*E*lectrical

*E*ngineering

*T*echnology

Frequency Response For Series ac Circuits

Spring 2019 (2185)

Frequency Response For Series ac Circuits

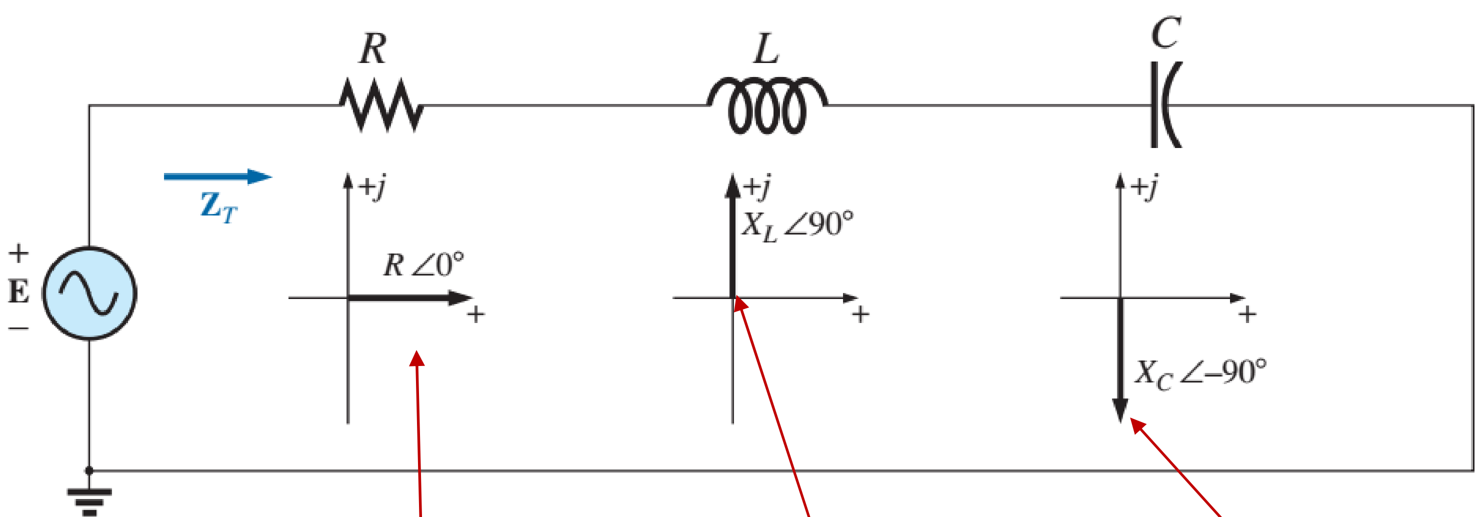
- Frequency Response for Basic Elements
  - Introduction
  - RLC Series Circuit (qualitatively)
  - RLC Series Circuit Impedance Example/**ICP** – Find the equation and sketch
- Frequency Response for a Series RC Circuit
  - Total Impedance and example/**ICP**
  - Voltage V_C and example/**ICP**

How does the response of a series circuit change as the frequency changes?

- Parameters of Interest
  - Impedance, **Z**
  - Voltage, **V**
  - Current, **I**

Frequency Response of Basic Elements

- The impedance and impedance diagram of **ideal circuit elements** are shown in the RLC circuit below



