9.2 SUPERPOSITION THEOREM

- ANOTHER TOOL WE CAN USE TO ANALYZE CIRCUITS WITH MULTIPLE SOURCES.

SUPERPOSITION: THE CURRENT THROUGH, OR VOLTAGE ACROSS, AN ELEMENT IN A LINEAR BILATERAL NETWORK IS EQUAL TO THE ALGEBRAIC SUM OF THE CURRENTS OR VOLTAGES PRODUCED <u>INDEPENDENTLY</u> BY <u>EACH</u> <u>SOURCE</u>.

LINEAR → THE CHARACTERISTICS OF THE NETWORK ELEMENTS ARE INDEPENDENT OF THE VOLTAGE ACROSS OR CURRENT THROUGH THEM. ("R" DOESN'T CHANGE WITH APPLIED VOLTAGE OR CURRENT)

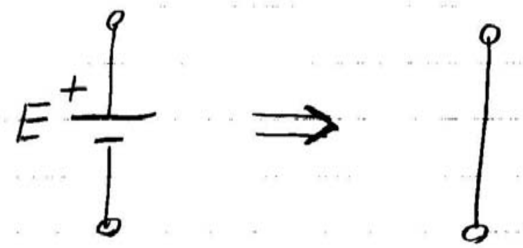
BILATERAL → NO CHANGE IN THE CHARACTERISTICS OF AN ELEMENT IF THE CURRENT THROUGH IT OR VOLTAGE ACROSS IT IS REVERSED. ("R" DOESN'T CHANGE IF WE REVERSE THE APPLIED VOLTAGE OR CURRENT)

* NOTE: WORKS FOR VOLTAGE + CURRENT, <u>NOT</u> POWER (DIRECTLY).

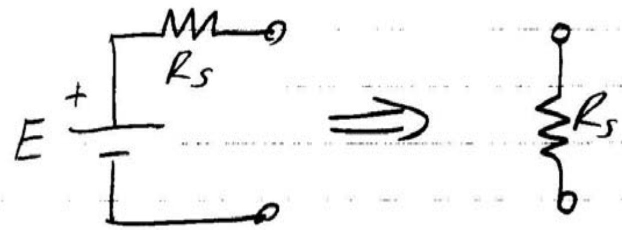
WE NEED TO FIND V OR I DUE TO EACH SOURCE INDEPENDENTLY. HOW?

(1) KEEP THE SOURCE OF INTEREST
(2) REMOVE OTHER SOURCES
(3) ANALYZE THE CIRCUIT
(4) REPEAT STEPS (1) TO (3) FOR EACH SOURCE
(5) ALGEBRAICALLY SUM THE CURRENTS (OR VOLTAGES)

TO REMOVE A VOLTAGE SOURCE:

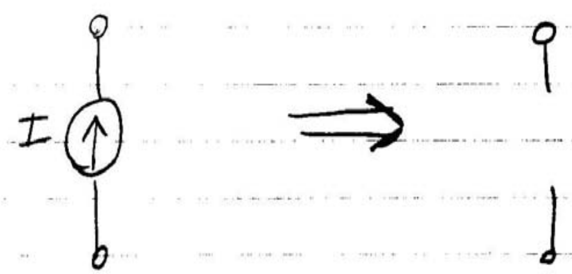


E S/C, NO VOLTAGE

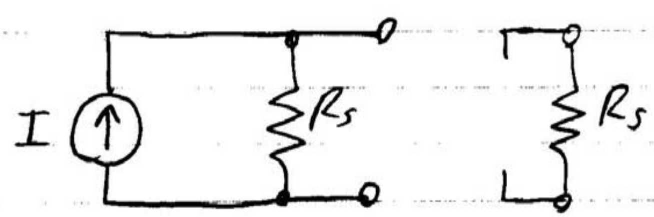


E S/C AGAIN

TO REMOVE A CURRENT SOURCE:

