# Basic Elements (Reactance and Freq Response)

- The Derivative
  - Intro and understanding (qualitative)
  - Effects of frequency
  - Calculus (the details)
- Response of R,L,C to Sinusoids (as forcing functions)
  - Resistor
  - Inductor
  - **Inductor ICPs**
  - Capacitor
  - **Capacitor ICPs**
- R,L,C Over Frequency (Intro)
  - Ideal vs “practical” models

## Understanding the Derivative

**Here: x(t) = sin(wt)**

**dx/dt -> The rate of change of x wrt t (wrt: with respect to)**

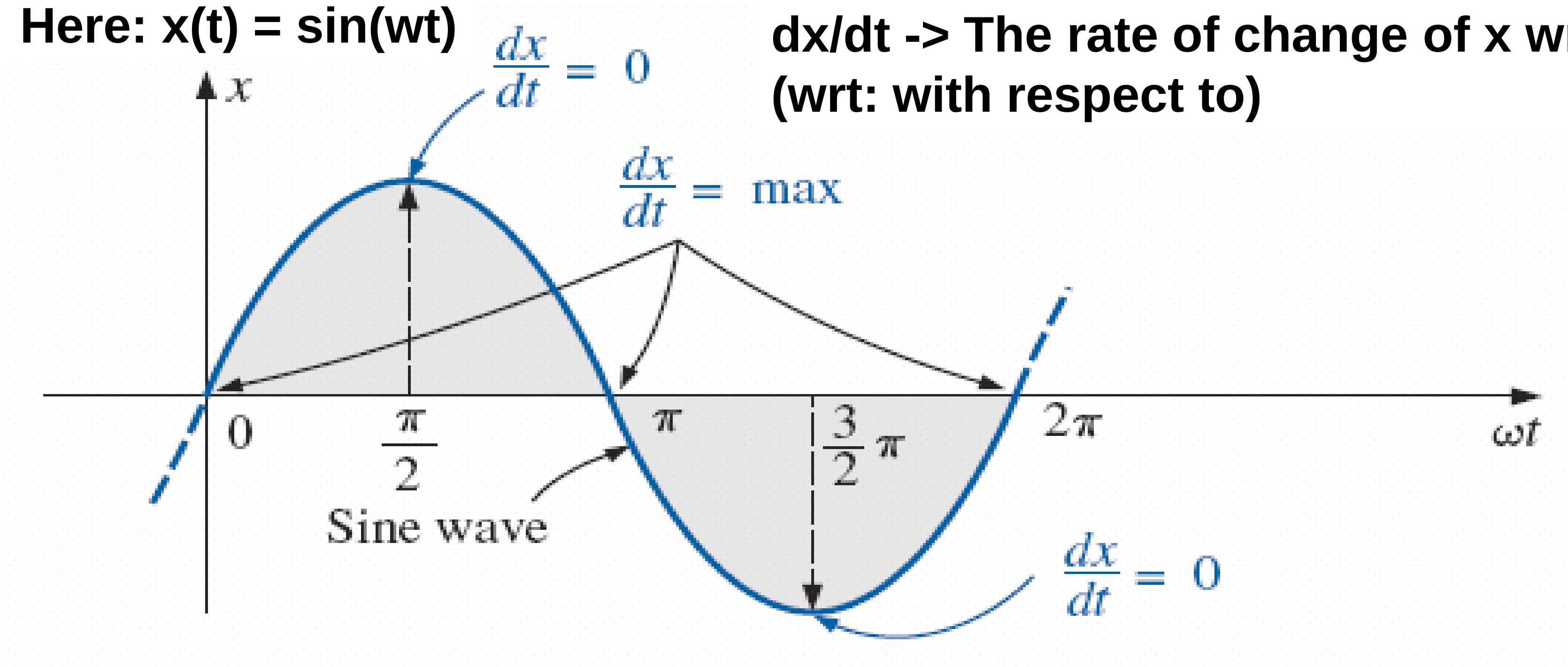
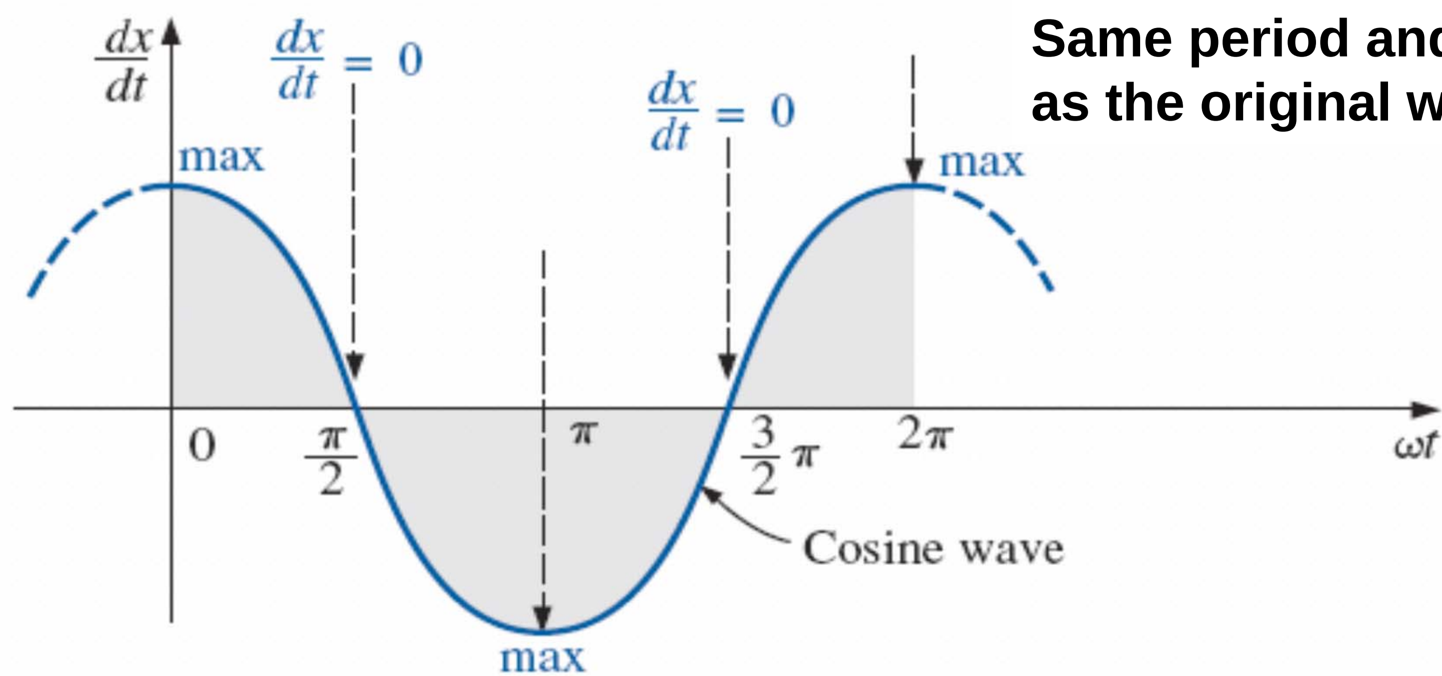



