*E*lectrical

*E*ngineering

*T*echnology

Thevenin's Theorem

Spring 2019 (2185)

Thevenin's Theorem

- Introduction and Detailed Approach
- Example 1
  - Illustrative example
  - Only independent sources
- Example 2
  - Only independent sources, more complex
  - **Work as we go in your calculator**
- Example 3
  - Includes a <u>dependent source controlled in-network</u>
  - <u>Requires a different method to determine **Z_{TH}** (test source)</u>
- In Class Problem
  - Includes a dependent source controlled out-of network
  - Use the standard approach

Thevenin's Theorem

- Thévenin's theorem, as stated for sinusoidal ac circuits, is changed only to include the term impedance instead of resistance; that is, any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an impedance in series.

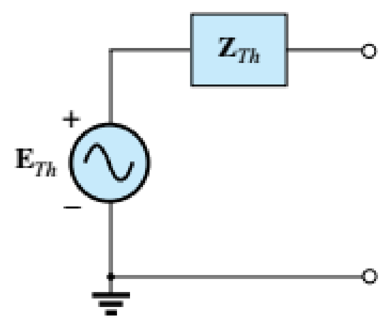


FIG. 19.23
Thévenin equivalent circuit for ac networks.

Thevenin's Theorem - Approach

Independent ac and dc Sources

1. *Remove that portion of the network across which the Thévenin equivalent circuit is to be found.*
2. *Mark (* **A and B or a-a'** *) the terminals of the remaining two-terminal network.*
3. *Calculate Z_{Th} by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.*
4. *Calculate E_{Th} by first replacing the voltage and current sources and then finding the open-circuit voltage between the marked terminals.*
5. *Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thévenin equivalent circuit.*

For dependent sources – Must use an alternate method to find **Z_{TH}** if the controlling variable is IN the network under investigation.

Thevenin's Theorem – Example 1 (Find the Thevenin Equivalent Circuit)

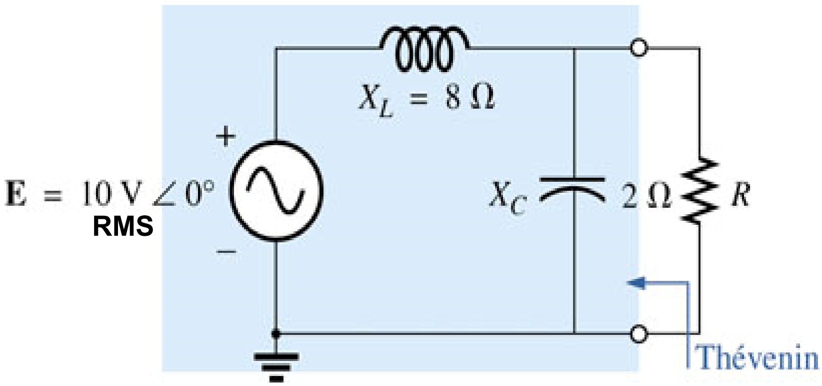


FIG. 19.24 *Example 19.7.*

Steps 1,2: Redraw and label the terminals

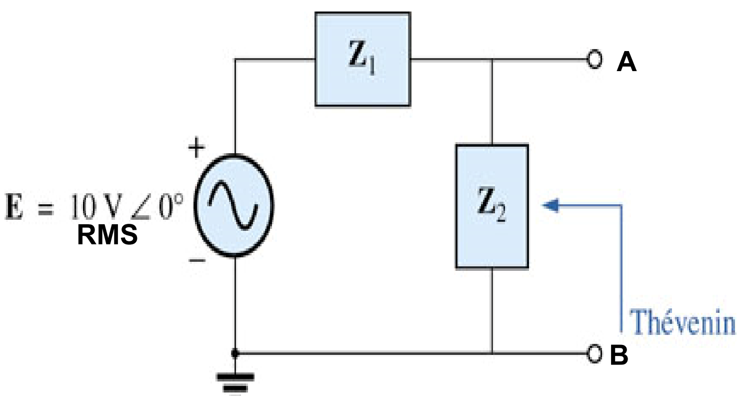
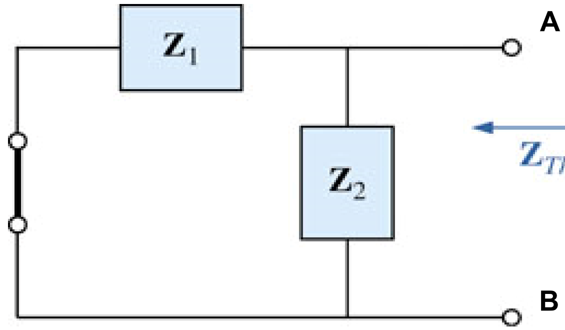