$Z_R = (R\angle 0°)$ $Z_L = (X_L\angle 90°)$ $Z_C = (X_C\angle -90°)$

Frequency Response of Basic Elements

- The response of the **ideal** elements to changes in frequency are shown below

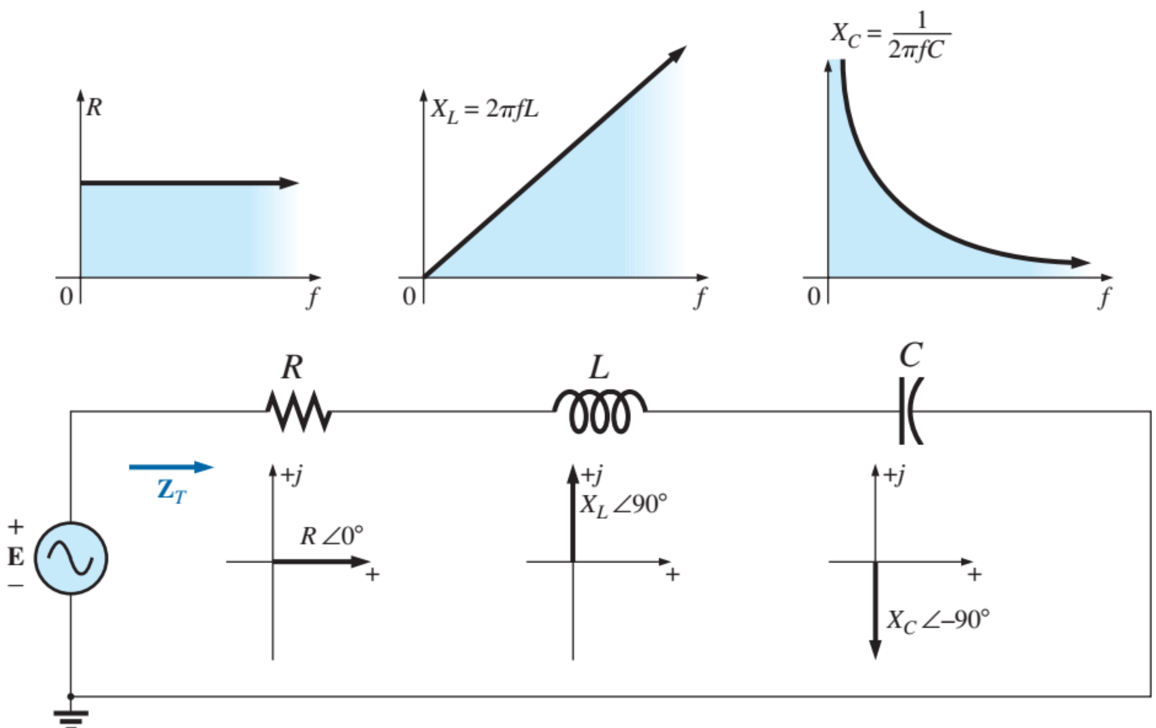


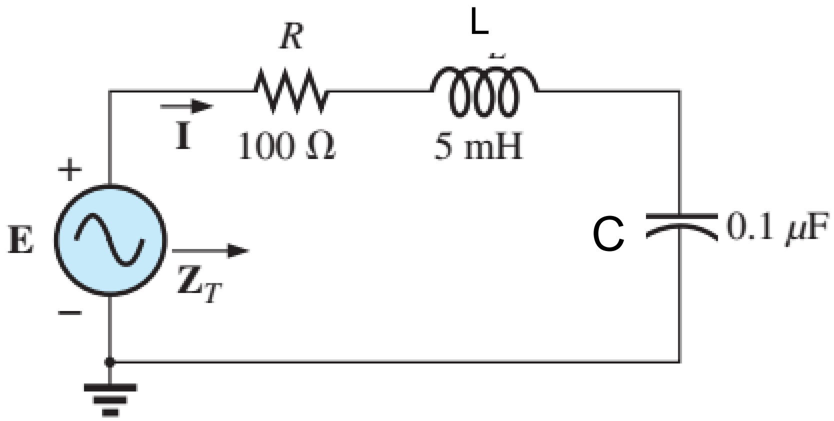
FIG. 15.49
Reviewing the frequency response of the basic elements.

Note:

	Low Freq. $(f = 0)$	**High Freq.** $(f \to \infty)$
Z_R	No changes	No changes
Z_L	Short-circuit	Open-circuit
Z_C	Open-circuit	Short-circuit

Frequency Response of Basic Elements

- What do $|\mathbf{Z}_T|$ and $|\mathbf{I}_T|$ look like for a series RLC Circuit?



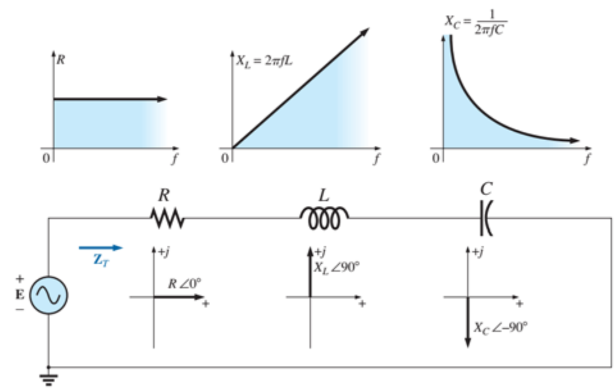
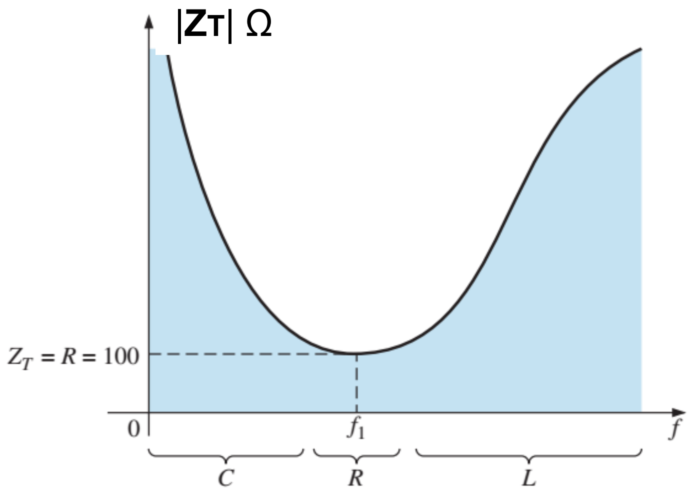
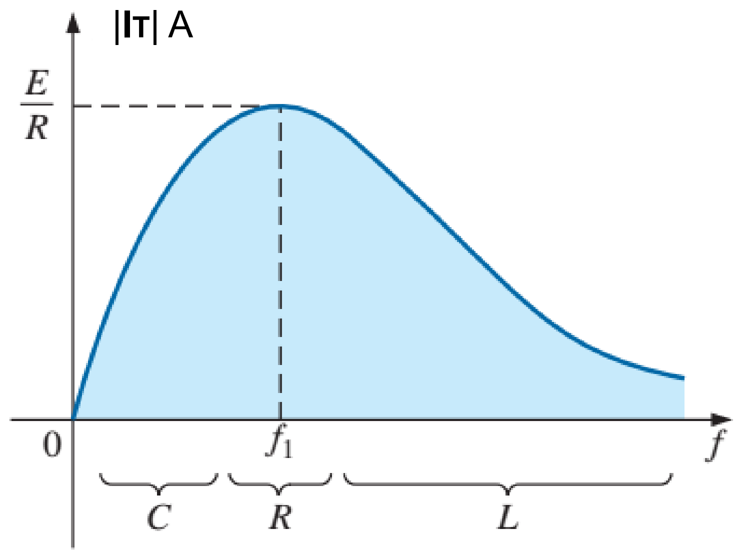


