Summary of Equations to Accompany

INTRODUCTORY CIRCUIT ANALYSIS, Thirteenth Edition, by Robert L. Boylestad

ac

Sinusoidal Alternating Waveforms

Sine wave $v = V_m \sin \alpha$, $\alpha = \omega t = 2\pi f t$, $f = 1/T$, 1 radian = 57.3°, radians = $(\pi/180°) \times$ (degrees), degrees = $(180°/\pi) \times$ (radians)
Identities $\sin(\omega t + 90°) = \cos \omega t$, $\sin \omega t = \cos(\omega t - (\pi/2))$, $\sin(-\alpha) = -\sin \alpha$, $\cos(-\alpha) = \cos \alpha$
Average value G = algebraic sum of areas/length of curve
Effective (rms) value $I_{rms} = 0.707 I_m$, $I_m = \sqrt{2} I_{rms}$, $I_{rms} = \sqrt{\text{area } [i(t)]^2 / T}$

The Basic Elements and Phasors

R: $I_m = V_m/R$, in phase **L:** $X_L = \omega L$, v_L leads i_L by 90°
C: $X_C = 1/\omega C$, i_C leads v_C by 90° **Power** $P = (V_m I_m/2)\cos\theta = V_{rms} I_{rms} \cos\theta$ **R:** $P = V_{rms} I_{rms} = I^2_{rms} R = V^2_{rms}/R$
Power factor $F_p = \cos\theta = P/V_{rms} I_{rms}$ **Rectangular form** $\mathbf{C} = A \pm jB$
Polar form $\mathbf{C} = C\angle\theta$ **Conversions** $C = \sqrt{A^2 + B^2}$, $\theta = \tan^{-1}(B/A)$, $A = C\cos\theta$, $B = C\sin\theta$ **Operations** $j = \sqrt{-1}$, $j^2 = -1$, $1/j = -j$, $\mathbf{C}_1 \pm \mathbf{C}_2 = (\pm A_1 \pm A_2) + j(\pm B_1 \pm B_2)$, $\mathbf{C}_1 \cdot \mathbf{C}_2 = C_1 C_2 \angle(\theta_1 + \theta_2)$, $\mathbf{C}_1/\mathbf{C}_2 = (C_1/C_2)\angle(\theta_1 - \theta_2)$

Series and Parallel ac Circuits

Elements $R\angle 0°$, $X_L \angle 90°$, $X_C \angle -90°$
Series $\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N$, $\mathbf{I}_s = \mathbf{E}/\mathbf{Z}_T$, $F_p = R/Z_T$
Voltage divider rule $\mathbf{V}_x = \mathbf{Z}_x \mathbf{E}/\mathbf{Z}_T$ **Parallel** $\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N$, $\mathbf{Z}_T = \mathbf{Z}_1\mathbf{Z}_2/(\mathbf{Z}_1 + \mathbf{Z}_2)$, $G\angle 0°$, $B_L\angle -90°$, $B_C\angle 90°$, $F_p = \cos\theta_T = G/Y_T$ **Current divider rule** $\mathbf{I}_1 = \mathbf{Z}_2\mathbf{I}_T/(\mathbf{Z}_1 + \mathbf{Z}_2)$, $\mathbf{I}_2 = \mathbf{Z}_1\mathbf{I}_T/(\mathbf{Z}_1 + \mathbf{Z}_2)$
Equivalent circuits $R_s = R_p X_p^2/(X_p^2 + R_p^2)$, $X_s = R_p^2 X_p/(X_p^2 + R_p^2)$, $R_p = (R_s^2 + X_s^2)/R_s$, $X_p = (R_s^2 + X_s^2)/X_s$

Series-Parallel ac Networks:

Employ block impedances and obtain general solution for reduced network. Then substitute numerical values. General approach similar to that for dc networks.

Methods of Analysis and Selected Topics (ac)

Source conversions $\mathbf{E} = \mathbf{I}\mathbf{Z}_p$, $\mathbf{Z}_s = \mathbf{Z}_p$, $\mathbf{I} = \mathbf{E}/\mathbf{Z}_s$
Bridge networks $\mathbf{Z}_1/\mathbf{Z}_3 = \mathbf{Z}_2/\mathbf{Z}_4$
Δ-Y, Y-Δ conversions See dc coverage, replacing R by Z.