FIG. 19.25 *Assigning the subscripted impedances to the network in Fig. 19.24.*

$$\mathbf{Z}_1 = jX_L = j\,8\ \Omega$$

$$\mathbf{Z}_2 = -jX_C = -j\,2\ \Omega$$

Step 3: Find **ZTH** (relax the independent sources)



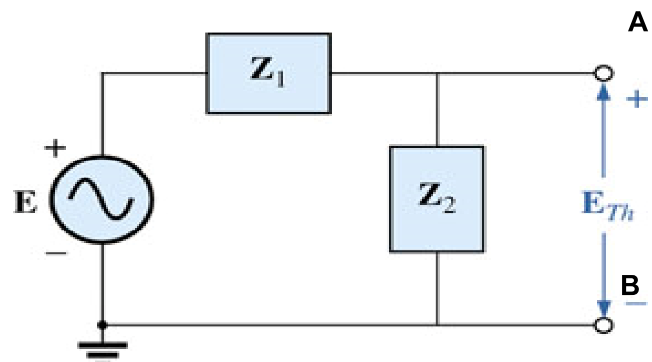
$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$2.67\ \Omega\angle -90^\circ$

$(0 - j2.67)\ \Omega$

FIG. 19.26 *Determining the Thévenin impedance for the network in Fig. 19.24.*

Step 4: Find **ETH** (**VAB** open-circuit)



$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$3.33\ \text{V}\angle -180^\circ$

-3.33 V

FIG. 19.27 *Determining the open-circuit Thévenin voltage for the network in Fig. 19.24.*

Thevenin's Theorem – Example 1 (Find the Thevenin Equivalent Circuit)

$$\mathbf{Z}_{Th} = 2.67\ \Omega \angle -90°$$

(0 - j2.67) Ω

$$\mathbf{E}_{Th} = 3.33\ \mathrm{V} \angle -180°$$

-3.33 V

Step 5: Draw the Thevenin Equivalent Circuit

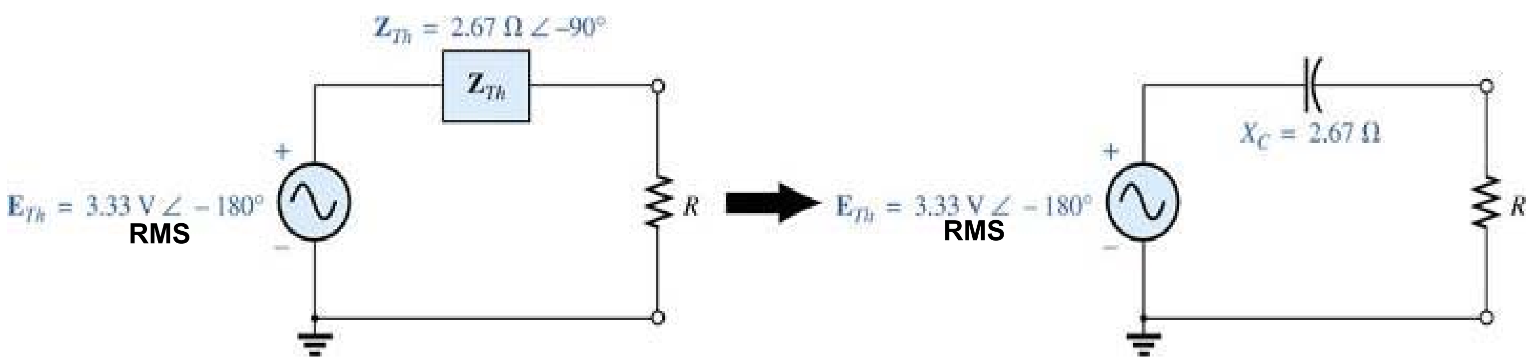
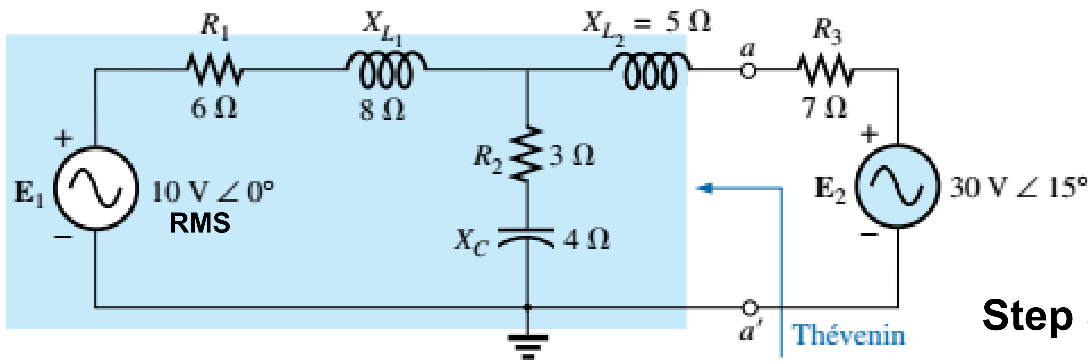


