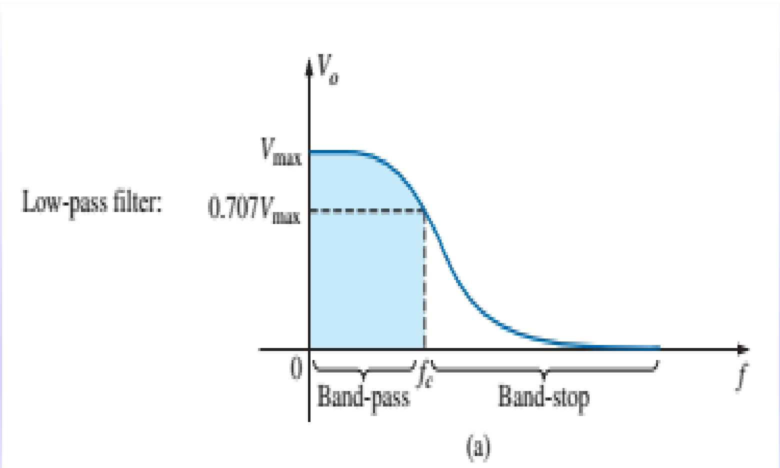
RC Low Pass Filter

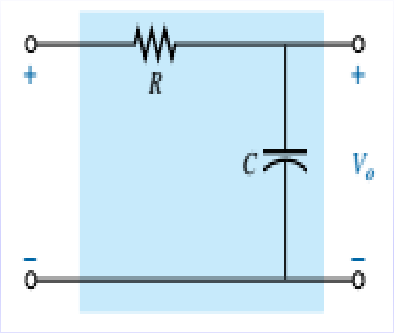
- Brief Review
  - Schematic, magnitude frequency response
  - Voltage gain (in dB) equation
- RC LPF Analysis
  - Transfer function
  - Magnitude response
  - Phase response
  - Cutoff (or critical) frequency, fc
  - Sketching the magnitude and phase responses (Bode Analysis)
- RC LPF – In Class Problem
  - Calculations and sketches
  - Simulation verification (normalized and dB y-axis)

RC Low Pass Filter (brief review)

- Be able to calculate the cutoff frequencies and sketch the frequency response of a **low-pass**, high-pass, band-pass (pass-band) or band-reject (stop-band) filter.
- Develop skills in interpreting and establishing the Bode response (**frequency response**) of any filter.
- The unit decibel (dB), defined by a logarithmic expression, is used throughout the industry to define levels of audio, **voltage gain**, energy, field strength, and so on.

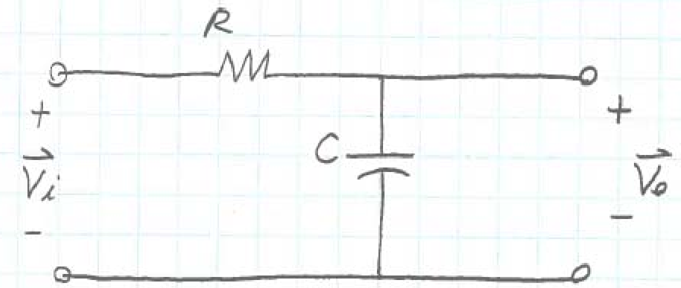


(a)



$$dB_v = 20 \log_{10} \frac{V_2}{V_1} \quad \text{(dB)}$$

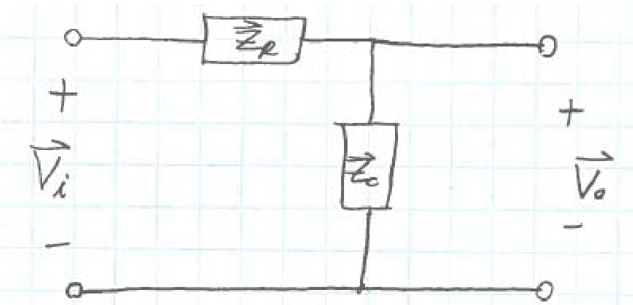
RC LPF Analysis



R-C LOW PASS FILTER

FIND: $\dfrac{\vec{V}_o}{\vec{V}_i}(f)$ Also called the **TRANSFER FUNCTION**

Converting to the phasor domain:



$$\vec{Z}_R = R$$

$$\vec{Z}_C = \underbrace{-j\left(\frac{1}{2\pi f C}\right)}_{X_C}$$

Using voltage divider:

$$\vec{V}_o = \vec{V}_i\left(\frac{\vec{Z}_C}{\vec{Z}_C + \vec{Z}_R}\right)$$

$$\frac{\vec{V}_o}{\vec{V}_i}(f) = \frac{-j\left(\frac{1}{2\pi f C}\right)}{-j\left(\frac{1}{2\pi f C}\right) + R}$$