**FIG. 14.1** *Defining those points in a sinusoidal waveform that have maximum and minimum derivatives.*

**Note max dx/dt at wt=0,pi and 2pi**
**And dx/dt = 0 at wt = pi/2 and 3*pi/2**

## Understanding the Derivative




**Same period and frequency as the original waveform**

**FIG. 14.2** *Derivative of the sine wave of Fig. 14.1.*

**Notice that x(t) = cos(wt) and that**
**d/dt (sinx) = cos(x)**

## Effect of Frequency on the Peak Value

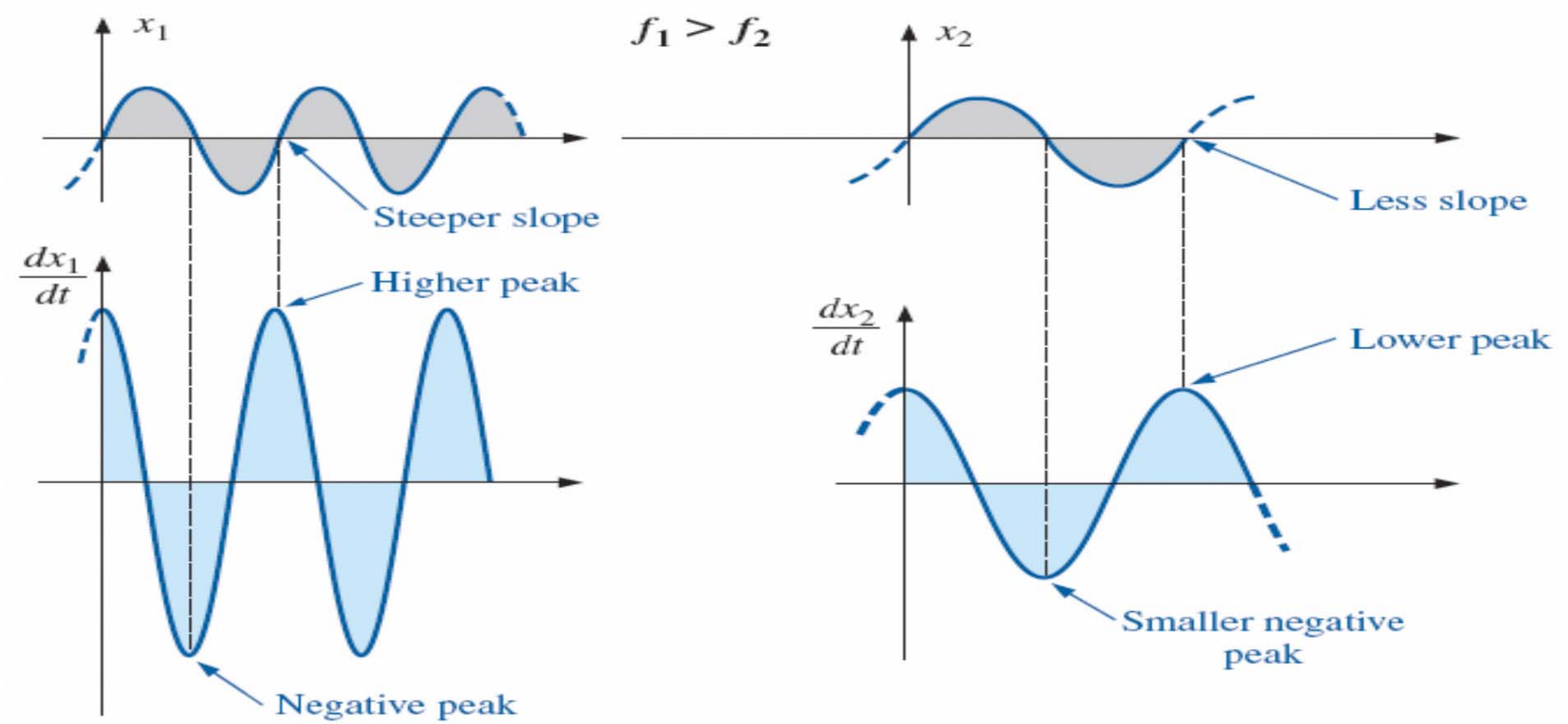



**FIG. 14.3** *Effect of frequency on the peak value of the derivative.*

**Steeper slope = higher peak in the derivative or Higher frequency = higher peak in the derivative**

## Effect of Frequency on the Peak Value

CALCULUS

$\rightarrow$ $D_x \sin(u) = \cos(u)\, D_x u$

CONSIDER : $v(t) = E_m \sin(\omega t + \theta)$

$$\frac{dv(t)}{dt} = E_m \cos(\omega t + \theta) \cdot \frac{d}{dt}(\omega t + \theta)$$

$$\boxed{v'(t) = E_m \omega \cos(\omega t + \theta)}$$

$\rightarrow$ $D_x \cos(u) = -\sin(u)\, D_x u$

CONSIDER : $v(t) = E_m \cos(\omega t + \theta)$

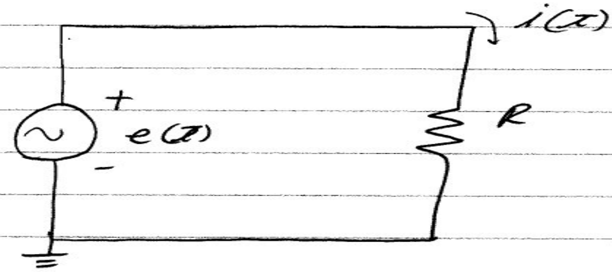
$$\frac{dv(t)}{dt} = E_m(-\sin(\omega t + \theta)) \cdot \frac{d}{dt}(\omega t + \theta)$$

