Average Power, Power factor, Complex Number Intro

- Average Power and Power Factor
  - Intro and background
  - Instantaneous vs average power
  - Average power delivered (R,L,C)
  - Power factor definition
  - **ICP – Average power and power factor**
- Lab #2 – Capacitor Charge and Discharge
  - Overview and prelab discussion
- Complex Numbers
  - Introduction and forms
  - R->P, P-R conversions
  - Math with complex numbers

Power Delivered to a Load by A Sinusoidal Forcing Function

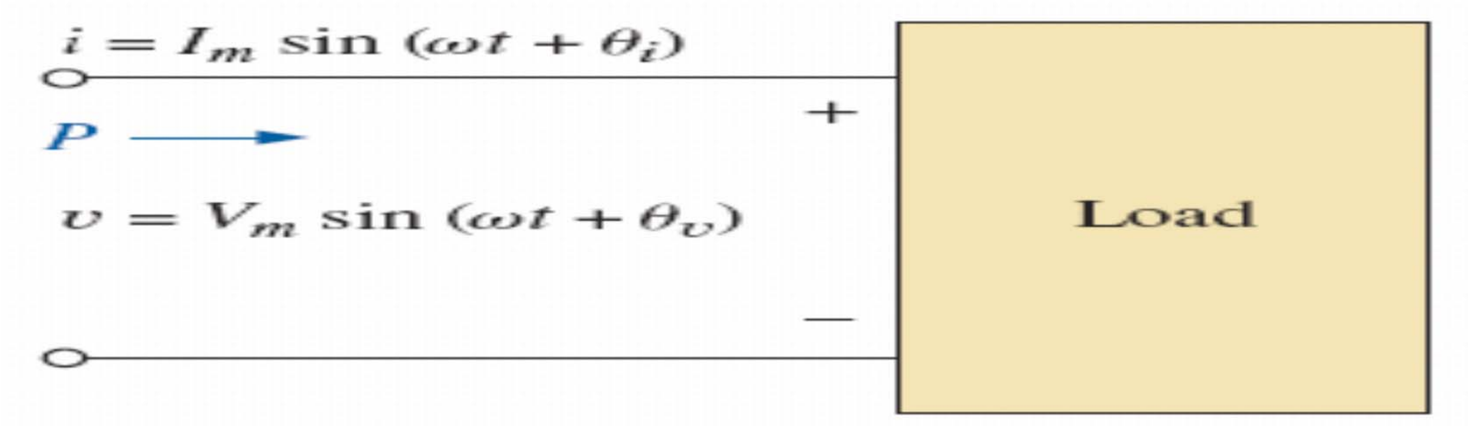


FIG. 14.28 *Determining the power delivered in a sinusoidal ac network.*

$$p(t) = v(t)\, i(t) = V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$$

But: $\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$

$$\therefore\ p(t) = \frac{V_m I_m}{2}\left[\cos(\omega t + \theta_v - \omega t - \theta_i) - \cos(\omega t + \theta_v + \omega t + \theta_i)\right]$$