FIG. 15.49

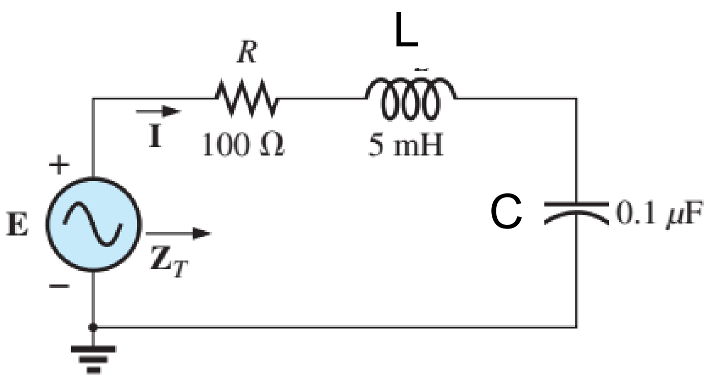
Reviewing the frequency response of the basic elements.

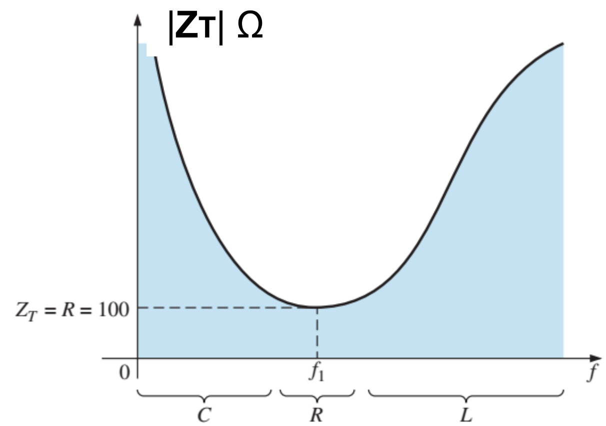




Frequency Response of Basic Elements

- Find $|\mathbf{Z_T}|$ for this circuit as a function of frequency (as the frequency changes)





ICP:
1) Calculate $|\mathbf{Z_T}|$ at 100Hz, 10kHz and 100kHz
2) At what frequency does $|\mathbf{Z_T}|$ hit its minimum?
3) Sketch $|\mathbf{Z_T}|$ as a function of frequency (calculator…)

Frequency Response of a Series R-C Circuit

- **Example**: Determine the frequency response of a series R-C circuit
  - Frequency range: 0 to 20 kHz
  - Frequency response of the individual elements are key
    - But - We are more interested in the frequency response of the entire circuit ($\boldsymbol{Z_T}$ and $\mathbf{V}$c here)