$$\boxed{v'(t) = -E_m \omega \sin(\omega t + \theta)}$$

## RESPONSE OF R, L, C ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

### RESISTOR

RECALL: $\frac{E}{I \; R}$



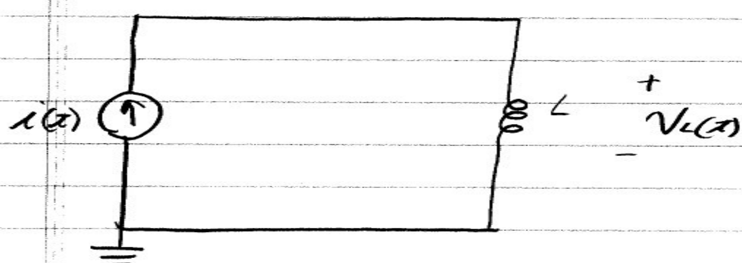

$$e(t) = V_m \sin(\omega t) \quad V$$

VOLTAGE & CURRENT IN-PHASE

FIND $i(t)$:

$$i(t) = \frac{e(t)}{R} = \frac{V_m}{R} \sin(\omega t) \quad A$$

### INDUCTOR




$$i(t) = I_m \sin(\omega t)$$

# Response of the Inductor to a Sinusoidal Forcing Function

RECALL: $v_L(t) = L\frac{di}{dt}$

$\therefore \quad v_L(t) = L\frac{d}{dt}\left[I_m \sin(\omega t)\right]$

$v_L(t) = \underbrace{I_m L \omega}_{V_m} \cos(\omega t)$

RECALL: $\cos(\omega t) = \sin(\omega t + 90°)$

$\therefore \quad v_L(t) = \omega L I_m \sin(\omega t + 90°)$

COMPARE: $i_L(t) = I_m \sin(\omega t)$

$v_L(t) = \underbrace{\omega L I_m}_{V_m} \sin(\omega t + 90°)$

$V_m = \omega L I_m$, PEAK VALUE OF $v_L(t)$ IS DIRECTLY RELATED TO $\omega$ & $L$

$\sin(\omega t + 90°)$, THE PHASE ANGLE OF $v_L(t)$ LEADS THAT OF $i_L(t)$ BY $90°$

RECALL: $\frac{E}{I/R} \rightarrow \frac{V_m}{I_m}$ = REACTANCE, $X_L$ FOR THE INDUCTOR — INDUCTIVE REACTANCE