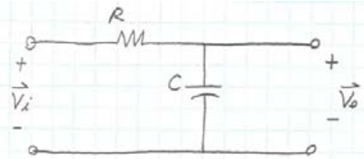
Simplifying:

$$= \frac{-j}{-j + R(2\pi f)(C)}$$

$$\frac{\vec{V}_o}{\vec{V}_i}(f) = \boxed{\frac{1}{1 + j(2\pi f R C)}}$$

Can be split into the **MAGNITUDE** response and the **PHASE** response

RC LPF Analysis



R-C LOW PASS FILTER

FIND: $\frac{\vec{V}_o}{\vec{V}_i}(f)$

$$\frac{\vec{V}_o}{\vec{V}_i}(f) = \boxed{\frac{1}{1 + j(2\pi f RC)}}$$

Magnitude Response

$$\left|\frac{\vec{V}_o}{\vec{V}_i}(f)\right| = \left|\frac{1}{1 + j(2\pi f RC)}\right|$$

$$= \frac{|1|}{|1 + j\,2\pi f RC|}$$

$$= \boxed{\frac{1}{\sqrt{1^2 + (2\pi f RC)^2}}}$$

MAGNITUDE RESPONSE

Phase Response

$$\measuredangle \frac{\vec{V}_o}{\vec{V}_i}(f) = \measuredangle NUM - \measuredangle DEN$$

$$= 0^\circ - TAN^{-1}(2\pi f RC)$$

$$= \boxed{-TAN^{-1}(2\pi f RC)}$$

$\leftarrow$ PHASE RESPONSE

½ Power Point

HALF POWER POINT $\left|\frac{\vec{V}_o}{\vec{V}_i}\right| = 0.707$

@ $\boxed{f_c = \frac{1}{2\pi RC}}$

CHECK :

$$0.707 = \frac{1}{\sqrt{1 + \left(\frac{2\pi RC}{2\pi RC}\right)^2}}$$

$$0.707 = \frac{1}{\sqrt{2}}$$

RC LPF – "Sketch" The Magnitude Response

$$\left|\frac{\vec{V}_o}{\vec{V}_i}(f)\right| = \boxed{\frac{1}{\sqrt{1^2 + (2\pi fRC)^2}}}$$

MAGNITUDE RESPONSE

Investigating this function (in dB):

$$\left|\frac{\vec{V}_o}{\vec{V}_i}(f)\right|_{dB} = 20\log_{10}\left(\frac{1}{\sqrt{1+(2\pi fRC)^2}}\right)$$

@ $f \to 0Hz$: $20\log_{10}\left(\frac{1}{\sqrt{1+0^2}}\right)$

$\approx 20\log_{10}(1) \sim 0\,dB$

@ $f = f_c$: $20\log_{10}\left(\frac{1}{\sqrt{1+1}}\right)$

$= 20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3\,dB$

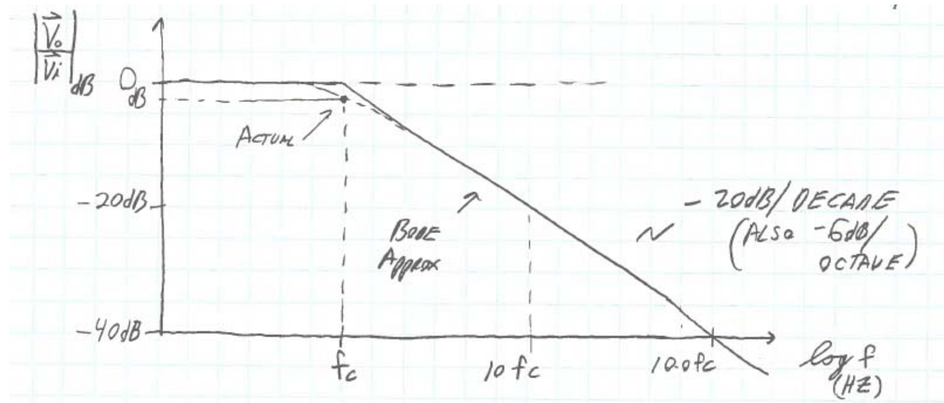
@ $f = 10f_c$ (ONE DECADE AWAY)

