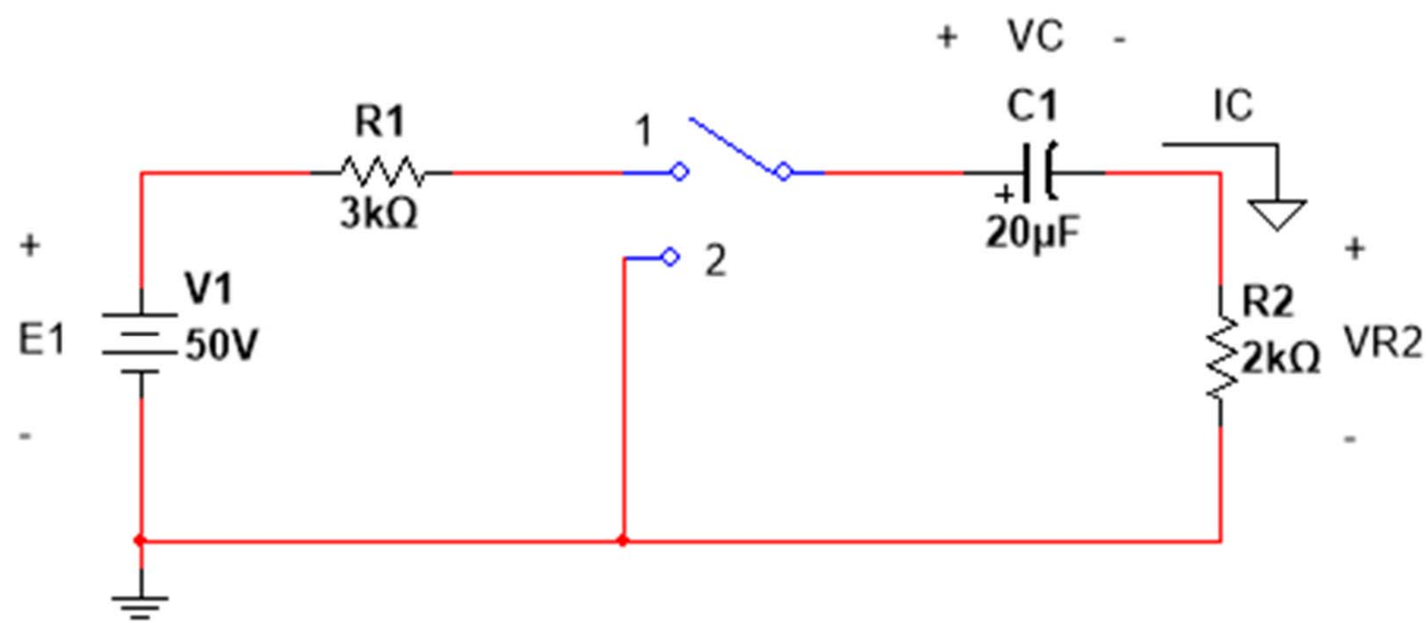
$$i_C(t) = -25 \cdot 10^{-3} \cdot e^{\frac{-(t-1)}{0.04}} A, t > 1$$

t (sec)	$i_C(t)$		$v_C(t)$		$v_{R2}(t)$	
	Calc	Multsim	Calc	Multisim	Calc	Multisim
1^+	-25 mA	-24.93 mA	50 V	49.85 V	-50 V	-50 V
1.04 (τ)	-9.20 mA	**-9.2 mA**	18.39 V	**18.4 V**	-18.39 V	**-18.4 V**
1.12 (3τ)	-1.24 mA	-1.23 mA	2.49 V	2.47 V	-2.49	-2.47 V

In Class Problem

Find t for $V_C(t)$ = 25 V for charge and discharge

- No initial charge on C1
- Switch to pos 1 at t = 0
- Switch to pos 2 at t = 1 sec



Same circuit as earlier