$$= \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)\right]$$

Power Delivered to a Load by A Sinusoidal Forcing Function

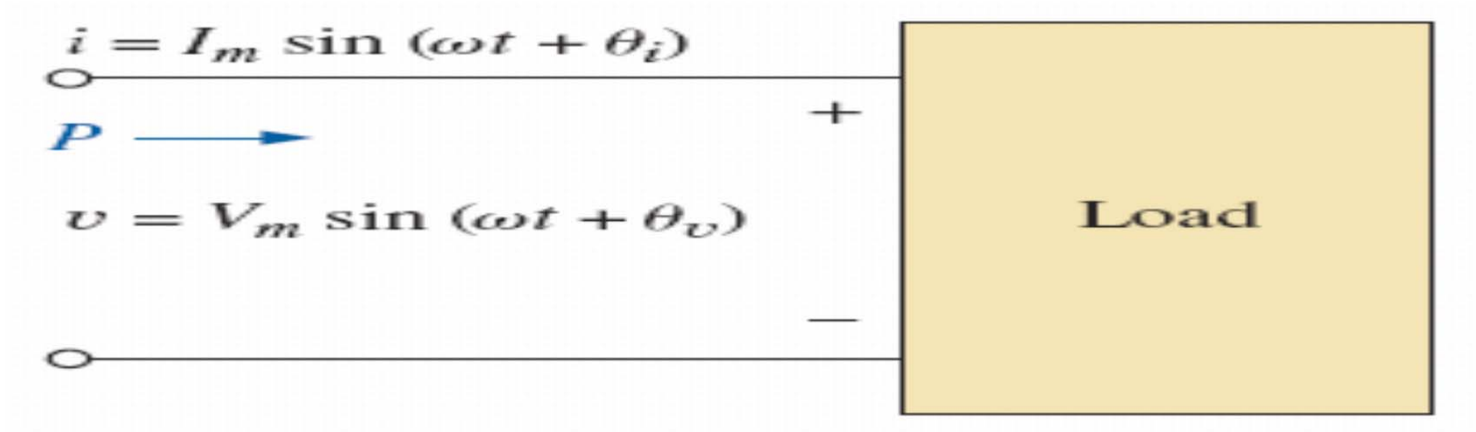


FIG. 14.28 *Determining the power delivered in a sinusoidal ac network.*

$$= \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)\right]$$

OR

$$\boxed{p(t) = \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}_{\text{FIXED}} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)}_{\text{TIME VARYING}}}$$