$20\log_{10}\left(\frac{1}{\sqrt{1+\left(\frac{10}{1}\right)^2}}\right) = -20.0\,dB$

@ $f = 100f_c$ ANOTHER DECADE OUT

$20\log_{10}\left(\frac{1}{\sqrt{1+\left(\frac{100}{1}\right)^2}}\right) = -40.0\,dB$

Sketching:



<u>The Bode approximation for this LPF</u>

- Sketch => Approximation
- Plot => Exact (use the equation)
- X-axis: logarithmic
- -20dB/decade = -6dB/octave (starts at fc)
- Largest error at fc

Note: -3dB at fc, 20dB/decade rolloff for f>fc

RC LPF – "Sketch" The Phase Response

$$\measuredangle \frac{\vec{V_o}}{\vec{V_i}}(f) \text{ IN DEGREES } = \boxed{-\mathrm{TAN}^{-1}(2\pi f RC)} \leftarrow \text{PHASE RESPONSE}$$

Investigating this function

$$\text{FOR } f \rightarrow 0\,\mathrm{Hz} : -\mathrm{TAN}^{-1}(0) \sim 0^\circ$$

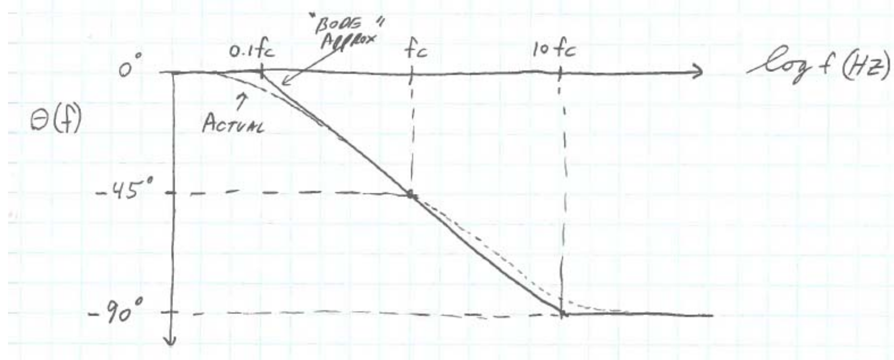
$$@\ f = 0.1 f_c : -\mathrm{TAN}^{-1}(0.1) = -5.71^\circ$$

$$@\ f = f_c : -\mathrm{TAN}^{-1}\left(\frac{2\pi RC}{2\pi RC}\right) = -\mathrm{TAN}^{-1}(1) = -45^\circ$$

$$@\ f = 10 f_c : -\mathrm{TAN}^{-1}(10) = -84.3^\circ \leftarrow \text{APPROACHING } -90^\circ$$

Note: -45 degrees at fc, close to 0 degrees at 0.1fc and almost -90 degrees at 10fc

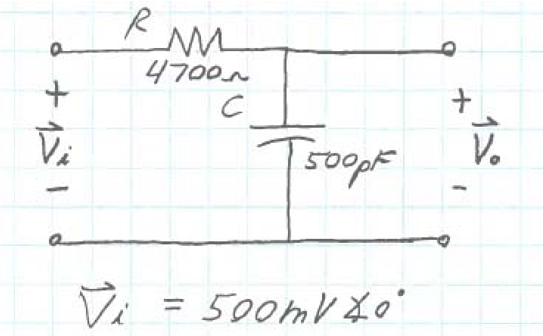
Sketching:



<u>The Bode approximation for this LPF</u>

- Sketch => Approximation
- Plot => Exact (use the equation)
- X-axis: logarithmic
- -45 Degrees at fc, 0 degrees at 0.1fc and -90 degrees at 10fc
- Largest error at 0.1fc and 10fc