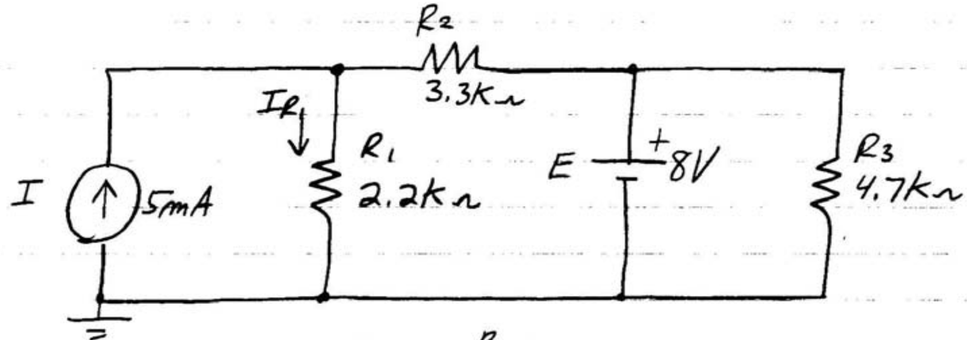


I O/C, NO CURRENT

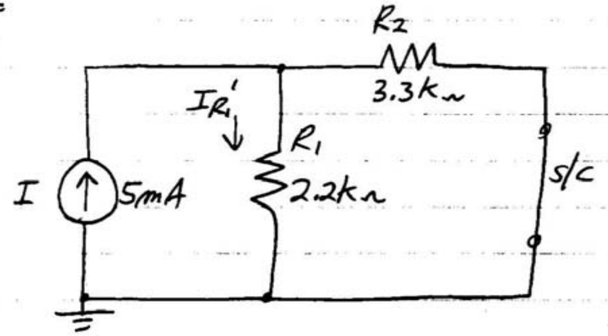


I O/C AGAIN

(EXAMPLE) FIND I_{R_1} :