Power Delivered to a Load by A Sinusoidal Forcing Function

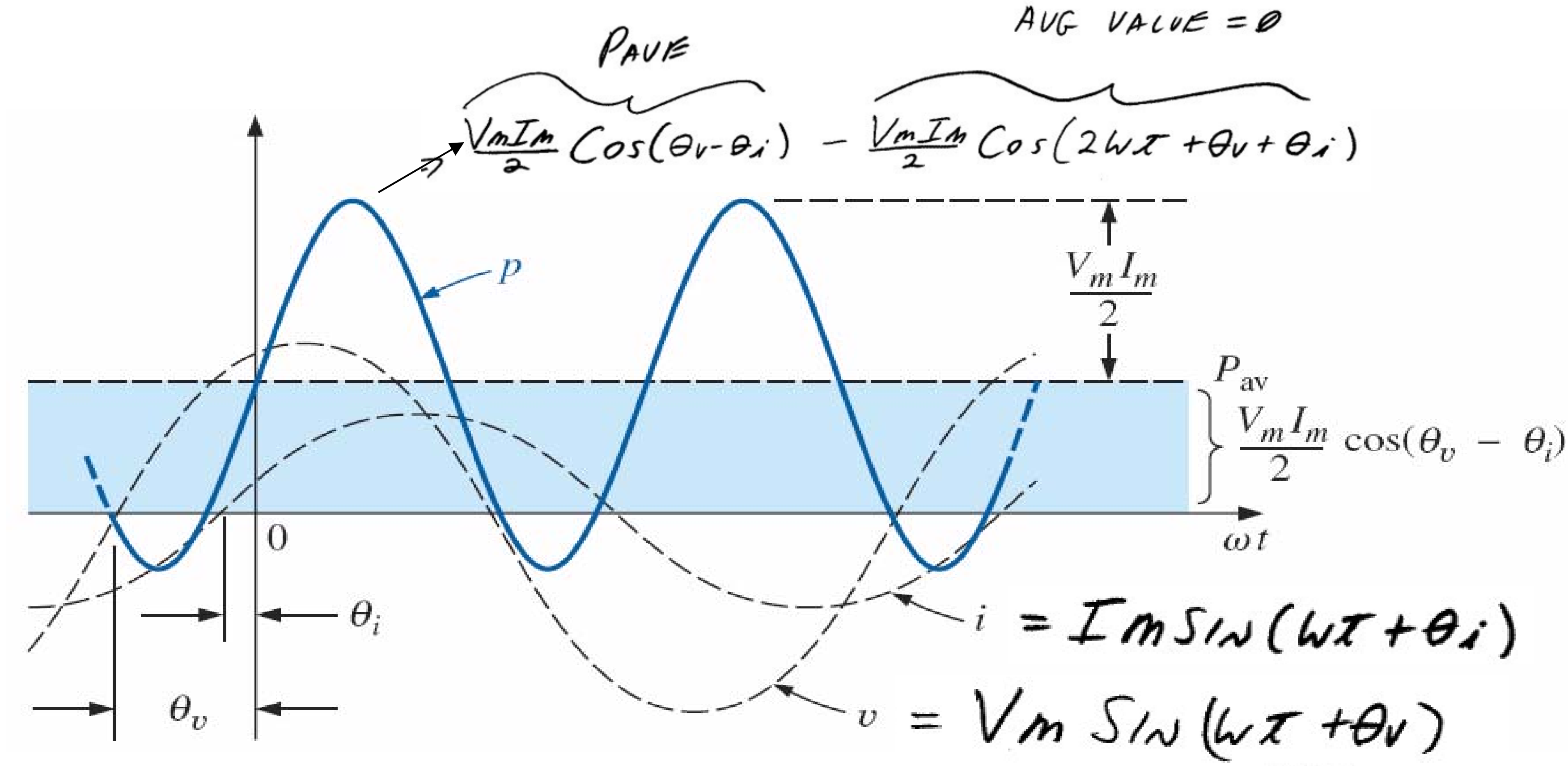


FIG. 14.29 *Defining the average power for a sinusoidal ac network.*

$$P_{AVE} = \frac{V_m I_m}{2} \cos(\theta) \ \text{WATTS}$$

$$\theta = |\theta_v - \theta_i|$$

Average Power for R,L,C

FOR VARIOUS LOADS $\therefore$ $P_{AVE} = \frac{V_m I_m}{2} \cos(\theta)$

$\theta = |\theta_v - \theta_i|$

RESISTOR

$|\theta_v - \theta_i| = 0$, SINCE $v(t)$ + $i(t)$ ARE IN PHASE

$$\boxed{P_{AVE} = \frac{V_m I_m}{2}}$$

BUT $V_m = V_{RMS} \cdot \sqrt{2}$

+ $I_m = I_{RMS} \cdot \sqrt{2}$

$$\therefore \boxed{P_{AVE} = V_{RMS} I_{RMS}}$$

OR RECALL : $I_{RMS} = \frac{V_{RMS}}{R}$

$$\therefore \boxed{P_{AVE} = \frac{V_{RMS}^2}{R} = I_{RMS}^2 R}$$

INDUCTOR

$|\theta_v - \theta_i| = 90°$ $\therefore$ $P_{AVE} = \frac{V_m I_m}{2} \underbrace{\cos(90°)}_{0}$

OR $\boxed{P_{AVE} = 0W}$

CAPACITOR

$|\theta_v - \theta_i| = 90°$ $\therefore$ $\boxed{P_{AVE} = 0W}$

* THE AVE POWER DISSIPATED BY AN IDEAL INDUCTOR OR IDEAL CAPACITOR IS 0 WATTS

POWER FACTOR

POWER FACTOR $\equiv \boxed{F_p \equiv \cos(\theta)}$

WHERE $\theta = |\theta_v - \theta_i|$

RECALL: $P_{AVE} = V_{RMS} I_{RMS} \cos(\theta)$

$$\therefore \boxed{\cos(\theta) = F_p = \frac{P_{AVE}}{V_{RMS} I_{RMS}}}$$

F_p: 0 → PURELY REACTIVE LOAD, NO POWER DELIVERED

1 → PURELY RESISTIVE LOAD, MAX POWER DELIVERED

WE LOOK AT THE CURRENT THROUGH THE LOAD:

→ IF $i(t)$ LEADS $v(t)$, LEADING POWER FACTOR

→ IF $i(t)$ LAGS $v(t)$, LAGGING POWER FACTOR

CAPACITIVE

INDUCTIVE

ICPs – Pave and Power Factor

A CIRCUIT DISSIPATES 100W (PAVE) AT 150V (VEFF) + 2A (IEFF).