RC LPF – In Class Problem



Find:

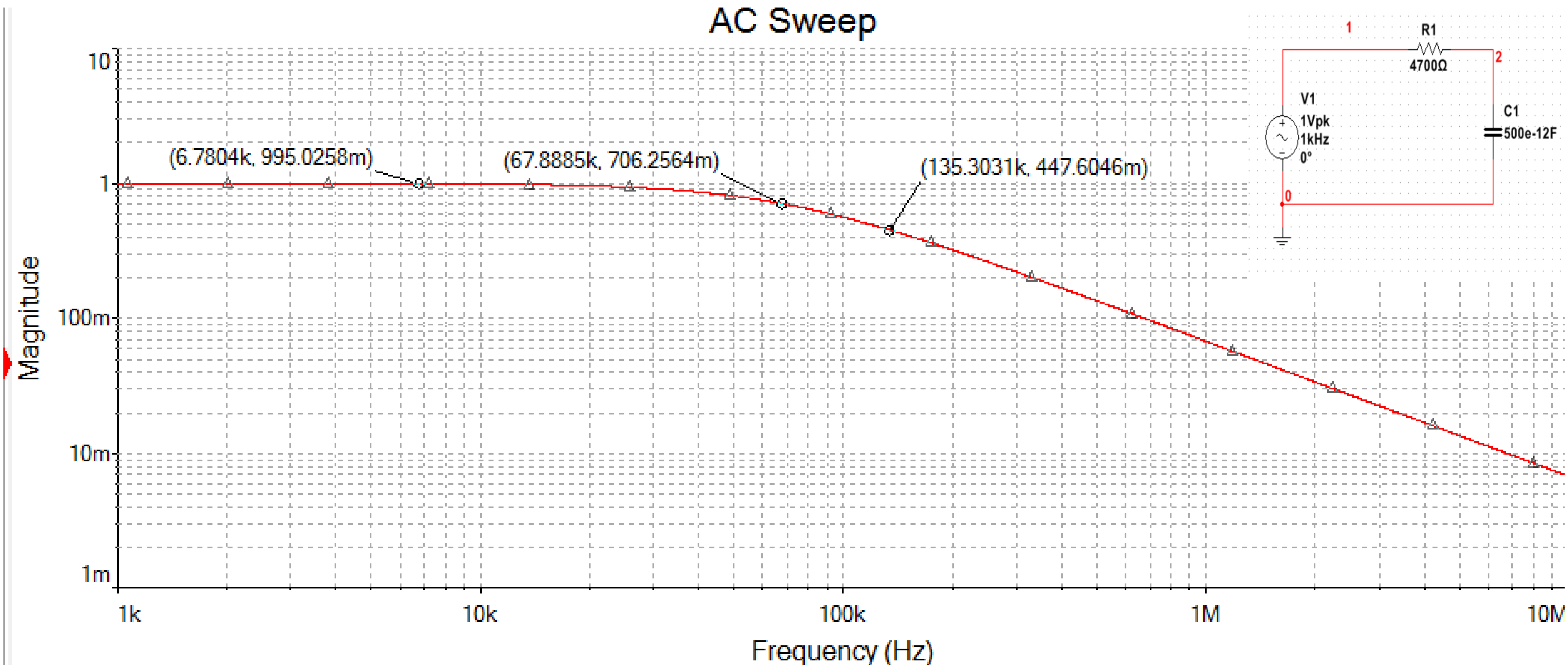
a) fc

b) Sketch the magnitude response (dB) and phase response

c) **Vo** one octave above fc

d) **Vo** one decade below fc

RC LPF – Simulation (1V AC Magnitude, normalized)



Notes (from this plot):

fc ~ 67.9kHz (|Vo/Vi| ~ 0.707)

