Parallel Resonance

- Introduction and General Circuit
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- Selectivity and Branch Currents
- Example

Parallel Resonance- Introduction and Circuit

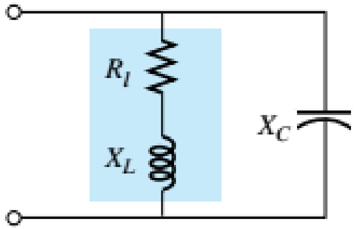
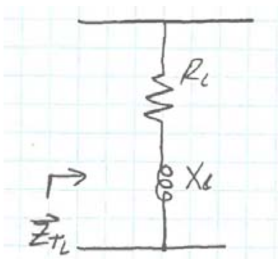


FIG. 21.23
Practical parallel L-C network.

Not just a simple R-L-C parallel network since practical inductors have to take into account series resistance

Taking the inverse of each component yields:

$$\vec{Z}_{TL} = \underbrace{\frac{R_L^2 + X_L^2}{R_L}}_{R_p} \parallel \underbrace{\frac{R_L^2 + X_L^2}{X_L}}_{X_{Lp}} \angle 90^\circ$$



$$\vec{Z}_{TL} = R_L + jX_L$$

$$\vec{Y}_{TL} = \frac{1}{\vec{Z}_{TL}} = \frac{1}{R_L + jX_L}$$

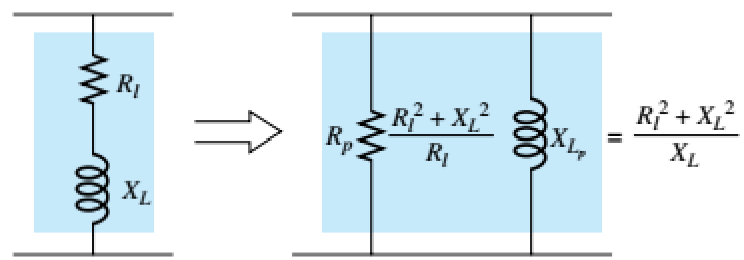
$$\vec{Y}_{TL} = \frac{1}{(R_L + jX_L)} \cdot \frac{(R_L - jX_L)}{(R_L - jX_L)}$$

$$= \frac{R_L - jX_L}{R_L^2 + X_L^2}$$

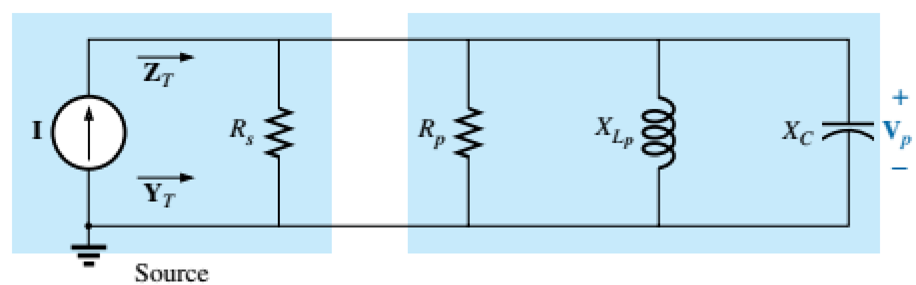
In rectangular form:

$$\vec{Y}_{TL} = \underset{G \angle 0^\circ (S)}{\frac{R_L}{R_L^2 + X_L^2}} - j \underbrace{\frac{X_L}{R_L^2 + X_L^2}}_{B_L \angle -90^\circ (S)}$$

So we have:

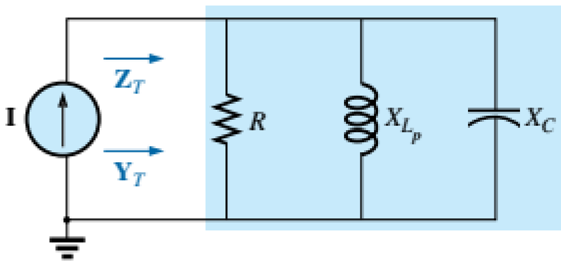


We will consider:



Parallel Resonance - Resonant Frequency

Combining Rs and Rp yields:



Analyzing

$$\mathbf{Y}_T = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{R} + \frac{1}{jX_{L_p}} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} - j\left(\frac{1}{X_{L_p}}\right) + j\left(\frac{1}{X_C}\right)$$

$$\boxed{\mathbf{Y}_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{L_p}}\right)}$$

For Fp =1, the imaginary component of **Y**T must be equal to zero, therefore (at resonance):

$$\frac{1}{X_C} = \frac{1}{X_{L_p}}$$

$$\boxed{X_{L_p} = X_C}$$

Substituting for XLP, yields:

$$\boxed{\frac{R_l^2 + X_L^2}{X_L} = X_C}$$

$$R_l^2 + X_L^2 = X_C X_L = \left(\frac{1}{\omega C}\right)\omega L = \frac{L}{C}$$

Solving for fp, reduces to (text section 21.10):

$$\boxed{f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}}$$

$$\boxed{f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}}}$$

For low RL values, fp ~ fs

Looking closer at the (RL²C/L) term:

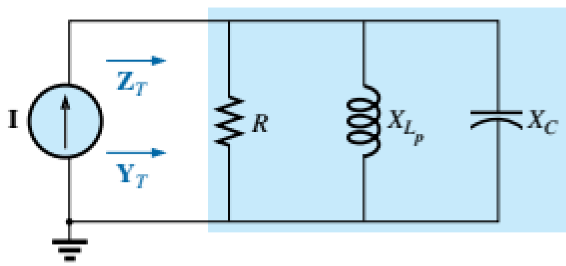
$$\frac{R_l^2 C}{L} = \frac{1}{\frac{L}{R_l^2 C}} = \frac{1}{\frac{(\omega)}{(\omega)}\frac{L}{R_l^2 C}} = \frac{1}{\frac{\omega L}{R_l^2 \omega C}} = \frac{1}{\frac{X_L X_C}{R_l^2}}$$

At resonance, XL=XC, therefore:

$$\frac{1}{\frac{X_L X_C}{R_l^2}} = \frac{1}{\frac{X_L^2}{R_l^2}} = \frac{1}{Q_l^2}$$

$$\boxed{f_p = f_s\sqrt{1 - \frac{1}{Q_l^2}}}\quad Q_l \geq 10$$

Parallel Resonance - Quality Factor



$$\boxed{f_p = f_s\sqrt{1 - \frac{1}{Q_l^2}}}_{Q_l \geq 10}$$

Or:

$$\boxed{f_p \cong f_s = \frac{1}{2\pi\sqrt{LC}}}_{Q_l \geq 10}$$

Recall:

$$\boxed{Q_s = \frac{\text{reactive power}}{\text{average power}}}$$

So here:

$$Q_p = \frac{V_p^2/X_{L_p}}{V_p^2/R}$$

$$\boxed{Q_p = \frac{R}{X_{L_p}} = \frac{R_s \| R_p}{X_{L_p}}}$$

But XLP=XC at resonance, so we have:

$$\boxed{Q_p = \frac{R_s \| R_p}{X_C}}$$

$$\underbrace{\frac{R_L^2 + X_L^2}{R_L}}_{R_p}$$

Expanding:

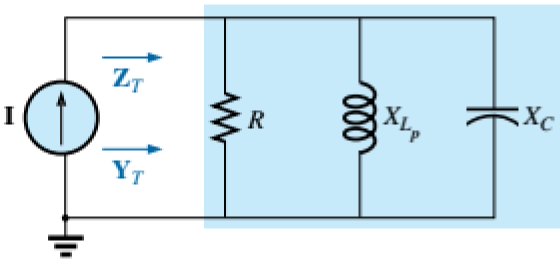
$$R_p = \frac{R_l^2 + X_L^2}{R_l} = R_l + \frac{X_L^2}{R_l}\left(\frac{R_l}{R_l}\right) = R_l + \frac{X_L^2}{R_l^2}R_l$$

$$= R_l + Q_l^2 R_l = (1 + Q_l^2)R_l$$

Or, for Q>=10:

$$\boxed{R_p \cong Q_l^2 R_l}_{Q_l \geq 10}$$

Parallel Resonance - Q>=10



$$\boxed{R_p \cong Q_l^2 R_l}_{Q_l \geq 10}$$

Also, recall:

$$X_{L_p} = \frac{R_l^2 + X_L^2}{X_L} :$$

$$= \frac{R_l^2(X_L)}{X_L(X_L)} + X_L$$

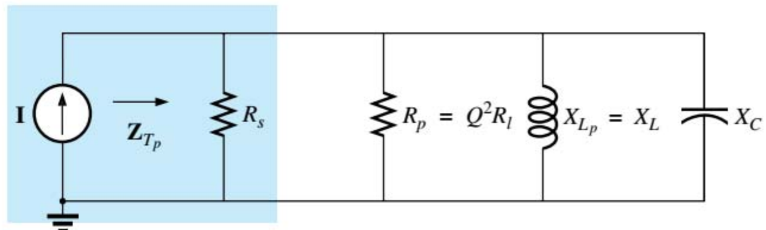
$$= \frac{X_L}{Q_l^2} + X_L$$

$$= X_L\left(\frac{1}{Q_l^2} + 1\right)$$

Therefore:

$$\boxed{X_{L_p} \cong X_L}_{Q_l \geq 10}$$

So, for Q>=10, we have:



$$R_p \cong Q_l^2 R_l = \left(\frac{X_L}{R_l}\right)^2 R_l = \frac{X_L^2}{R_l}$$

But, <u>at resonance</u> X_L=X_C, so:

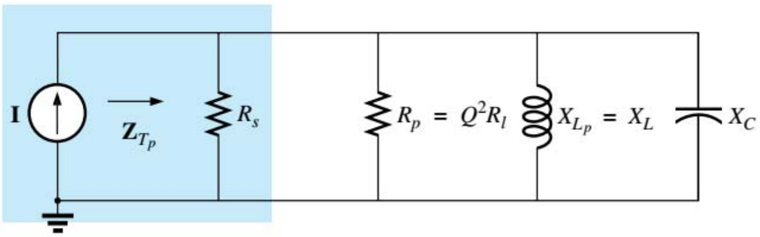
$$\frac{X_L^2}{R_l} = \frac{X_L X_C}{R_l} = \frac{2\pi f L}{R_l(2\pi f C)}$$

$$\boxed{R_p \cong \frac{L}{R_l C}}_{Q_l \geq 10}$$

$$\boxed{Z_{T_p} \cong R_s \| R_p = R_s \| Q_l^2 R_l}_{Q_l \geq 10}$$

$$\boxed{Q_p = \frac{R}{X_{L_p}} \cong \frac{R_s \| Q_l^2 R_l}{X_L}}$$

Parallel Resonance – Selectivity and Branch Currents



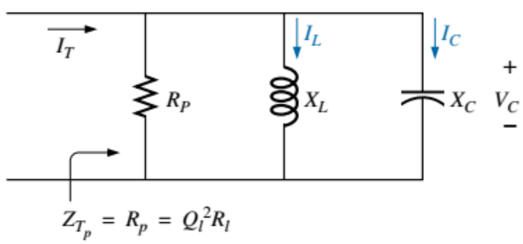
As before, BW is defined as:

$$\boxed{BW = f_2 - f_1 = \frac{f_p}{Q_p}}$$

Or can be calculated as:

$$\boxed{BW = f_2 - f_1 \cong \frac{1}{2\pi}\left[\frac{R_l}{L} + \frac{1}{R_s C}\right]}$$