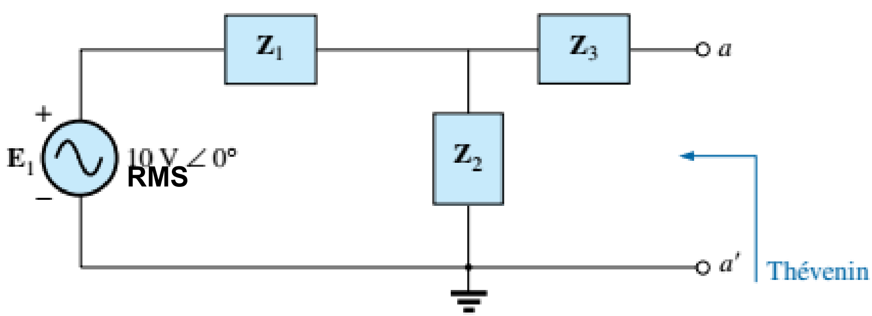
FIG. 19.28 *The Thévenin equivalent circuit for the network in Fig. 19.24.*

Thevenin's Theorem – Example 2 (use your calculator)



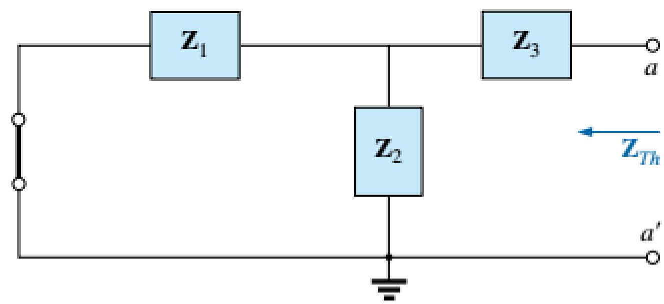
Find: the Thevenin equivalent circuit looking into terminals a-a'

Steps 1&2: Redraw, removing E2 and R3 (external to the network terminals), keep track of a-a'



$$Z_1 = R_1 + jX_{L_1} = 6\ \Omega + j\,8\ \Omega$$
$$Z_2 = R_2 - jX_C = 3\ \Omega - j\,4\ \Omega$$
$$Z_3 = +jX_{L_2} = j\,5\ \Omega$$