Network Theorems

Review dc content on other side.

Thévenin's theorem (dependent sources) $\mathbf{E}_{oc} = \mathbf{E}_{Th}$, $\mathbf{Z}_{Th} = \mathbf{E}_{oc}/\mathbf{I}_{sc}$, $\mathbf{Z}_{Th} = \mathbf{E}_g/\mathbf{I}_g$ **Norton's theorem** (dependent sources) $\mathbf{I}_{sc} = \mathbf{I}_N$, $\mathbf{Z}_N = \mathbf{E}_{oc}/\mathbf{I}_{sc}$, $\mathbf{Z}_N = \mathbf{E}_g/\mathbf{I}_g$ **Maximum power transfer theorem** $Z_L = Z_{Th}$, $\theta_L = -\theta_{Th_Z}$, $P_{max} = E^2_{Th}/4R_{Th}$

Power (ac)

R: $P = VI = V_m I_m/2 = I^2 R = V^2/R$ **Apparent power** $S = VI$, $P = S\cos\theta$, $F_p = \cos\theta = P/S$ **Reactive power** $Q = VI\sin\theta$
L: $Q_L = VI = I^2 X_L = V^2/X_L$, C: $Q_C = VI = I^2 X_C = V^2/X_C$, $S_T = \sqrt{P_T^2 + Q_T^2}$, $F_p = P_T/S_T$

Resonance

Series $X_L = X_C$, $f_s = 1/(2\pi\sqrt{LC})$, $Z_{T_s} = R$, $Q_l = X_L/R_l$, $Q_s = X_L/R = (1/R)\sqrt{L/C}$, $V_{L_s} = Q_s E$, $V_{C_s} = Q_s E$, $P_{HPF} = (1/2)P_{max}$, $f_1 = (1/2\pi)[-R/2L + (1/2)\sqrt{(R/L)^2 + 4/LC}]$, f_2 (use $-R/2L$), $BW = f_2 - f_1 = R/2\pi L = f_s/Q_s$ **Parallel** $X_{L_p} = X_C$, $X_{L_p} = (R_l^2 + X_L^2)/X_L$, $f_p = [1/(2\pi\sqrt{LC})]\sqrt{1 - (R_l^2 C/L)}$, $Z_{T_p} = R_s \| R_p$, $R_p = (R_l^2 + X_L^2)/R_l$, $Q_p = (R_s \| R_p)/X_{L_p}$, $BW = f_2 - f_1 = f_p/Q_p$
$Q \geq 10$: $Z_{T_p} \cong R_s \| Q^2 R_l$, $X_{L_p} \cong X_L$, $X_L = X_C$, $f_p \cong 1/(2\pi\sqrt{LC})$, $Q_p = Q_l$, $I_L = I_C \cong QI_T$, $BW = f_p/Q_p = R_l/2\pi L$

Decibels, Filters, and Bode Plots

Logarithms $N = b^x$, $x = \log_b N$, $\log_e x = 2.3 \log_{10} x$, $\log_{10} ab = \log_{10} a + \log_{10} b$, $\log_{10} a/b = \log_{10} a - \log_{10} b$, $\log_{10} a^n = n\log_{10} a$, $\text{dB} = 10\log_{10} P_2/P_1$, $\text{dB}_v = 20\log_{10} V_2/V_1$

***R-C* filters** (high-pass) $f_c = 1/(2\pi RC)$, $\mathbf{V}_o/\mathbf{V}_i = R/\sqrt{R^2 + X_C^2}\ \angle\tan^{-1}(X_C/R)$ (low-pass) $f_c = 1/(2\pi RC)$, $\mathbf{V}_o/\mathbf{V}_i = X_C/\sqrt{R^2 + X_C^2}\ \angle -\tan^{-1}\dfrac{R}{X_C}$
Octave 2 : 1, 6 dB/octave **Decade** 10 : 1, 20 dB/decade

Transformers

Mutual inductance $M = k\sqrt{L_p L_s}$ **Iron-core** $E_p = 4.44 f N_p \Phi_m$, $E_s = 4.44 f N_s \Phi_m$, $E_p/E_s = N_p/N_s$, $a = N_p/N_s$, $I_p/I_s = N_s/N_p$, $\mathbf{Z}_p = a^2\mathbf{Z}_L$, $E_p I_p = E_s I_s$, $P_i = P_o$(ideal)
Air-core $\mathbf{Z}_i = \mathbf{Z}_p + (\omega M)^2/(\mathbf{Z}_s + \mathbf{Z}_L)$)