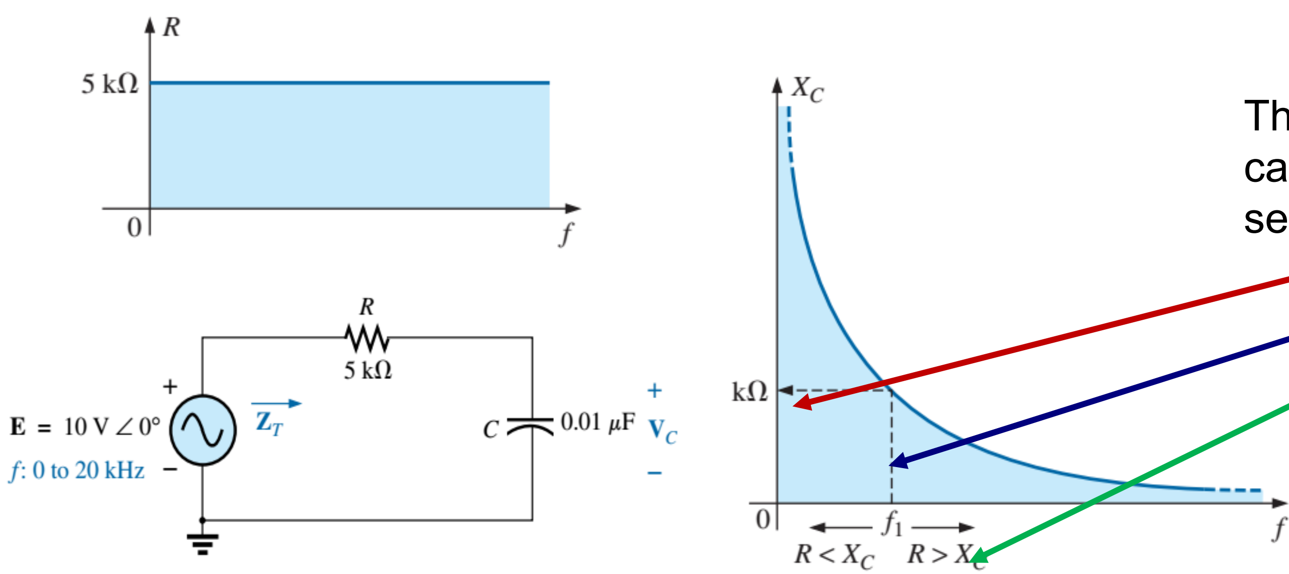


FIG. 15.51

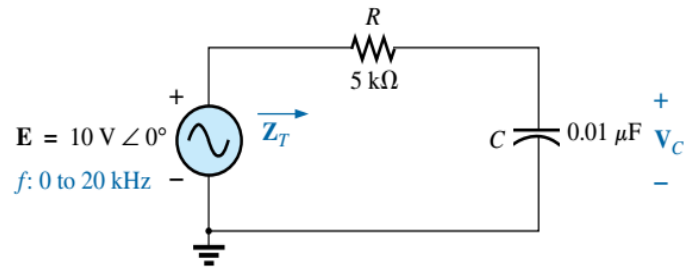
The frequency response can be subdivided into 2 sections

- Section 1: $X_C > R$
- Divider : $X_C = R$
- Section 2: $X_C < R$

Frequency Response of a Series R-C Circuit

- Frequency for $X_R = X_C$

$$X_C = \frac{1}{2\pi f_1 C} = R \qquad \therefore f_1 = \frac{1}{2\pi RC}$$



- Total impedance, $\boldsymbol{Z_T}$
  - Rectangular form

$$\boldsymbol{Z_T} = \boldsymbol{Z_R} + \boldsymbol{Z_C}$$
$$\boldsymbol{Z_T} = (\boldsymbol{R} + \boldsymbol{j0}) + (\boldsymbol{0} - \boldsymbol{jX_C})$$
$$\boldsymbol{Z_T} = \boldsymbol{R} - \boldsymbol{jX_C}$$

  - Polar form

$$\boldsymbol{Z_T} = Z_T \angle \theta$$
$$\boldsymbol{Z_T} = \sqrt{\left(R^2 + {X_C}^2\right)} \angle - \tan^{-1}\left(\frac{X_C}{R}\right)$$

Frequency Response of a Series R-C Circuit

Magnitude: $\sqrt{(R^2 + {X_C}^2)}$ $\qquad Z_T \qquad$ Angle: $\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$

