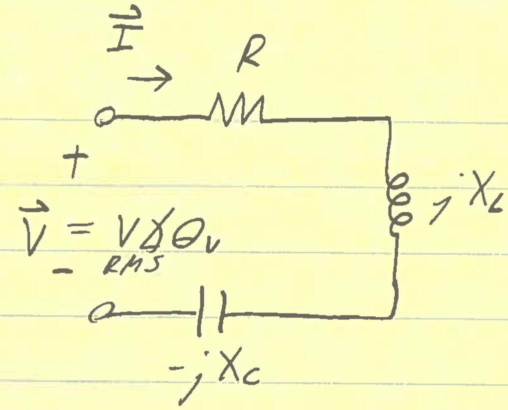
CONSIDER:

- ASSUME INDUCTIVE

$(\theta_V > \theta_I)$



$$\vec{I} = \frac{\vec{V}}{Z_T} = \frac{\vec{V}}{R + j(X_L - X_C)}$$

, CONVERT TO POLAR FORM:

$$|\vec{I}| = \frac{|\vec{V}|}{|R + j(X_L - X_C)|} = \frac{V_{RMS}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \measuredangle \vec{V} - \measuredangle [R + j(X_L - X_C)]$$

$$\measuredangle \vec{I} = \theta_V - \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$\therefore \vec{I} = \frac{V_{RMS}}{\sqrt{R^2 + (X_L - X_C)^2}} \measuredangle \left[\theta_V - \tan^{-1}\left(\frac{X_L - X_C}{R}\right)\right]$$

RECALL:

$$P = V_{RMS} I_{RMS} \cos(\theta)$$

$$= \frac{V_{RMS} V_{RMS}}{\sqrt{R^2 + (X_L - X_C)^2}} \cos\left[\theta_V - \left[Q_V - \tan^{-1}\left(\frac{X_L - X_C}{R}\right)\right]\right]$$

OR $$P = \frac{V_{RMS}^2}{\sqrt{R^2 + (X_L - X_C)^2}} \cos\left[\tan^{-1}\left(\frac{X_L - X_C}{R}\right)\right]$$

FIND Q:

$$Q = V_{RMS} I_{RMS} \sin(\theta)$$

$$\therefore \quad Q_L = \frac{V_{RMS} V_{RMS}}{\sqrt{R^2 + (X_L - X_C)^2}} \sin\left[\theta_V - \left[\theta_V - \tan^{-1}\left(\frac{X_L - X_C}{R}\right)\right]\right]$$

OR $$Q_L = \frac{V_{RMS}^2}{\sqrt{R^2 + (X_L - X_C)^2}} \sin\left[\tan^{-1}\left(\frac{X_L - X_C}{R}\right)\right]$$

RECALL:

$$\vec{S} = P + jQ_L$$

$$\therefore \quad \vec{S} = \frac{V_{RMS}^2}{\sqrt{R^2 + (X_L - X_C)^2}} \cos\Bigg[\overbrace{\tan^{-1}\left(\frac{X_L - X_C}{R}\right)}^{\text{CALL "}\alpha\text{"}}\Bigg]$$

$$+ j \frac{V_{RMS}^2}{\sqrt{R^2 + (X_L - X_C)^2}} \sin\Bigg[\overbrace{\tan^{-1}\left(\frac{X_L - X_C}{R}\right)}^{\alpha}\Bigg]$$

OR $$\vec{S} = \frac{V_{RMS}^2}{\sqrt{R^2 + (X_L - X_C)^2}} \left[\cos(\alpha) + j\sin(\alpha)\right]$$

BUT $\cos(\alpha) + j\sin(\alpha) = e^{j\alpha}$, EULER'S IDENTITY

$$\therefore \quad \vec{S} = \frac{V_{RMS}^2 \, e^{j\alpha}}{\sqrt{R^2 + (X_L - X_C)^2}}, \quad \alpha = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

CONSIDER $\vec{V} \cdot \vec{I}^* \Rightarrow$

$$\underbrace{(V_{RMS} \angle \theta_V)}\ \underbrace{\frac{V_{RMS}}{\sqrt{R^2+(X_L-X_C)^2}} \angle(-\theta_V+\alpha)}$$

$$\therefore \vec{V}\vec{I}^* = \frac{V_{RMS}^2}{\sqrt{R^2+(X_L-X_C)^2}} \angle \alpha$$

OR

$$\vec{V}\vec{I}^* = \frac{V_{RMS}^2 \, e^{j\alpha}}{\sqrt{R^2+(X_L-X_C)^2}}, \left(\text{SINCE } R\angle\theta = Re^{j\theta}\right)$$

BUT THIS IS ALSO $\vec{S}$

$$\therefore \boxed{\vec{S} = \vec{V}\vec{I}^*} \qquad (\text{EQ } 19.29)$$