$$\boxed{X_L = \frac{V_m}{I_m} = \omega L \text{, OHMS}}$$

## Response of the Inductor to a Sinusoidal Forcing Function

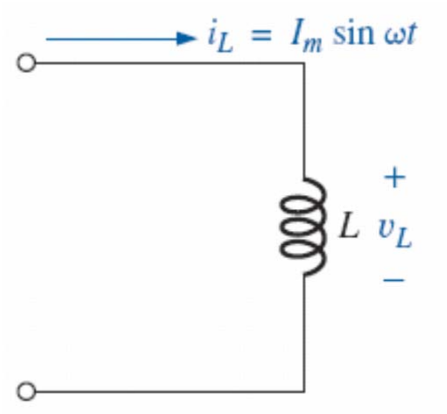
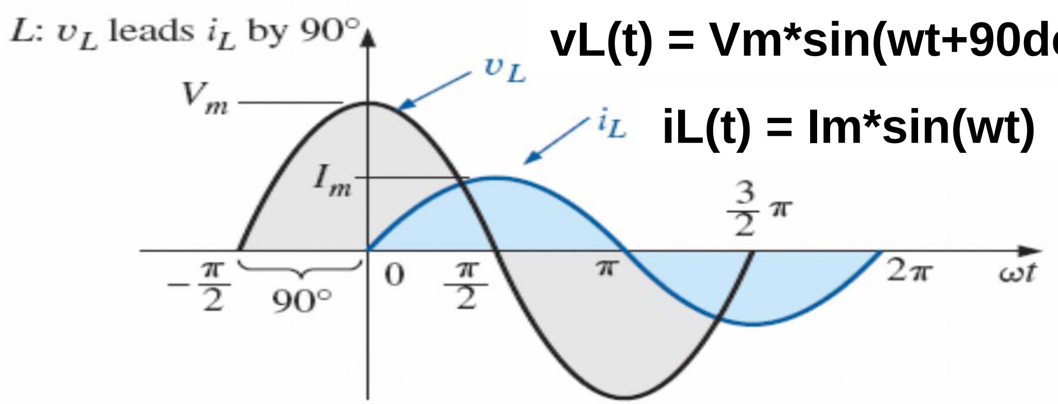



**FIG. 14.6** *Investigating the sinusoidal response of an inductive element.*




**vL(t) = Vm*sin(wt+90deg)**

**iL(t) = Im*sin(wt)**

**FIG. 14.7** *For a pure inductor, the voltage across the coil leads the current through the coil by 90°.*

# ICPs – Inductive Reactance

**P1 – Find the reactance of a 2H inductor at**

a) F=60Hz
b) F=2kHz

**P2 – Determine the inductance of a coil with a reactance of**

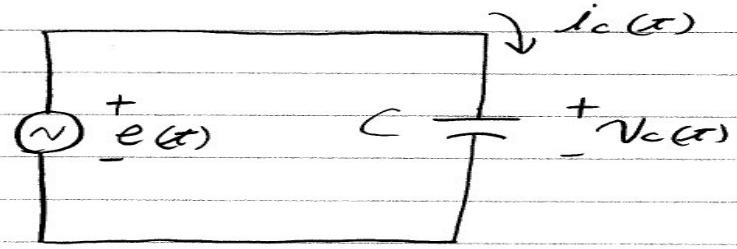
a) 20 Ohms @ 2Hz
b) 5280 Ohms @ 1000Hz

**P3 – Determine the frequency (in Hz) for a 10H inductor with**

a) XL = 50 Ohms
b) XL = 3770 Ohms

## Response of the Capacitor to a Sinusoidal Forcing Function

CAPACITOR




$$e(t) = V_m \sin(\omega t)$$

FIND $i_c(t)$:

$$i_c(t) = C \frac{dV_c(t)}{dt}$$

$$= C \frac{d}{dt}\left[V_m \sin(\omega t)\right]$$

$$i_c(t) = \omega C V_m \cos(\omega t)$$

$$\therefore\ i_c(t) = \omega C V_m \sin(\omega t + 90^\circ)$$

COMPARE:

$$V_c(t) = V_m \sin(\omega t)$$

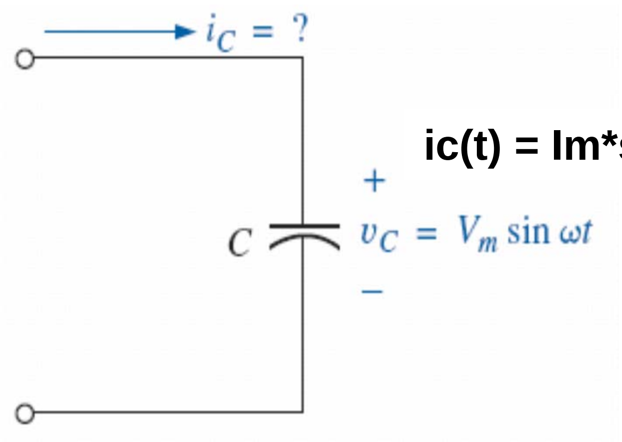
$$i_c(t) = \underbrace{\omega C V_m}_{I_m} \sin(\omega t + 90^\circ)$$

$I_m = \omega C V_m$, PEAK VALUE OF $i_c(t)$ IS DIRECTLY RELATED TO $\omega$ & $C$