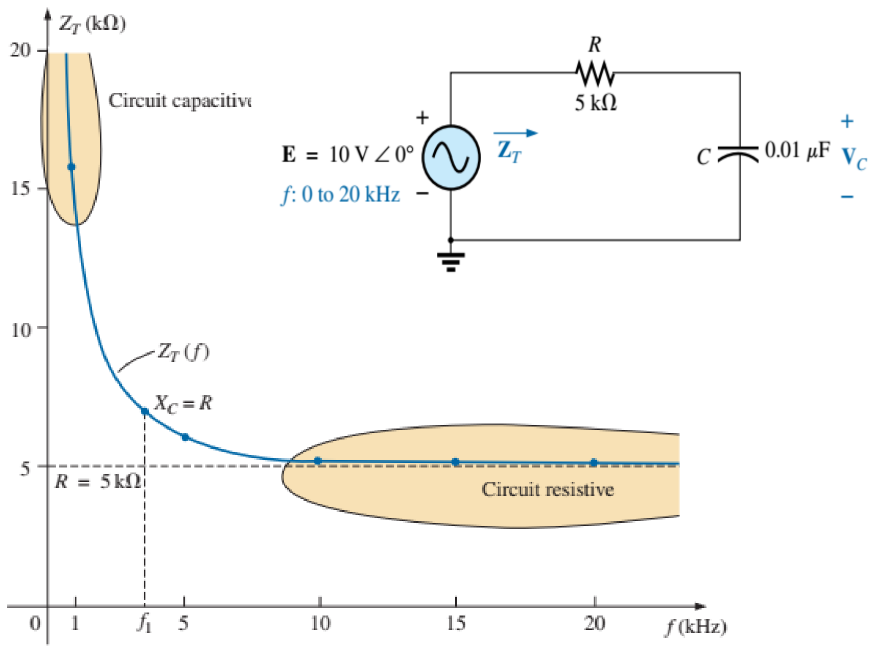


FIG. 15.53
The magnitude of the input impedance versus frequency for the circuit in Fig. 15.50.

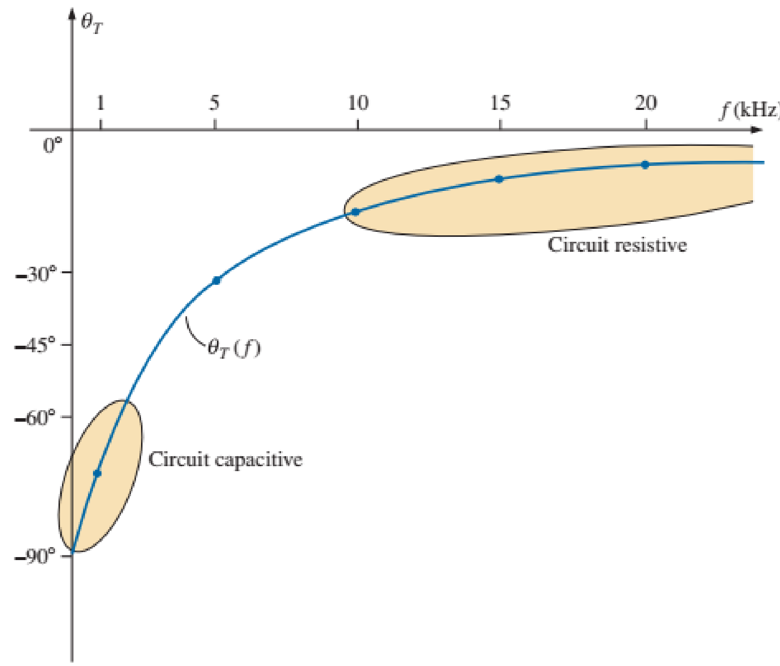


FIG. 15.54
The phase angle of the input impedance versus frequency for the circuit in Fig. 15.50.

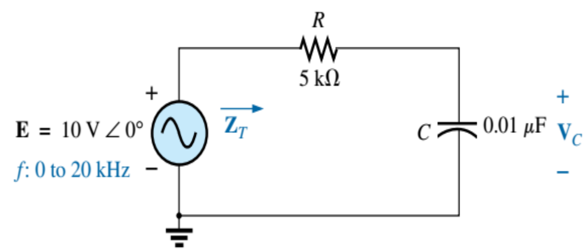
ICP - Calculate |**ZT**| and <**ZT** at: 100Hz, 3.18kHz, 20kHz

frequency	ZT
100 Hz	159.2KΩ < -88.2º
3.18 kHz	7.07KΩ < -45º
20 kHz	5.06KΩ < -9.04º

Frequency Response of a Series R-C Circuit

- Find the voltage $\boldsymbol{V_C}$ using voltage divider

$$\boldsymbol{V_C} = \frac{\boldsymbol{Z_C E}}{\boldsymbol{Z_T}} = \frac{\boldsymbol{Z_C E}}{\boldsymbol{Z_R}+\boldsymbol{Z_C}} = \frac{(X_C\angle{-90^\circ})(E\angle 0^\circ)}{(R+j0)+(0-jX_C)} = \frac{(X_C E\angle{-90^\circ})}{(R-jX_C)}$$



$$= \frac{(X_C E\angle{-90^\circ})}{\sqrt{(R^2+X_C{}^2)}\angle\ -\tan^{-1}\left(\frac{X_C}{R}\right)}$$

$$\boldsymbol{V_C} = |\boldsymbol{V_C}|\angle\theta_C = \frac{X_C E}{\sqrt{R^2+X_C{}^2}}\angle{-90^\circ} - \left(-\tan^{-1}\left({}^{X_C}/_{R}\right)\right)$$