(1) FIND: THE POWER FACTOR

(2) IS THE LOAD RESISTIVE, REACTIVE OR BOTH? FIND $\theta = |\theta_v - \theta_i|$ IN DEGREES

(3) CAN WE TELL IF THE LOAD IS INDUCTIVE OR CAPACITIVE IN NATURE WITH THE GIVEN INFO?

Lab #2 – Capacitor Charge and Discharge

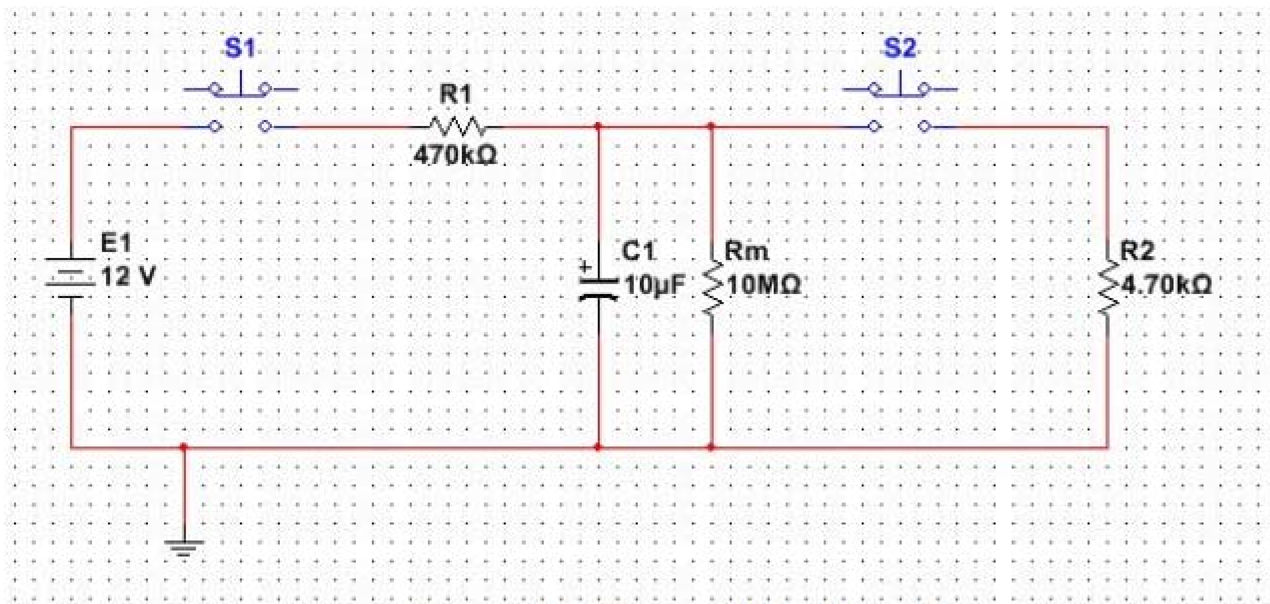
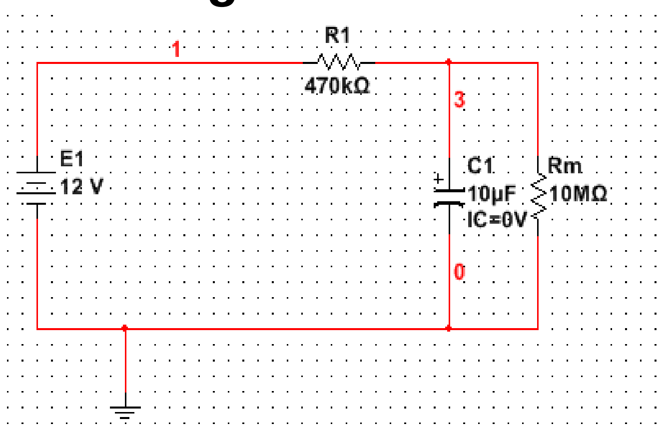
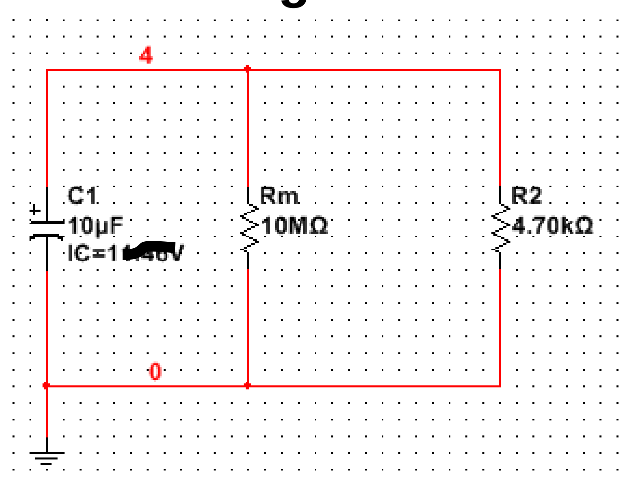