Polyphase Systems

Y-Y system $I_{\phi g} = I_L = I_{\phi L}$, $V_\phi = E_\phi$, $E_L = \sqrt{3}\,V_\phi$ **Y-Δ system** $V_\phi = E_L$, $I_L = \sqrt{3} I_\phi$ **Δ-Δ system** $V_\phi = E_L = E_\phi$, $I_L = \sqrt{3} I_\phi$ **Δ-Y system** $E_L = \sqrt{3}\,V_\phi$, $I_\phi = I_L$, $E_L = E_\phi$ **Power** $P_T = 3P_\phi$, $Q_T = 3Q_\phi$, $S_T = 3S_\phi = \sqrt{3} E_L I_L$, $F_p = P_T/S_T$

Pulse Waveforms and the *R-C* Response

% tilt $= [(V_1 - V_2)/V] \times 100\%$ with $V = (V_1 + V_2)/2$
Pulse repetition frequency (prf) $= 1/T$
Duty cycle $= (t_p/T) \times 100\%$
V_{av} = (duty cycle)(peak value) + (1 − duty cycle) × (V_b)
***R-C* circuits** $v_C = V_i + (V_f - V_i)(1 - e^{-t/RC})$
Compensated attenuator $R_p C_p = R_s C_s$

Nonsinusoidal Circuits

Fourier series $f(\alpha) = A_0 + A_1 \sin\omega t + A_2 \sin 2\omega t + \cdots + A_n \sin n\omega t + B_1 \cos\omega t + B_2 \cos 2\omega t + \cdots + B_n \cos n\omega t$
Even function $f(\alpha) = f(-\alpha)$, no B_n terms **Odd function** $f(\alpha) = -f(-\alpha)$, no A_n terms, no odd harmonics if $f(t) = f[(T/2) + t]$, no even harmonics if $f(t) = -f((T/2) + t)$
Effective (rms) value
$V_{(rms)} = \sqrt{V_0^2 + (V_{m_1}^2 + \cdots + V'^2_{m_n} + V'^2_{m_1} + \cdots + V'^2_{m_n})/2}$
Power $P_T = V_0 I_0 + V_1 I_1 \cos\theta + \ldots + V_n I_n \cos\theta_n = I_{rms}^2 R = V_{rms}^2/R$

Standard Resistor Values

Ohms (Ω)					Kilohms (kΩ)		Megohms (MΩ)	
0.10	**1.0**	**10**	**100**	**1000**	**10**	**100**	**1.0**	**10.0**
0.11	1.1	11	110	1100	11	110	1.1	11.0
0.12	**1.2**	**12**	**120**	**1200**	**12**	**120**	**1.2**	**12.0**
0.13	1.3	13	130	1300	13	130	1.3	13.0
0.15	**1.5**	**15**	**150**	**1500**	**15**	**150**	**1.5**	**15.0**
0.16	1.6	16	160	1600	16	160	1.6	16.0
0.18	**1.8**	**18**	**180**	**1800**	**18**	**180**	**1.8**	**18.0**
0.20	2.0	20	200	2000	20	200	2.0	20.0
0.22	**2.2**	**22**	**220**	**2200**	**22**	**220**	**2.2**	**22.0**
0.24	2.4	24	240	2400	24	240	2.4	
0.27	**2.7**	**27**	**270**	**2700**	**27**	**270**	**2.7**	
0.30	3.0	30	300	3000	30	300	3.0	
0.33	**3.3**	**33**	**330**	**3300**	**33**	**330**	**3.3**	
0.36	3.6	36	360	3600	36	360	3.6	
0.39	**3.9**	**39**	**390**	**3900**	**39**	**390**	**3.9**	
0.43	4.3	43	430	4300	43	430	4.3	
0.47	**4.7**	**47**	**470**	**4700**	**47**	**470**	**4.7**	
0.51	5.1	51	510	5100	51	510	5.1	
0.56	**5.6**	**56**	**560**	**5600**	**56**	**560**	**5.6**	
0.62	6.2	62	620	6200	62	620	6.2	
0.68	**6.8**	**68**	**680**	**6800**	**68**	**680**	**6.8**	
0.75	7.5	75	750	7500	75	750	7.5	
0.82	**8.2**	**82**	**820**	**8200**	**82**	**820**	**8.2**	
0.91	9.1	91	910	9100	91	910	9.1	