- Voltage $\boldsymbol{V_C}$
  - Magnitude: $$|V_C| = \frac{X_C E}{\sqrt{R^2+X_C{}^2}}$$
  - Angle: $$\angle\theta_C = -90^\circ + \tan^{-1}\left({}^{X_C}/_{R}\right) = -\tan^{-1}\left({}^{R}/_{X_C}\right)$$

Frequency Response of a Series R-C Circuit

$$\boldsymbol{V_C}$$

Magnitude: $V_C = \dfrac{X_C E}{\sqrt{R^2 + {X_C}^2}}$

Angle: $\angle\theta_C$

$= -90° + \tan^{-1}\left({}^{X_C}/_{R}\right)$

$= -\tan^{-1}\left({}^{R}/_{X_C}\right)$

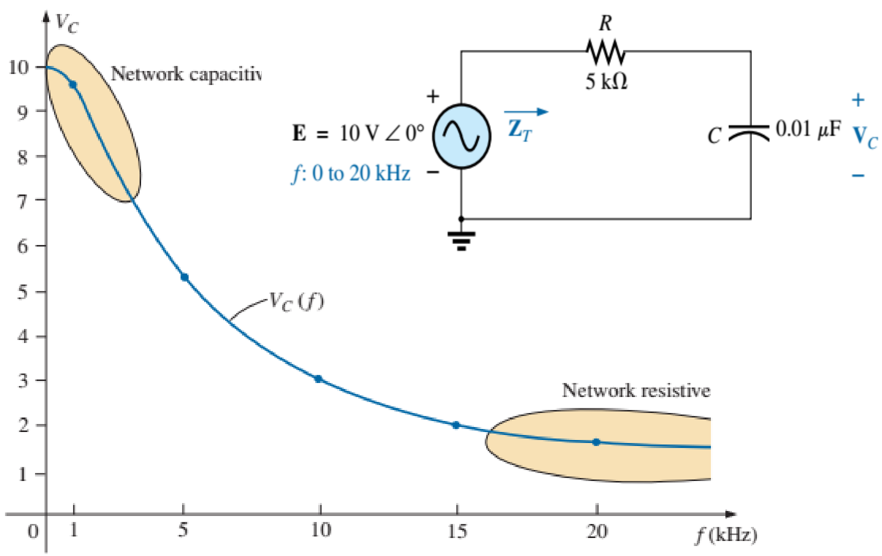


FIG. 15.58

The magnitude of the voltage V_C versus frequency for the circuit in Fig. 15.50.

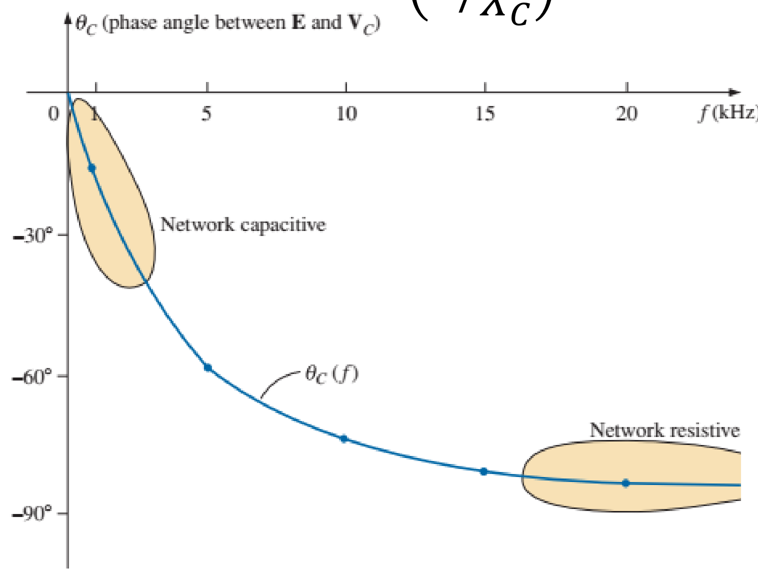


FIG. 15.59

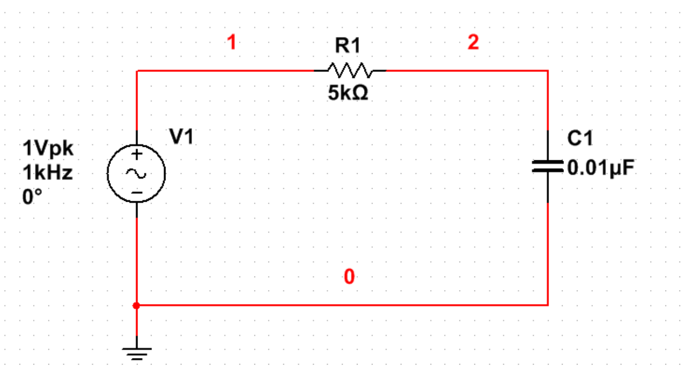
The phase angle between E and V_C versus frequency for the circuit in Fig. 15.50.