DUE TO I

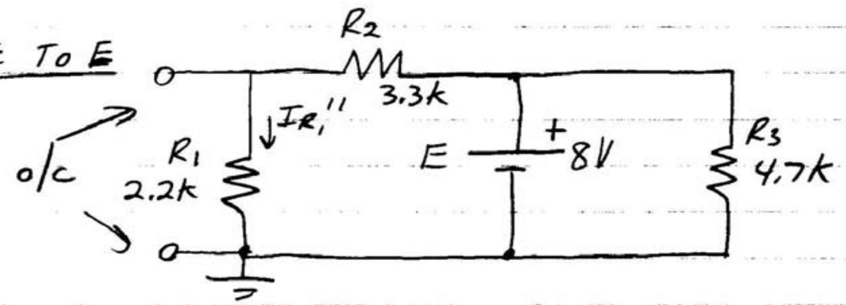


$$I_{R_1}' = I\left(\frac{R_2}{R_2+R_1}\right)$$

$$= 5mA\left(\frac{3.3k}{3.3k+2.2k}\right)$$

$$\underline{I_{R_1}' = 3mA}$$

DUE TO E

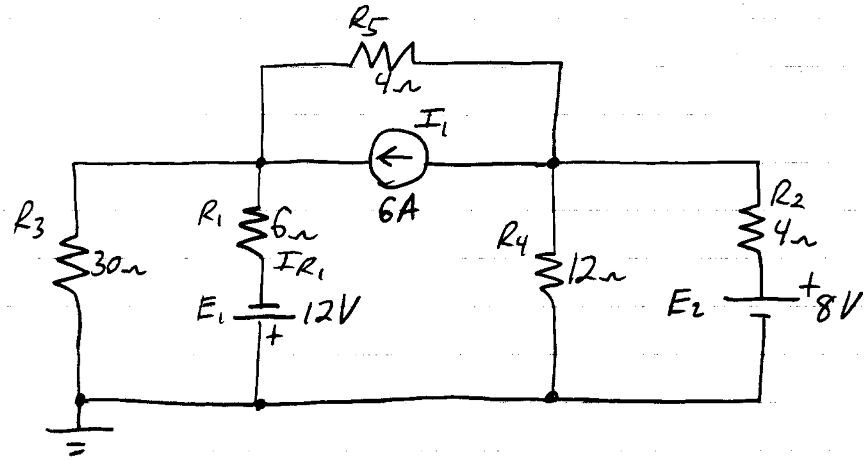


$$I_{R_1}'' = \frac{E}{R_1+R_2} = \frac{8V}{3.3k+2.2k} = \underline{1.455mA}$$

$$I_{R_1} = I_{R_1}' + I_{R_1}'' = \boxed{4.455mA}$$

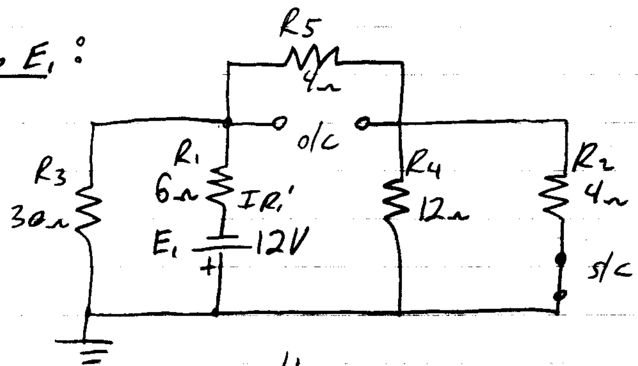
* NOTE : $\underline{\underline{2}}$ SOURCES ∴ $\underline{\underline{2}}$ CIRCUITS TO ANALYZE

(EXAMPLE) FIND I_{R_1} + P_{R_1}

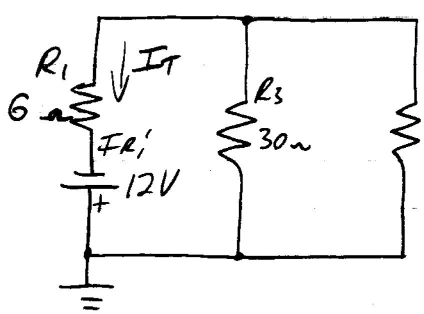


3 SOURCES ∴
3 CIRCUITS TO
ANALYZE

DUE TO E_1:



⇓ REDRAW, COMBINE $R_4 // R_2 + R_5$



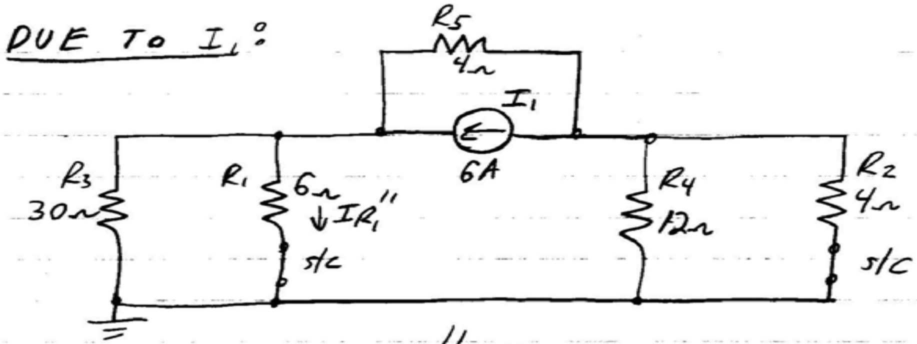
$$R_5 + (R_4 // R_2) = 4 + 12 // 4$$
$$= \underline{7\Omega}$$

$$R_T = R_1 + R_3 // 7$$
$$= 6 + 30 // 7$$
$$= \underline{11.67\Omega}$$

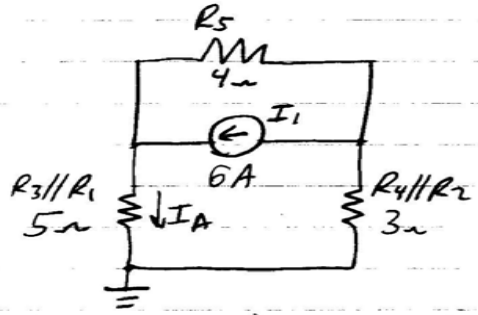
$$\therefore I_T = \frac{12V}{11.67\Omega} = \underline{1.028A}$$

$$\therefore \underline{\underline{I_{R_1}' = \downarrow 1.028A}}$$

DUE TO I_1:



⇓ REDRAW



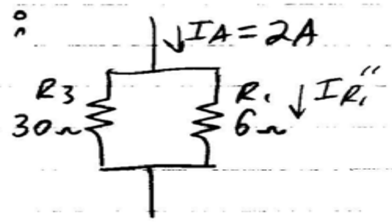
$I_A = I_T(R_T/R_X)$

$I_A = 6A[(8\Omega//4\Omega)/8\Omega]$

$I_A = 2A$

$$I_A = I_1\left(\frac{R_5}{R_5+5+3}\right) = 6A\left(\frac{4}{4+5+3}\right) = \underline{2A}$$

∴ WE HAVE:



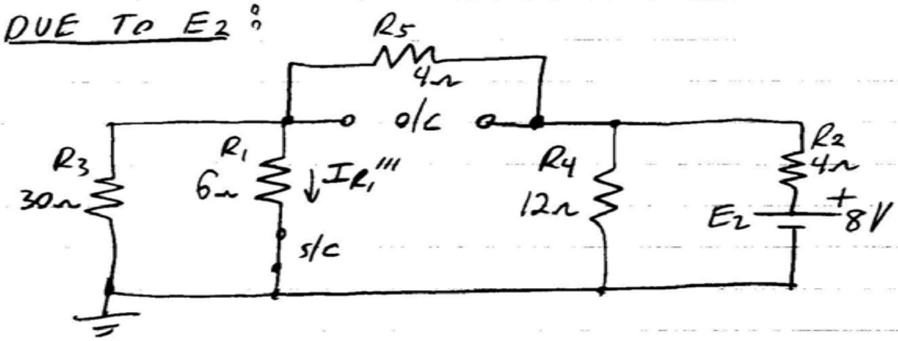
$$I_{R_1} = I_A\left(\frac{R_3}{R_3+R_1}\right) = 2A\left(\frac{30}{36}\right)$$

$I_{R1}'' = I_T(R_T/R_X)$

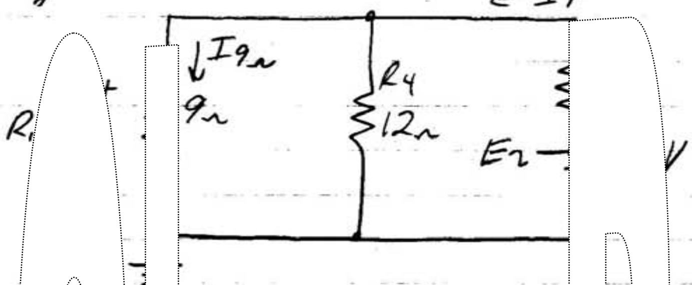
$I_{R1}'' = I_A[(6\Omega//30\Omega)/6\Omega]$

$I_{R1}'' = 1.67A$

DUE TO E_2:



REDRAW :

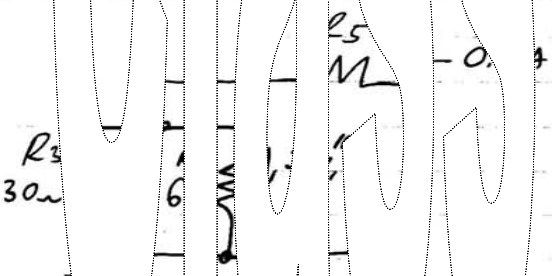


$I_T = \frac{E_2}{R \ldots}$ … $= 9\Omega \,//\, 12 \ldots$
$\ldots .143$

$\therefore I_T = \frac{8}{\ldots 3} = 8\ldots mA$

$I_{9\Omega} = I_T \left(\frac{12}{\ldots} \right) = 875m \ldots \frac{12}{21} \ldots$

$\therefore$ … E … AVE …



$I_{R_1}''' = 0.5 \left(\frac{R}{R_1 + \ldots} \right)$

$= 0.5 \left(\frac{3}{36} \right) = 4\ldots.6\, m$

$I_{R_1} = I_{R_1}' + I_{R_1}'' + I_{R_1}'''$

$= \ldots 28A + 1.6 \ldots A + 4 \ldots 6\, m \ldots$

$3 \ldots A$

$V_{R_2} = (I_{R_1})(R_1)$

$= (3 \ldots)(\ldots \Omega) = 1\ldots 3V$

In Class Problem