@ fc/10: |Vo| = (0.995)(500mV) ~498mV

@2fc: |Vo| = (0.448)(500mV) ~ 224mV

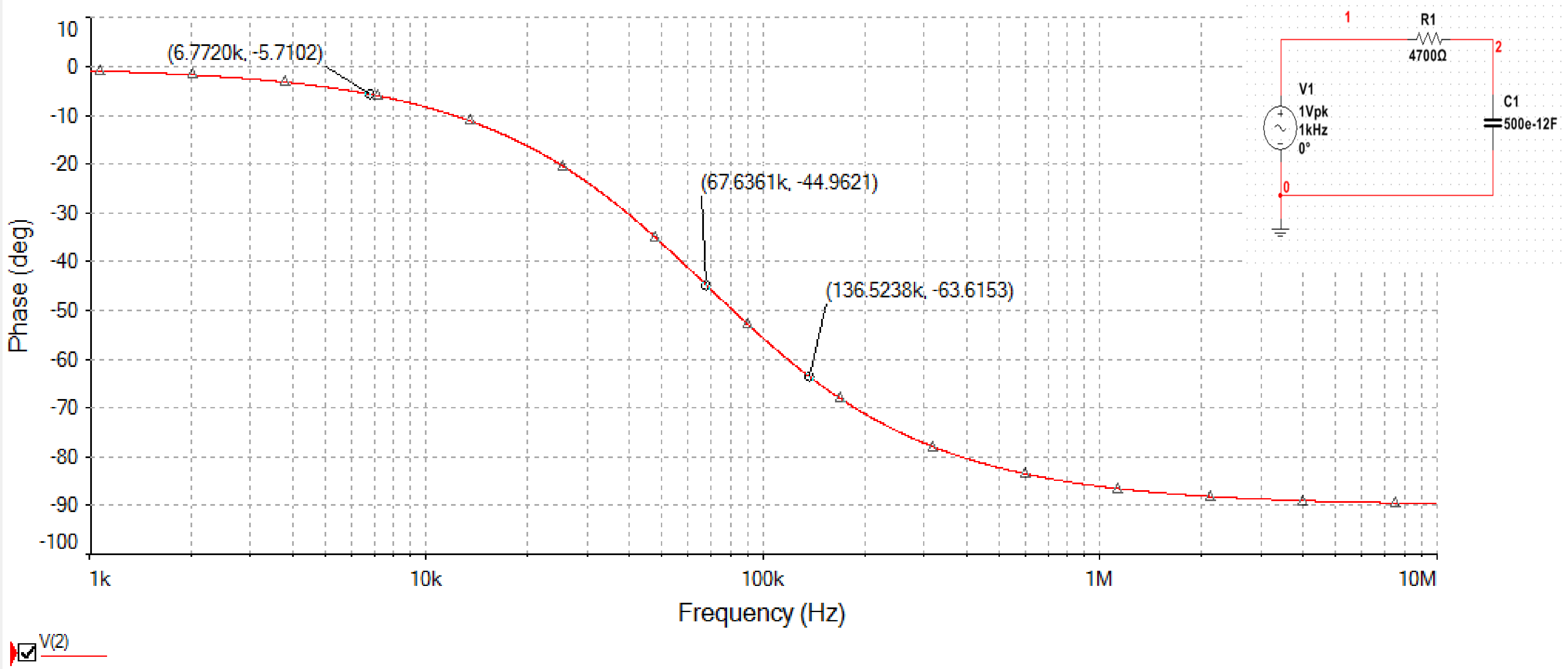
Recall (calculations)

$\boxed{67.73\,kHz}$

$\boxed{\vec{V}_o = 497.5\,mV \angle -5.71^\circ}$

$\boxed{\vec{V}_o = 0.224\,V \angle -63.4^\circ}$

RC LPF – Simulation (1V AC Magnitude, normalized)



Notes (from this plot):

fc ~ 67.6kHz (<(Vo/Vi) ~ -45 Deg)