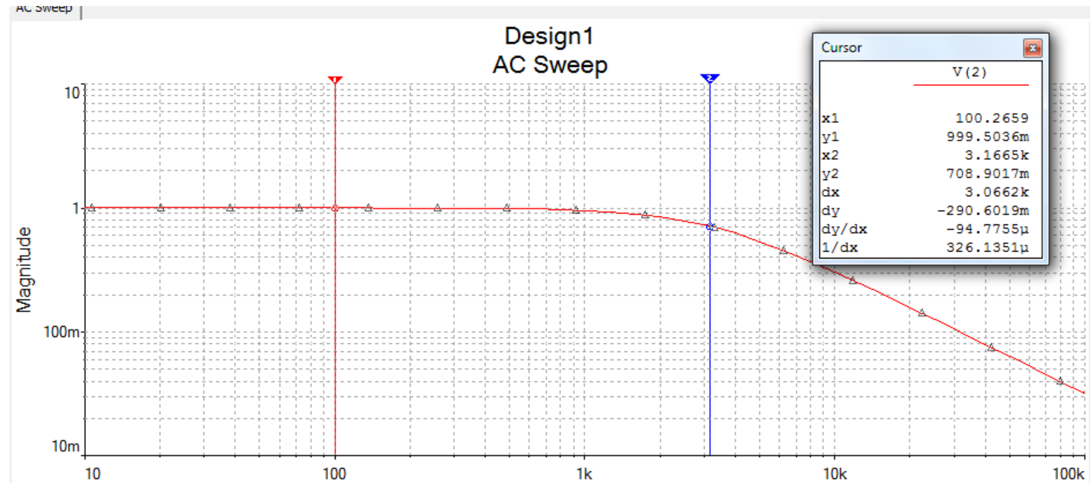
Calculate |**Vc**| and <**Vc** at:
100Hz, 3.18kHz, 20kHz

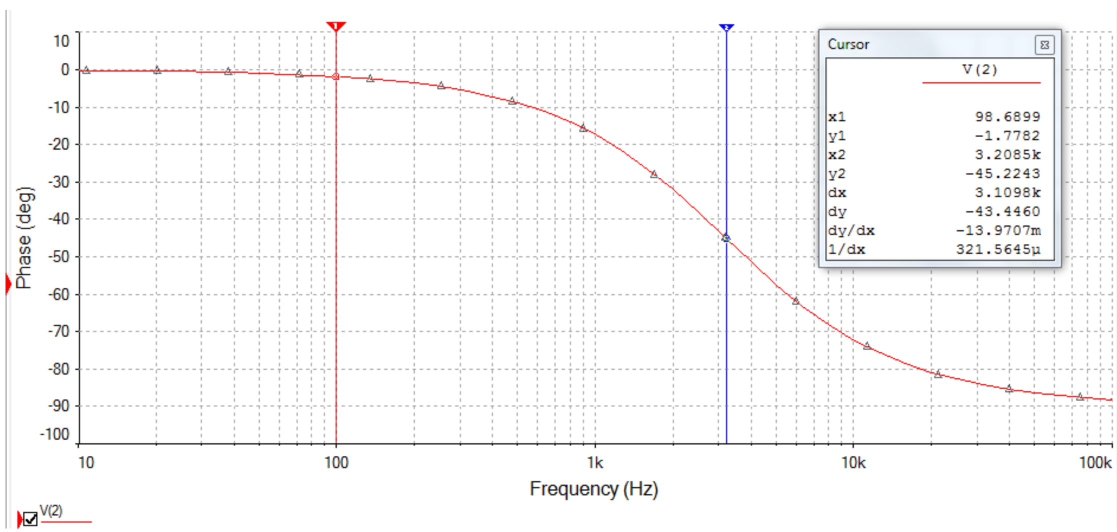
frequency	Vc
100 Hz	**10.0V < -1.8º**
3.18 kHz	**7.07V < -45º**
20 kHz	**1.57VΩ < -81.0º**

Frequency Response of a Series R-C Circuit – Multisim

- There is a great way to check your understanding and perform this kind of work more efficiently…