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dc

Introduction

Conversions 1 meter = 100 cm = 39.37 in., 1 in. = 2.54 cm, 1 yd = 0.914 m = 3 ft, 1 mile = 5280 ft, °F = 9/5°C + 32, °C = 5/9(°F − 32), K = 273.15 + °C **Scientific notation** 10^{12} = tera = T, 10^9 = giga = G, 10^6 = mega = M, 10^3 = kilo = k, 10^{-3} = milli = m, 10^{-6} = micro = μ, 10^{-9} = nano = n, 10^{-12} = pico = p **Powers of ten** $1/10^n = 10^{-n}$, $1/10^{-n} = 10^n$, $(10^n)(10^m) = 10^{n+m}$, $10^n/10^m = 10^{n-m}$, $(10^n)^m = 10^{nm}$

Voltage and Current

Coulomb's law $F = kQ_1Q_2/r^2$, $k = 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$, Q = coulombs (C), r = meters (m) **Current** $I = Q/t$ (amperes), t = seconds (s), $Q_e = 1.6 \times 10^{-19}$ C **Voltage** $V = W/Q$ (volts), W = joules (J)

Resistance

Circular wire $R = \rho l/A$ (ohms), ρ = resistivity, l = feet, $A_{CM} = (d_{mils})^2$, $\rho(\mathrm{Cu}) = 10.37$ **Metric units** l = cm, A = cm^2, $\rho(\mathrm{Cu}) = 1.724 \times 10^{-6}$ ohm-cm **Temperature** $(|T_i| + T_1)/R_1 = (|T_i| + T_2)/R_2$, $R_1 = R_{20}[1 + \alpha_{20}(T_1 - 20°\mathrm{C})]$, $\alpha_{20}(\mathrm{Cu}) = 0.00393$ **Color code** Bands 1–3: 0 = black, 1 = brown, 2 = red, 3 = orange, 4 = yellow, 5 = green, 6 = blue, 7 = violet, 8 = gray, 9 = white, Band 3: 0.1 = gold, 0.01 = silver, Band 4: 5% = gold, 10% = silver, 20% = no band, Band 5: 1% = brown, 0.1% = red, 0.01% = orange, 0.001% = yellow **Conductance** $G = 1/R$ siemens (S)

Ohm's Law, Power, and Energy

Ohm's law $I = E/R$, $E = IR$, $R = E/I$ **Power** $P = W/t = VI = I^2R = V^2/R$ (watts), 1 hp = 746 W
Efficiency $\eta\% = (P_o/P_i) \times 100\%$, $\eta_T = \eta_1 \cdot \eta_2 \cdot \eta_3 \cdot \cdots \cdot \eta_n$
Energy $W = Pt$, $W\,(\mathrm{kWh}) = [P(\mathrm{W}) \cdot t(\mathrm{h})]/1000$

Series Circuits

$R_T = R_1 + R_2 + R_3 + \cdots + R_N$, $R_T = NR$, $I = E/R_T$, $V = IR$
Kirchhoff's voltage law $\Sigma_C V = 0$, $\Sigma_C V_{rises} = \Sigma_C V_{drops}$
Voltage divider rule $V_x = R_xE/R_T$

Parallel dc Circuits

$R_T = 1/(1/R_1 + 1/R_2 + 1/R_3 + \cdots + 1/R_N)$, $R_T = R/N$, $R_T = R_1R_2/(R_1 + R_2)$, $I = EG_T = E/R_T$
Kirchhoff's current law $\Sigma I_{entering} = \Sigma I_{leaving}$
Current divider rule $I_x = (R_T/R_x)I$, (Two parallel elements): $I_1 = R_2I/(R_1 + R_2)$, $I_2 = R_1I/(R_1 + R_2)$

Series-Parallel Circuits

Potentiometer loading $R_L >> R_T$
Ammeter $R_{shunt} = R_mI_{CS}/(I_{max} - I_{CS})$
Voltmeter $R_{series} = (V_{max} - V_{VS})/I_{CS}$
Ohmmeter $R_s = (E/I_{CS}) - R_m$ − zero-adjust/2