Step 3: Find **ZTH**



$$\vec{Z}_{TH} = (\vec{Z}_1 // \vec{Z}_2) + \vec{Z}_3$$

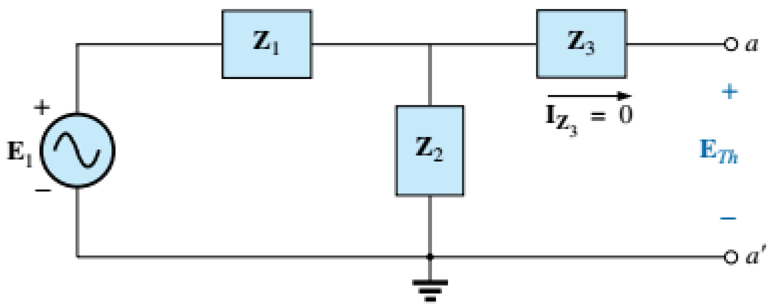
$$Z_{Th} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= j\,5\ \Omega + \frac{(10\ \Omega \angle 53.13^\circ)(5\ \Omega \angle -53.13^\circ)}{(6\ \Omega + j\,8\ \Omega) + (3\ \Omega - j\,4\ \Omega)}$$

$$Z_{Th} = 4.64\ \Omega + j\,2.94\ \Omega$$

Thevenin’s Theorem – Example 2 (use your calculator)

Step 4: Find **ETH**



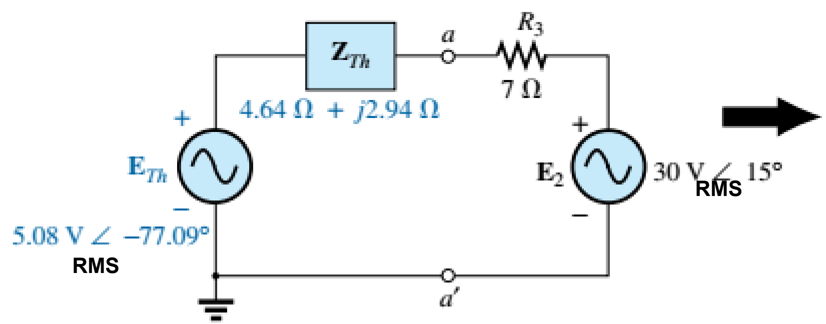
$$\mathbf{Z}_1 = R_1 + jX_{L_1} = 6\ \Omega + j\,8\ \Omega$$
$$\mathbf{Z}_2 = R_2 - jX_C = 3\ \Omega - j\,4\ \Omega$$
$$\mathbf{Z}_3 = +jX_{L_2} = j\,5\ \Omega$$

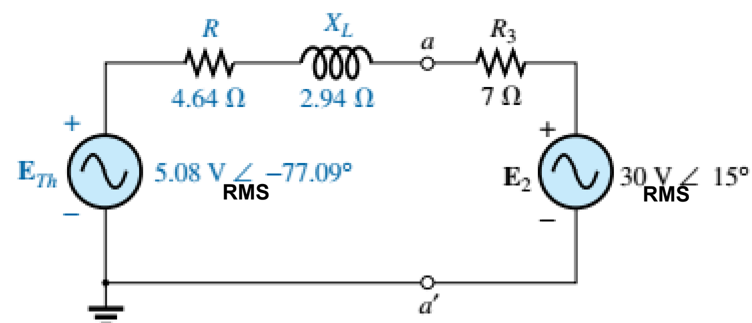
$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}_1}{\mathbf{Z}_2 + \mathbf{Z}_1}$$

$$= \frac{(5\ \Omega \angle -53.13^\circ)(10\ \text{V}_{\text{RMS}} \angle 0^\circ)}{9.85\ \Omega \angle 23.96^\circ}$$

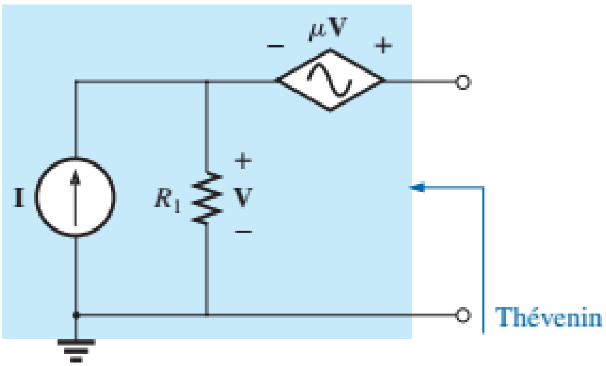
$$\mathbf{E}_{Th} = \frac{50\ \text{V} \angle -53.13^\circ}{9.85 \angle 23.96^\circ} = \mathbf{5.08\ V_{RMS} \angle -77.09^\circ}$$