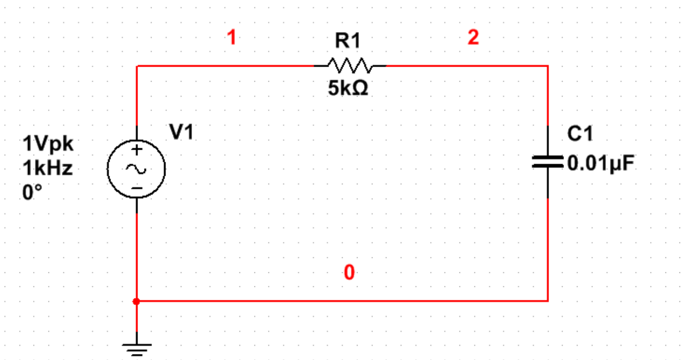


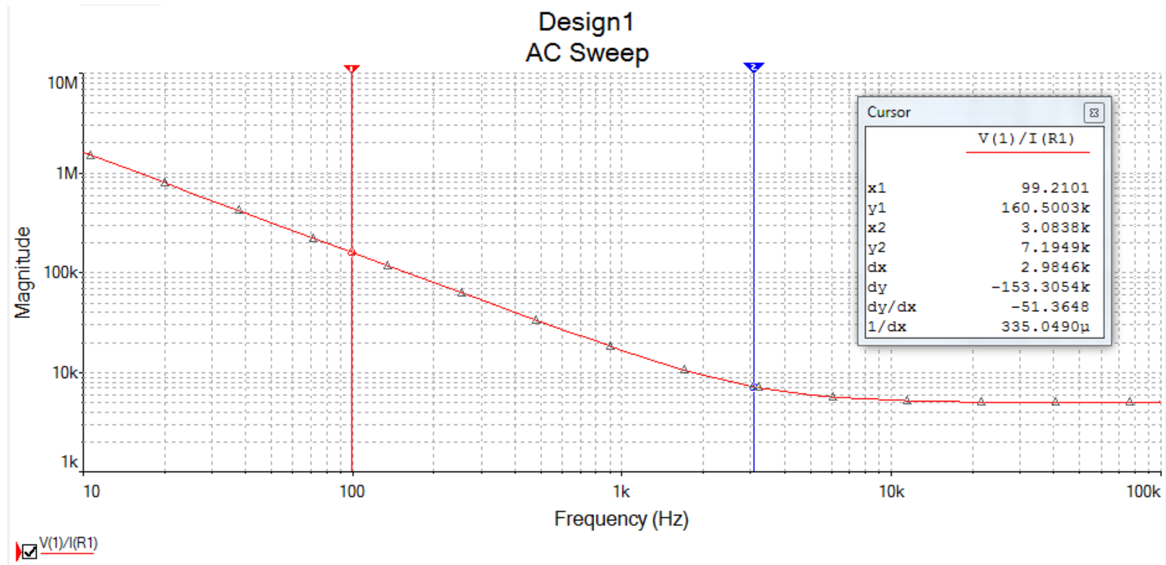


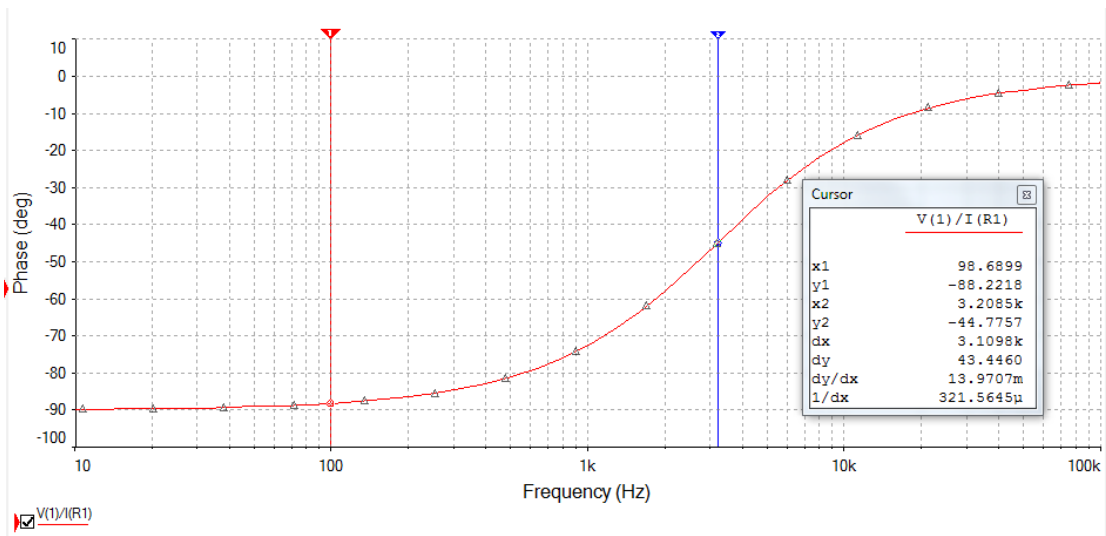
Frequency	V_C
100.2 Hz	**10.0V < -1.78º**
3.17 kHz	**7.09V < -45.2º**

Frequency Response of a Series R-C Circuit – Multisim

- What about **Z**T?







frequency	ZT
~99 Hz	**160.5KΩ < -88.2º**
~3.1 kHz	**7.19KΩ < -44.8º**

Frequency Response of a Series R-C Circuit – Multisim

- Get familiar with Multisim for this kind of work
  - See the posted file to get started
  - Use Multisim to check your HW answers and text examples