Methods of Analysis and Selected Topics (dc)

Source conversions $E = IR_p$, $R_s = R_p$, $I = E/R_s$

Determinants $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$

Bridge networks $R_1/R_3 = R_2/R_4$ **Δ-Y conversions** $R' = R_A + R_B + R_C$, $R_3 = R_AR_B/R'$, $R_2 = R_AR_C/R'$, $R_1 = R_BR_C/R'$, $R_Y = R_\Delta/3$
Y-Δ conversions $R'' = R_1R_2 + R_1R_3 + R_2R_3$, $R_C = R''/R_3$, $R_B = R''/R_2$, $R_A = R''/R_1$, $R_\Delta = 3R_Y$

Network Theorems

Superposition Voltage sources (short-circuit equivalent), current sources (open-circuit equivalent)
Thévenin's Theorem R_{Th}: (all sources to zero), E_{Th}: (open-circuit terminal voltage)
Maximum power transfer theorem $R_L = R_{Th} = R_N$, $P_{max} = E^2_{Th}/4R_{Th} = I^2_NR_N/4$

Capacitors

Capacitance $C = Q/V = \epsilon A/d = 8.85 \times 10^{-12}\epsilon_r A/d$ farads (F), $C = \epsilon_r C_o$ **Electric field strength** $\mathscr{E} = V/d = Q/\epsilon A$ (volts/meter) **Transients** (charging) $i_C = (E/R)e^{-t/\tau}$, $\tau = RC$, $v_C = E(1 - e^{-t/\tau})$, (discharge) $v_C = Ee^{-t/\tau}$, $i_C = (E/R)e^{-t/RC}$ i_C $i_{C_{av}} = C(\Delta v_C/\Delta t)$
Series $Q_T = Q_1 = Q_2 = Q_3$, $1/C_T = (1/C_1) + (1/C_2) + (1/C_3) + \cdots + (1/C_N)$, $C_T = C_1C_2/(C_1 + C_2)$ **Parallel** $Q_T = Q_1 + Q_2 + Q_3$, $C_T = C_1 + C_2 + C_3$ **Energy** $W_C = (1/2)CV^2$

Inductors

Self-inductance $L = N^2\mu A/l$ (henries), $L = \mu_r L_o$
Induced voltage $e_{L_{av}} = L(\Delta i/\Delta t)$ **Transients** (storage) $i_L = I_m(1 - e^{-t/\tau})$, $I_m = E/R$, $\tau = L/R$, $v_L = Ee^{-t/\tau}$ (decay), $v_L = [1 + (R_2/R_1)]Ee^{-t/\tau'}$, $\tau' = L/(R_1 + R_2)$, $i_L = I_me^{-t/\tau'}$, $I_m = E/R_1$
Series $L_T = L_1 + L_2 + L_3 + \cdots + L_N$ **Parallel** $1/L_T = (1/L_1) + (1/L_2) + (1/L_3) + \cdots + (1/L_N)$, $L_T = L_1L_2/(L_1 + L_2)$
Energy $W_L = 1/2(LI^2)$

Magnetic Circuits

Flux density $B = \Phi/A$ (webers/m^2) **Permeability** $\mu = \mu_r\mu_o$ (Wb/A · m)
Reluctance $\mathscr{R} = l/\mu A$ (rels) **Ohm's law** $\Phi = \mathscr{F}/\mathscr{R}$ (webers)
Magnetomotive force $\mathscr{F} = NI$ (ampere-turns) **Magnetizing force** $H = \mathscr{F}/l = NI/l$ **Ampère's circuital law** $\Sigma_C \mathscr{F} = 0$
Flux $\Sigma\,\Phi_{entering} = \Sigma\,\Phi_{leaving}$ **Air gap** $H_g = 7.96 \times 10^5\,B_g$

Greek Alphabet

Letter	Capital	Lowercase	Letter	Capital	Lowercase
Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π
Epsilon	E	ϵ	Rho	P	ρ
Zeta	Z	ζ	Sigma	Σ	σ
Eta	H	η	Tau	T	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

Prefixes

Multiplication Factors	SI Prefix	SI Symbol
1 000 000 000 000 000 000 = 10^{18}	exa	E
1 000 000 000 000 000 = 10^{15}	peta	P
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a