@ fc/10: <(Vo) ~ -5.7 Deg

@2fc: <(Vo) ~ -63.6 Deg

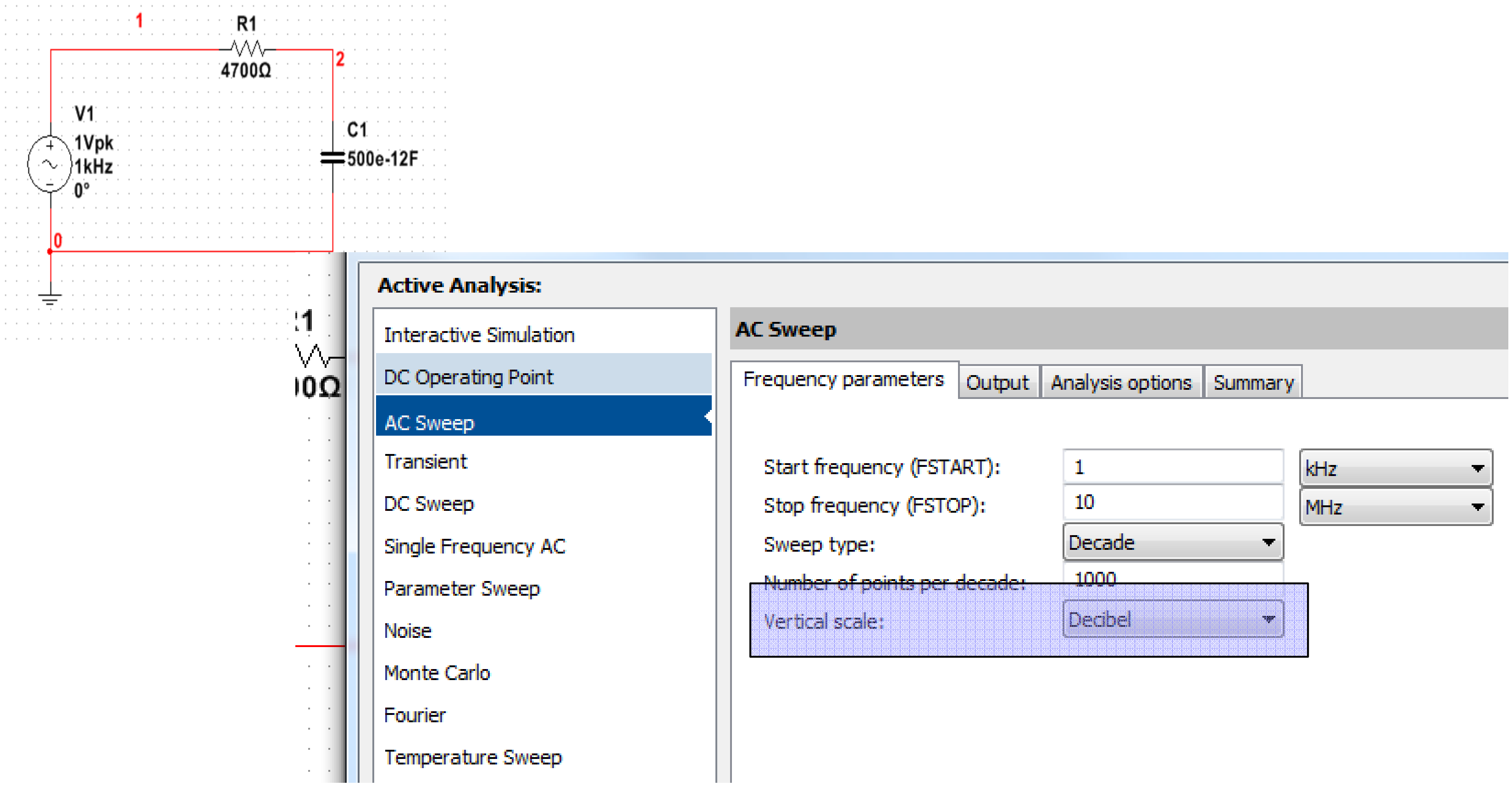
Recall (calculations)