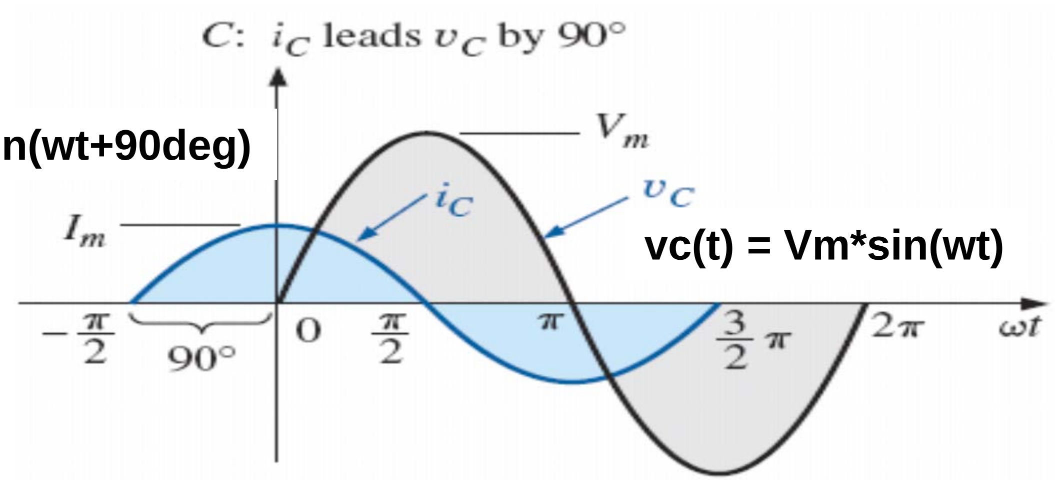
$\sin(\omega t + 90^\circ)$, THE PHASE ANGLE OF $i_c(t)$ LEADS THAT OF $V_c(t)$ BY $90^\circ$

$$\frac{E}{I/R} \longrightarrow \frac{V_m}{I_m} = \boxed{X_C = \frac{1}{\omega C}}$$ OHMS, FOR A CAPACITOR (CAPACITIVE REACTANCE)

## Response of the Capacitor to a Sinusoidal Forcing Function




ic(t) = Im*sin(wt+90deg)




vc(t) = Vm*sin(wt)

**FIG. 14.8** *Investigating the sinusoidal response of a capacitive element.*

**FIG. 14.9** *The current of a purely capacitive element leads the voltage across the element by 90° .*

*** ELI the ICE Man ***

# ICPs – Capacitive Reactance

**P1 – Find the reactance of a 5uF capacitor for**

a) F=DC
b) F=60Hz
c) F=24kHz

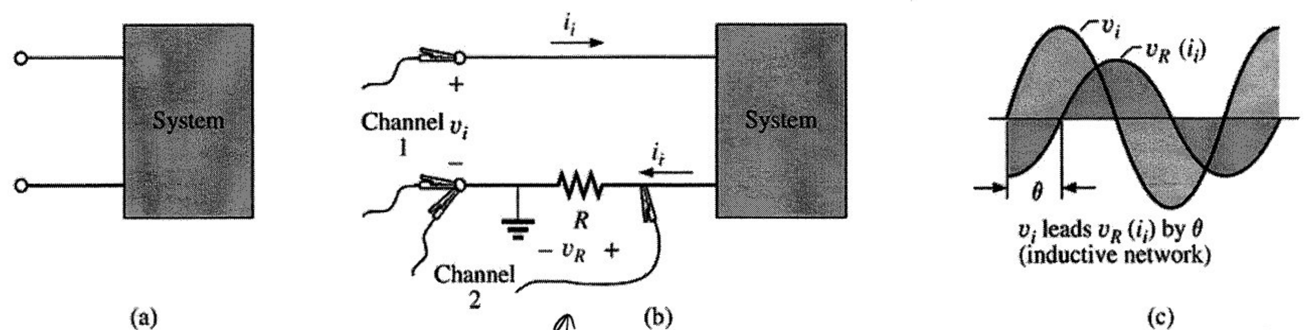
**P2 – Find the frequency at which a 50uF capacitor has**

a) Xc = 342 Ohms
b) Xc = 2000 Ohms

**P3 – Given Vc(t) = 30*sin(200t) Volts for a 1uF capacitor, find ic(t)**

**Using an Oscilloscope to Determine Network Characteristics**

**The oscilloscope is a 2-channel volt-meter**




$v_R(t) = i(t) \cdot R$

**By placing a "sense" resistor in series with the input current we can measure it indirectly**

## SUMMARY

| | R | L | C |
|---|---|---|---|
| REACTANCE | $\cancel{R}$ | $\omega L$ | $1/\omega C$ , $\Omega$ |
| REACTANCE AT DC ($\omega = 0$) | $\cancel{R}$ | 0 (SHORT) | $\infty$ (OPEN) , $\Omega$ |
| INCREASE FREQ ↓ | ↓ NO CHANGE | ↓ INCREASING REACTANCE | ↓ DECREASING REACTANCE |
| REACTANCE AT $\omega \to \infty$ (HIGH FREQ) | $\cancel{R}$ | $\infty$ (OPEN) | 0 , $\Omega$ |

