8.9 NODAL ANALYSIS (GENERAL APPROACH)

- USES KCL IN AN ORGANIZED MANNER TO SOLVE CIRCUITS PROBLEMS.

PROCEDURE: (1) DETERMINE THE # OF NODES

(2) CHOOSE A REFERENCE NODE, LABEL IT NODE 0. OFTEN WE CHOOSE GROUND AS NODE 0.

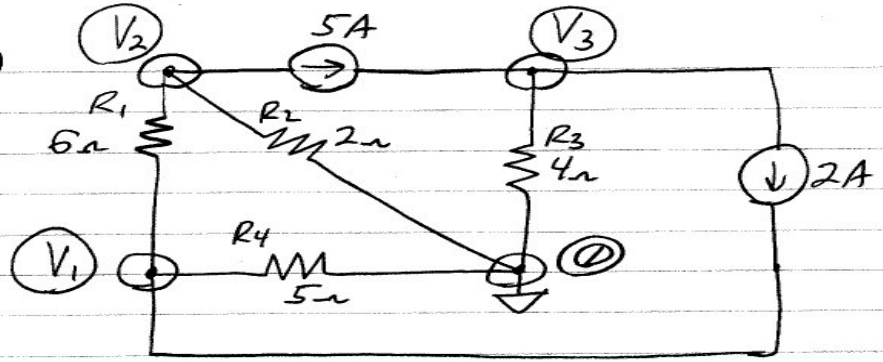
LABEL ALL OTHER NODES $V_1, V_2, \ldots, V_N$

(3) APPLY KCL AT EACH NODE (EXCEPT NODE 0). ASSUME UNKNOWN CURRENTS LEAVE THE NODE IN EACH CASE.

(4) SOLVE THE EQUATIONS FOR THE NODE VOLTAGES.

RECALL KCL: $\sum(\text{Currents Entering}) = \sum(\text{Currents Leaving})$

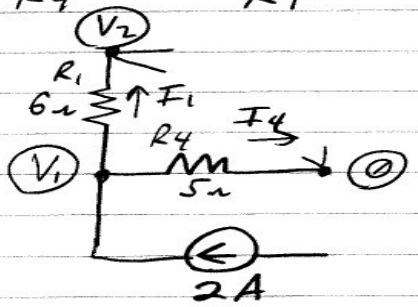
(EXAMPLE)



- 4 NODES
- LABELED 0, V_1, V_2, V_3

APPLY KCL AT EACH NODE:

NODE (V1) $\quad 2 = \frac{V_1}{R_4} + \frac{V_1 - V_2}{R_1}$



2A INTO NODE V_1 =

$(I_1 + I_4)$ SUM OF CURRENTS LEAVING

$I_1 = \frac{V_1 - V_2}{R_1}$

$+ I_4 = \frac{V_1 - 0}{R_4} = \frac{V_1}{R_4}$

$\therefore 2 = \frac{V_1 - V_2}{R_1} + \frac{V_1}{R_4}$

$2 = \frac{V_1}{5} + \frac{V_1}{6} - \frac{V_2}{6} \Rightarrow \quad 2 = 0.367 V_1 - 0.167 V_2 \qquad (1)$

NODE (V2) $\quad 0 = 5 + \frac{V_2}{R_2} + \frac{V_2 - V_1}{R_1} \Rightarrow \quad -5 = -\frac{V_1}{6} + \frac{V_2}{6} + \frac{V_2}{2}$

$-5 = -0.167 V_1 + 0.667 V_2 \qquad (2)$

NODE (V3) $\quad 5 = 2 + \frac{V_3}{R_3} \Rightarrow \quad 3 = \frac{V_3}{4}$

$3 = 0.25 V_3 \qquad (3)$

SOLVE EQ (1) → (3):

$$2 = 0.367V_1 - 0.167V_2 + 0V_3 \quad (1)$$

$$-5 = -0.167V_1 + 0.667V_2 + 0V_3 \quad (2)$$

$$3 = 0V_1 + 0V_2 + 0.25V_3 \quad (3)$$

$$V_1 = \frac{\begin{vmatrix} 2 & -0.167 & 0 \\ -5 & 0.667 & 0 \\ 3 & 0 & 0.25 \end{vmatrix}}{\begin{vmatrix} 0.367 & -0.167 & 0 \\ -0.167 & 0.667 & 0 \\ 0 & 0 & 0.25 \end{vmatrix}}$$

NUM ~ NUM

DEN ~ DEN

NUM →

$$\begin{vmatrix} 2 & -0.167 & 0 \\ -5 & 0.667 & 0 \\ 3 & 0 & 0.25 \end{vmatrix} = 2\begin{vmatrix} 0.667 & 0 \\ 0 & 0.25 \end{vmatrix} - (-0.167)\begin{vmatrix} -5 & 0 \\ 3 & 0.25 \end{vmatrix} + (0)\begin{vmatrix} -5 & 0.667 \\ 3 & 0 \end{vmatrix}$$

$$= 2[(0.667)(0.25) - (0)(0)] + 0.167[(-5)(0.25) - (0)(3)] + 0[(-5)(0) - (0.667)(3)]$$

$$= 0.3335 - 0.2088 + 0 = \underline{0.1248}$$

DEN → 54.23×10^{-3}

$$V_1 = \frac{NUM}{DEN} = \boxed{2.30\,V}$$

SIMILARLY: $\boxed{V_2 = -6.92V}$

$\boxed{V_3 = 12V}$

Write the nodal equations and, using determinants, solve for the nodal voltages.