$\boxed{67.73\,kHz}$

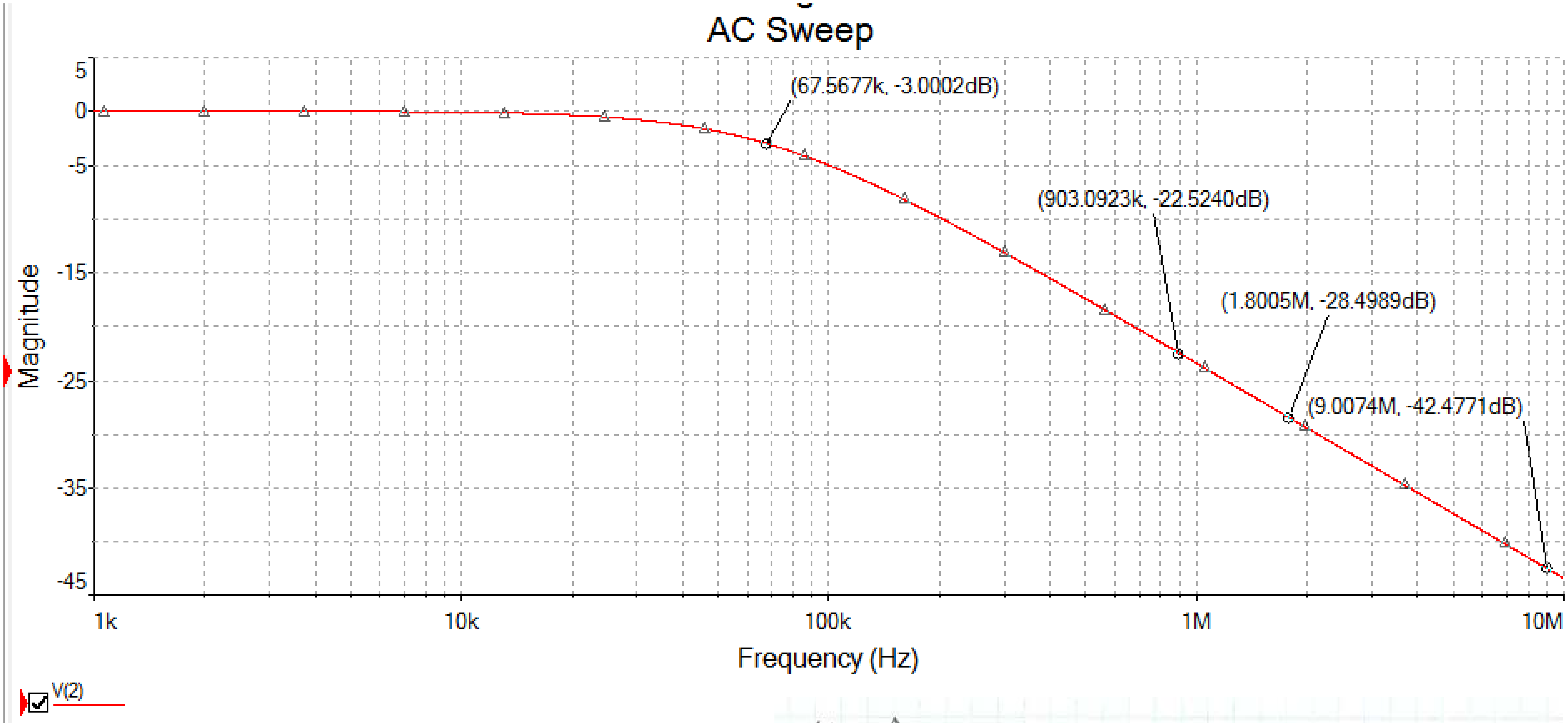
$\boxed{\vec{V}_o = 497.5\,mV \angle -5.71^\circ}$

$\boxed{\vec{V}_o = 0.224\,V \angle -63.4^\circ}$

RC LPF – Simulation (1V AC Magnitude, normalized, y-axis in dB)



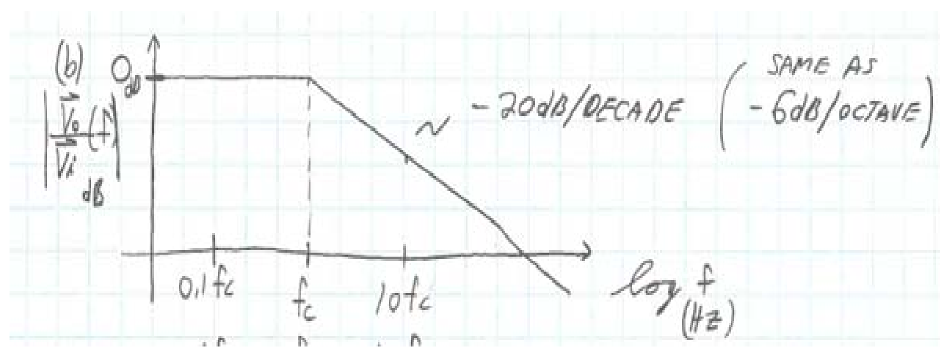
RC LPF – Simulation (1V AC Magnitude, normalized, y-axis in dB)