EXAMPLE FIND THE REACTANCE OF A 1μF CAPACITOR & A 1μH INDUCTOR IF THE APPLIED VOLTAGE IS $30 \sin(400t - 30^\circ)$

$\uparrow$ $\omega = 400\ r/s$

$\therefore\ \underline{f = 63.66\ Hz}$

$$X_L = \omega L = (400\ r/s)(1\mu H)$$

$$\boxed{X_L = 400 \times 10^{-6}\ \Omega} \leftarrow \text{LOW}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(400\ r/s)(1\mu F)}$$

$$\boxed{X_C = 2{,}500\ \Omega} \leftarrow \text{HIGH}$$

## Frequency Response of The Basic Elements (ideal)

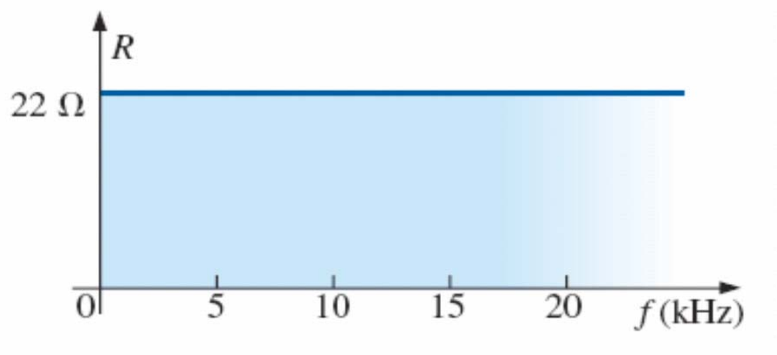



**FIG. 14.16** *R versus f for the range of interest.*

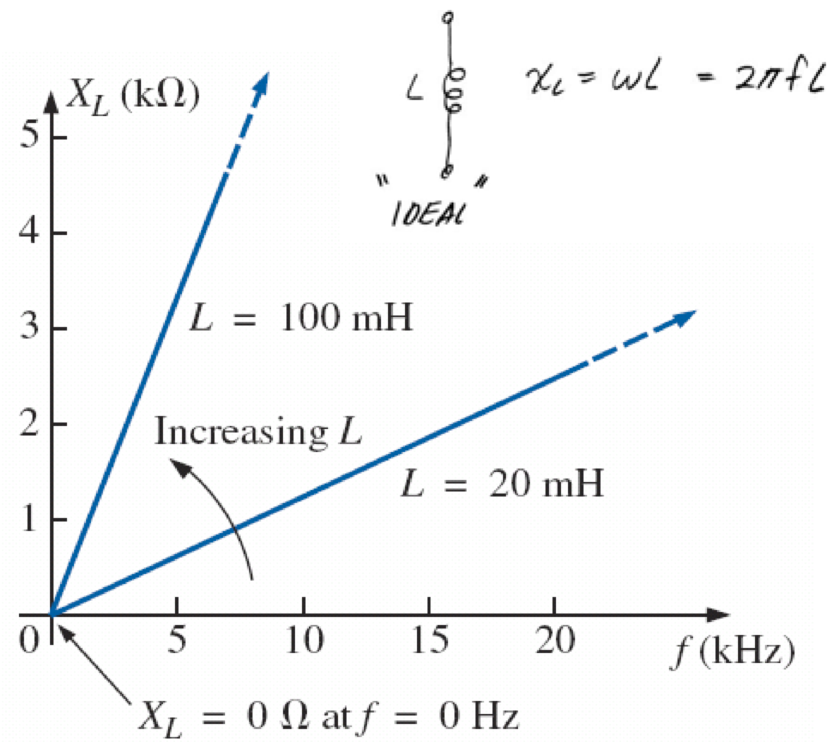



**FIG. 14.17** *$X_L$ versus frequency.*

## Frequency Response of The Basic Elements (ideal)

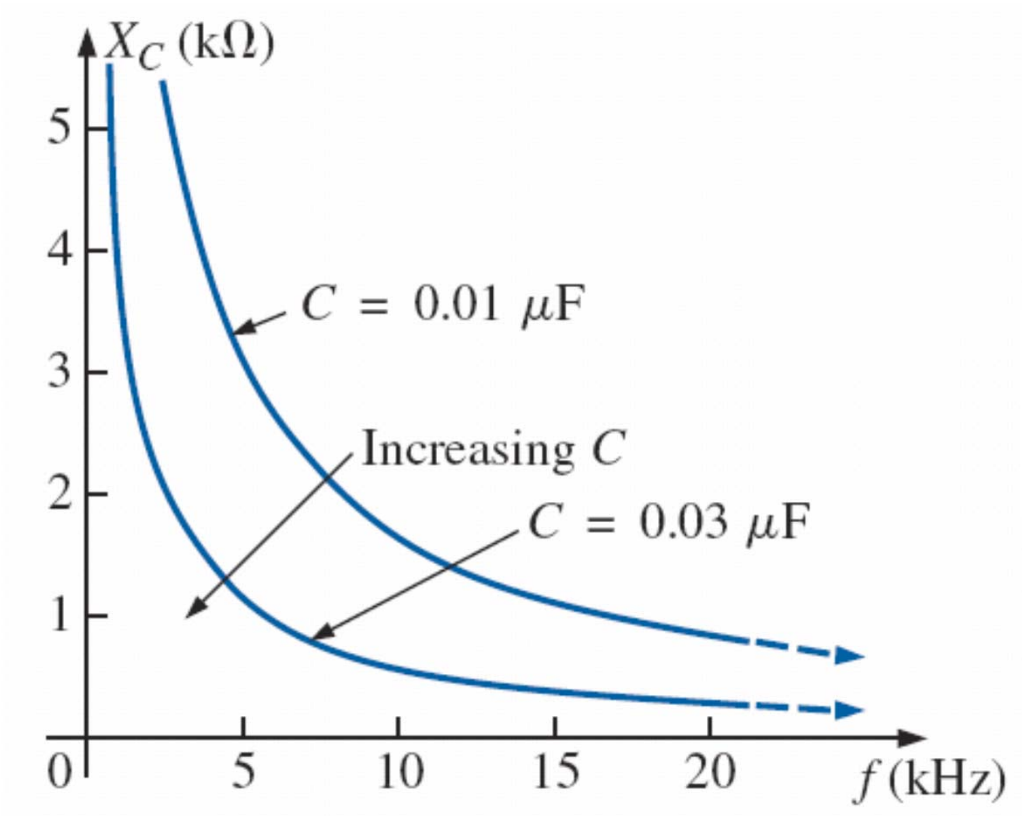


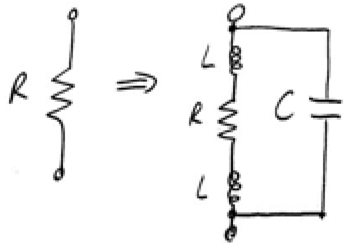

**FIG. 14.19** *$X_C$ versus frequency.*

C

"IDEAL"