Step 5: Draw the Thevenin Equivalent Circuit





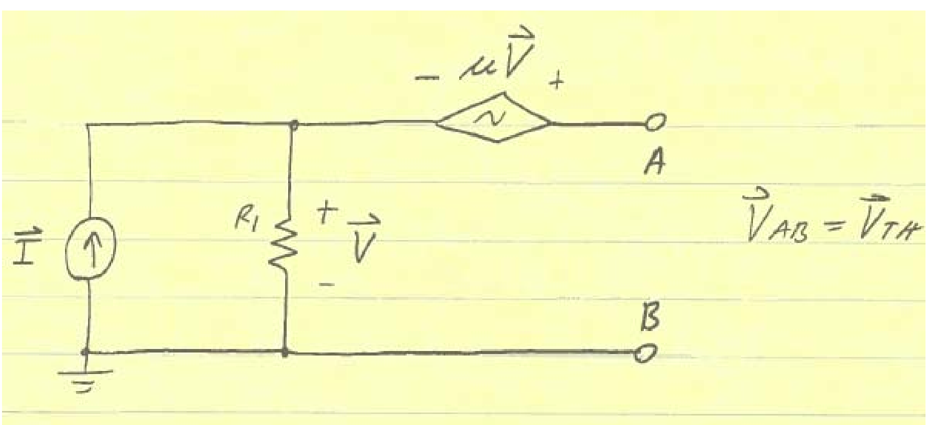
Thevenin's Theorem – Example 3 (dependent source controlled within network)



Note: The network contains a dependent source that is controlled by a voltage in the network

- Find **V_{TH}** as usual
- Find **Z_{TH}** using an alternate method (test source)

Steps 1&2: Redraw, keep track of a-b



Step 4: Find **V_{TH}**

KVL: $\vec{V} + \mu\vec{V} - \vec{V}_{TH} = 0$

$\therefore \vec{V}_{TH} = \vec{V} + \mu\vec{V}$

BUT $\vec{V} = \vec{I}R_1$

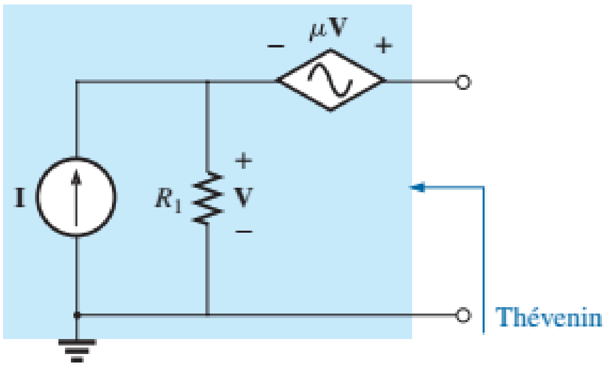
$\therefore \vec{V}_{TH} = \vec{I}R_1 + \mu\vec{I}R_1$

OR

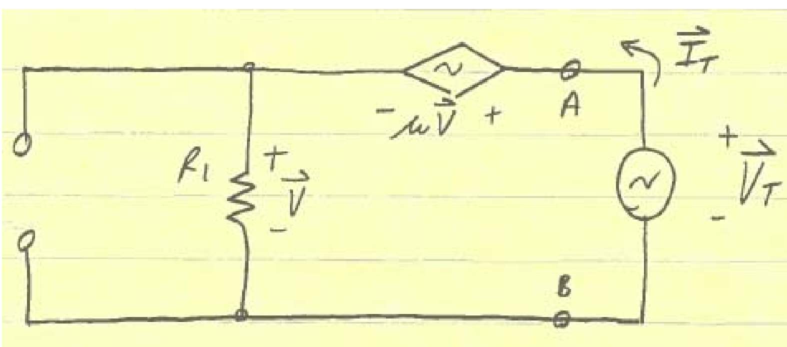
$$\boxed{\vec{V}_{TH} = \vec{I}R_1(1+\mu)}$$