Finally, consider:



<u>At resonance:</u>

$$V_C = V_L = V_R = I_T Z_{T_p} = I_T Q_l^2 R_l$$

Finding Ic=|**Ic**|:

$$I_C = \frac{V_C}{X_C} = \frac{I_T Q_l^2 R_l}{X_C}$$

$$I_C = \frac{I_T Q_l^2 R_l}{X_L} = I_T \frac{Q_l^2}{\frac{X_L}{R_l}} = I_T \frac{Q_l^2}{Q_l}$$

Hence:

$$\boxed{I_C \cong Q_l I_T}_{\,Q_l \geq 10}$$

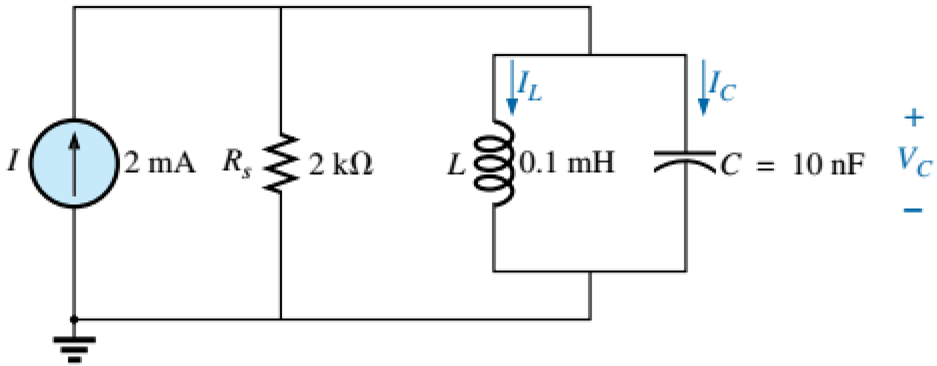
Similarly:

$$\boxed{I_L \cong Q_l I_T}_{\,Q_l \geq 10}$$

Parallel Resonance – Example

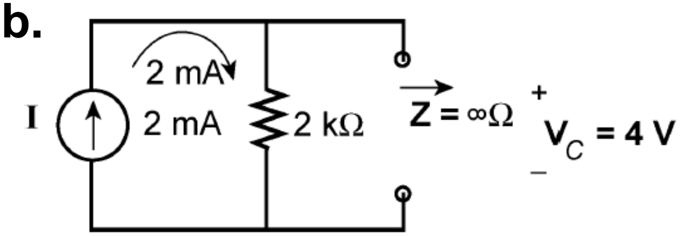
13. For the "ideal" parallel resonant circuit in Fig. 21.54:

a. Determine the resonant frequency (f_p).
b. Find the voltage V_C at resonance.
c. Determine the currents I_L and I_C at resonance.
d. Find Q_p.



a. $$f_p = \frac{1}{2\pi\sqrt{LC}} = \frac{2}{2\pi\sqrt{(0.1\,\text{mH})(10\,\text{nF})}} = \mathbf{159.16\ kHz}$$

b.



c.

$$I_L = \frac{V_L}{X_L} = \frac{4\,\text{V}}{2\pi f_p L} = \frac{4\,\text{V}}{100\,\Omega} = \mathbf{40\ mA}$$

$$I_C = \frac{V_L}{X_C} = \frac{4\,\text{V}}{1/2\pi f_p C} = \frac{4\,\text{V}}{100\,\Omega} = \mathbf{40\ mA}$$

Note that $I_L = I_C > I$ at resonance

d.

$$Q_p = \frac{R_s}{X_{L_p}} = \frac{2\,\text{k}\Omega}{2\pi f_p L} = \frac{2\,\text{k}\Omega}{100\,\Omega} = \mathbf{20}$$