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

## Frequency Response of The Basic Elements (more realistic)



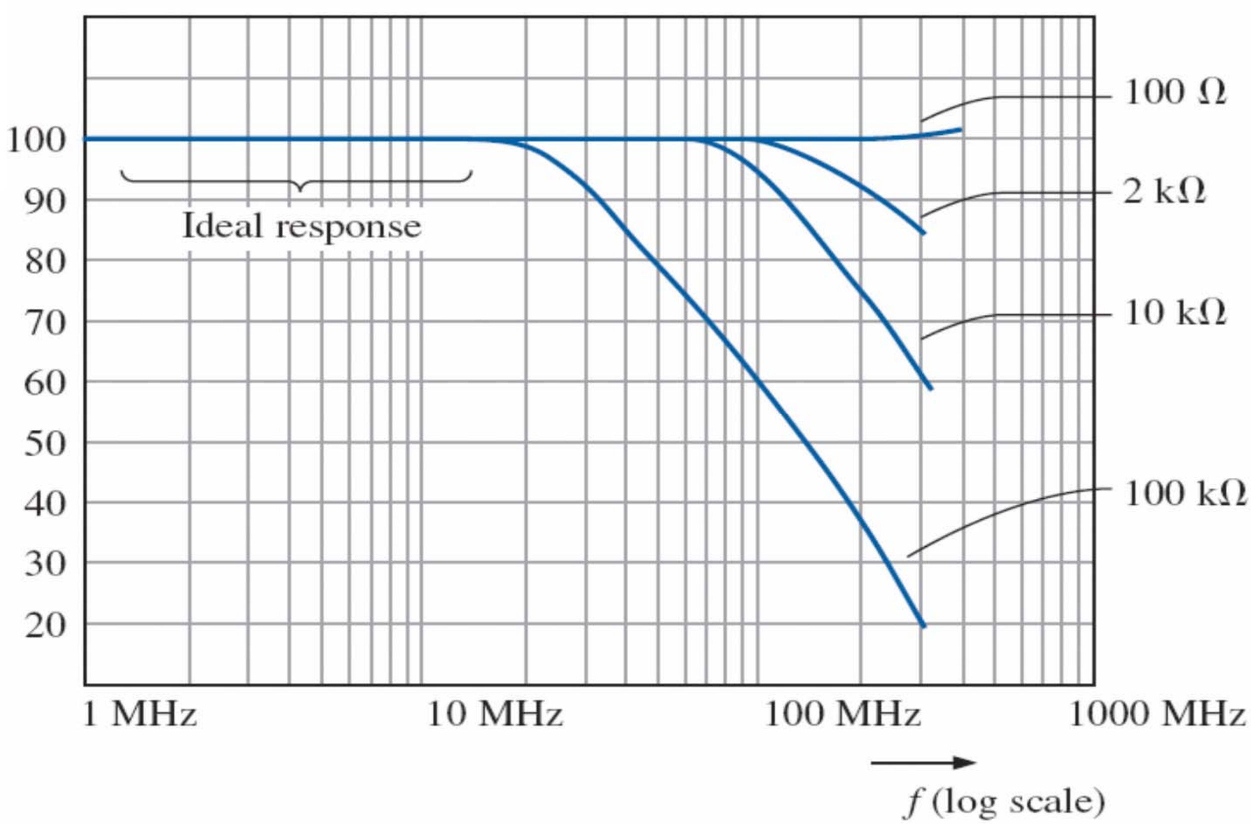




**FIG. 14.21** *Typical resistance-versus-frequency curves for carbon composition resistors.*

## Frequency Response of The Basic Elements (more realistic)

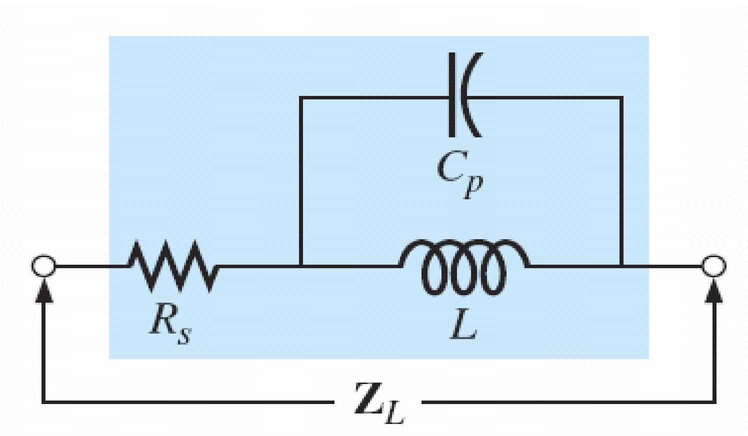