Note: **V_{TH}** (**E_{TH}**) is a dependent voltage source itself

Thevenin's Theorem – Example 3 (dependent source controlled within network)



Step 3: Find **Z_{TH}** by
- Relaxing the independent sources (I)
- Applying a test-source across a-b



$$\vec{Z}_{TH} = \frac{\vec{V}_T}{\vec{I}_T}$$

$$\text{KVL: } \vec{V}_T = \mu\vec{V} + \vec{V}$$

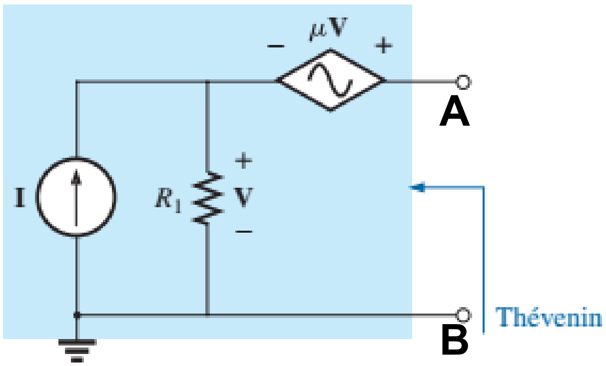
$$\text{BUT } \vec{V} = \vec{I}_T R_1$$

$$\therefore \vec{V}_T = \mu\vec{I}_T R_1 + \vec{I}_T R_1$$

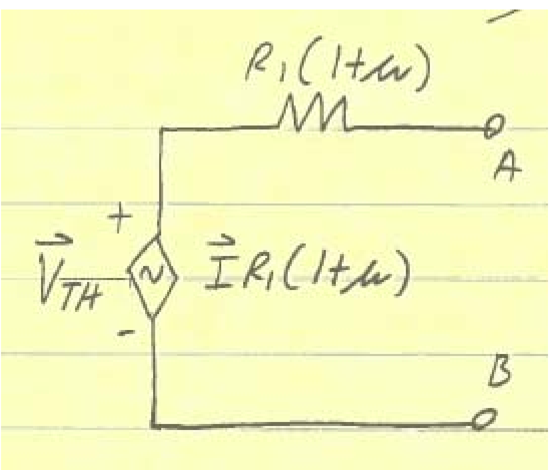
$$\frac{\vec{V}_T}{\vec{I}_T} = \mu R_1 + R_1$$

$$\vec{Z}_{TH} = \boxed{R_1(1+\mu)}$$

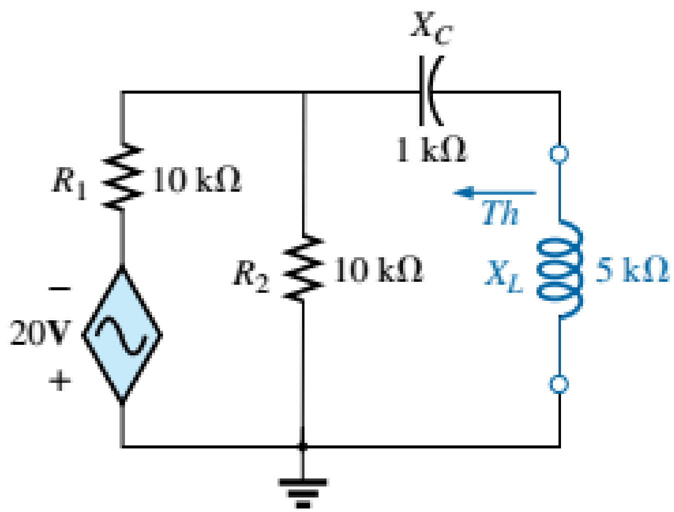
Thevenin's Theorem – Example 3 (dependent source controlled within network)



Step 5: Draw the Thevenin Equivalent Circuit



In Class Problem



Find:

- The Thevenin equivalent circuit for the network external to the inductor

Approach:

- Standard Thevenin approach
    - Dependent source is controlled out of the network of interest