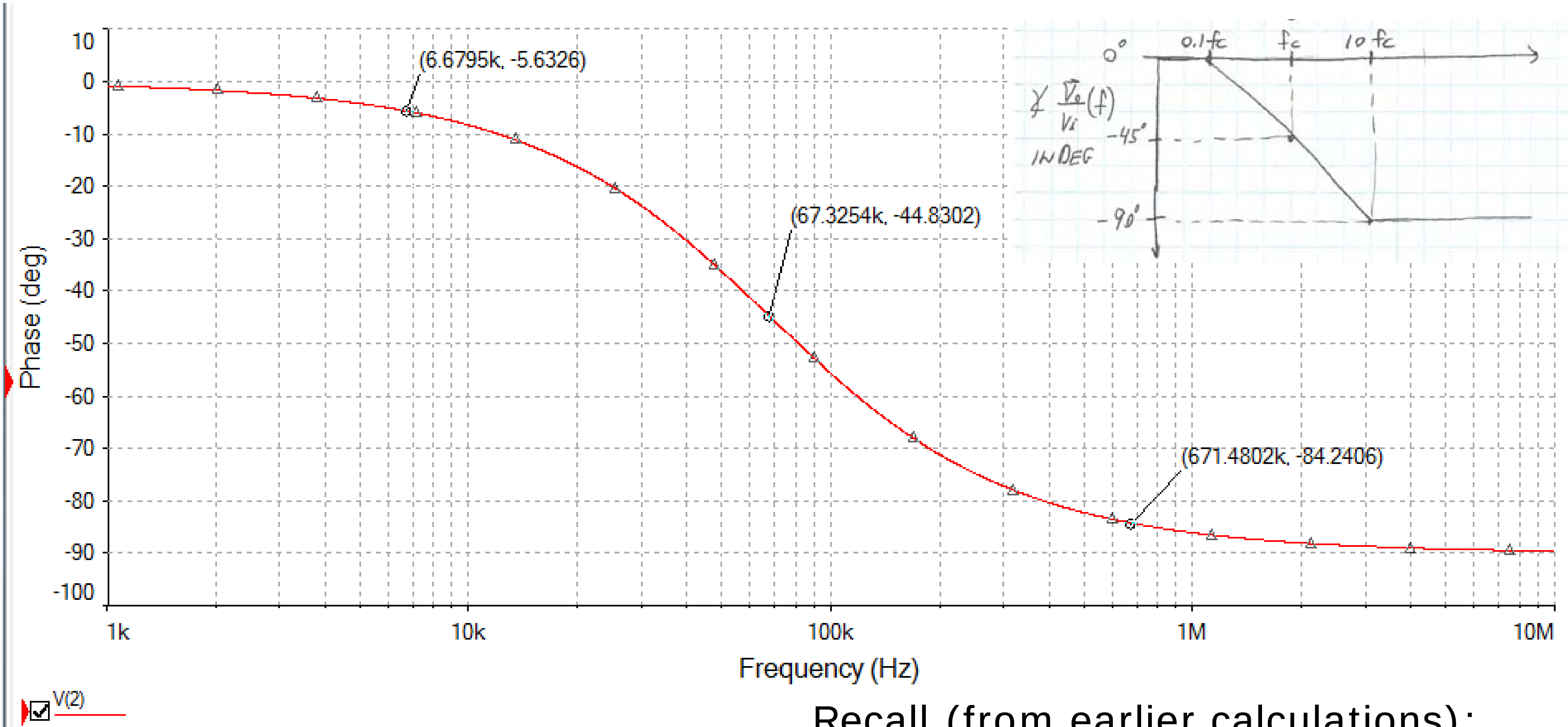
Notes (from this plot):

fc = -3dB point

- 20db/decade or -6dB/octave rolloff



RC LPF – Simulation (1V AC Magnitude, normalized, y-axis in dB)



Recall (from earlier calculations):

@ $f = 0.1 f_c$: $-\tan^{-1}(0.1) = -5.71°$

@ $f = 10 f_c$: $-\tan^{-1}(10)$

$= -84.3°$ ← APPROACHING $-90°$

Notes (from this plot):

fc = -45 Degree phase shift

0.1fc ~ -5.6 degree phase shift

10fc ~ -84.2 degree phase shift