**FIG. 14.22** *Practical equivalent for an inductor.*

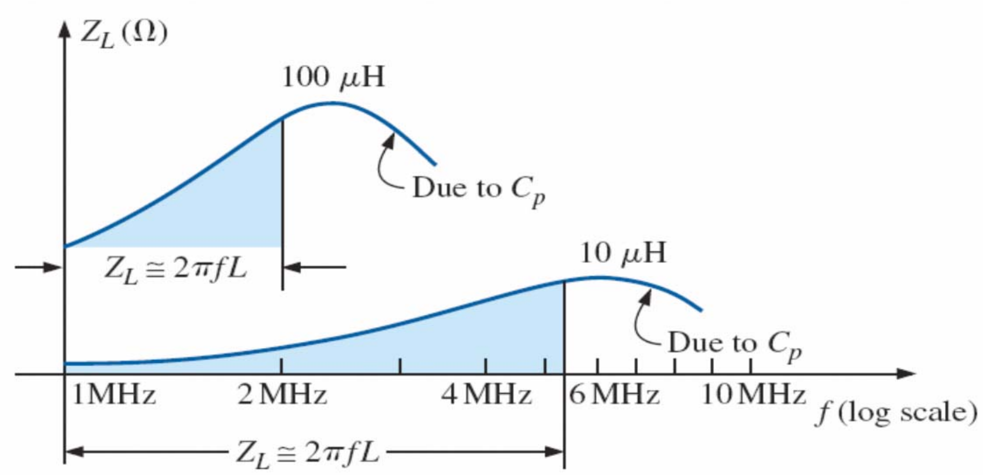



**FIG. 14.23** *$Z_L$ versus frequency for the practical inductor equivalent of Fig. 14.22.*

## Frequency Response of The Basic Elements (more realistic)

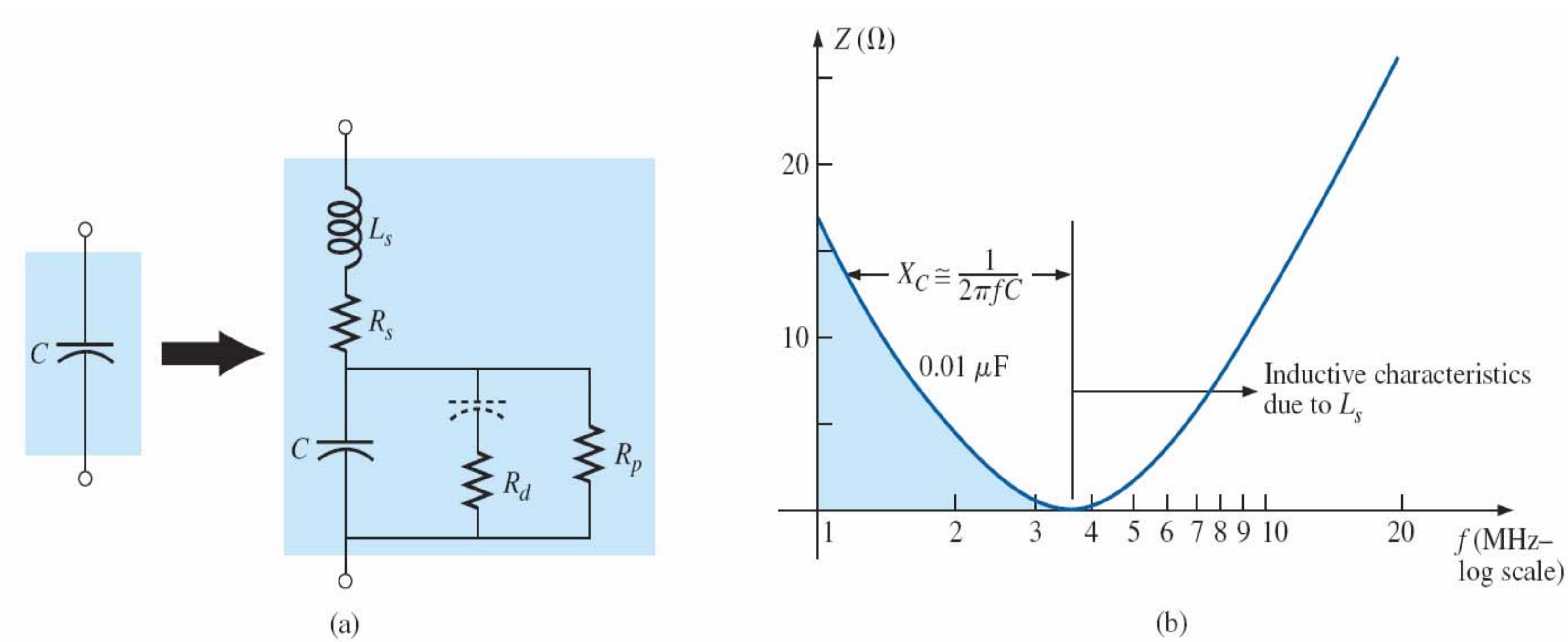



**FIG. 14.24** *Practical equivalent for a capacitor; (a) network; (b) response.*