Figure 1 - RC Charge and Discharge Circuit

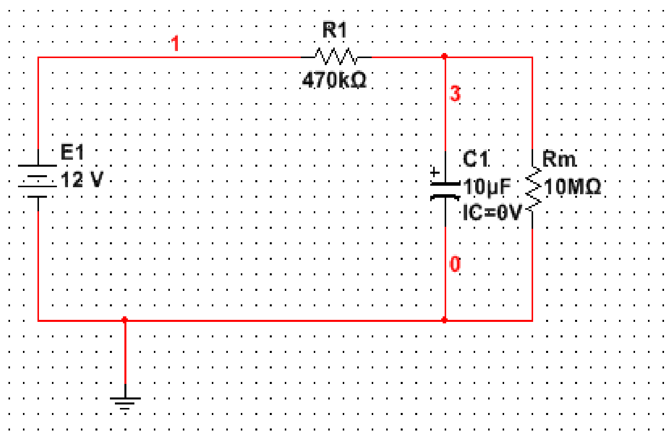
Charge Simulation



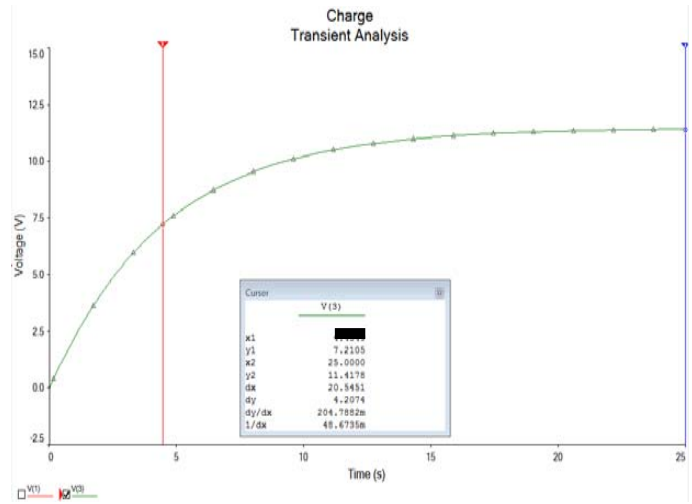
Discharge Simulation



Lab #2 – Capacitor Charge and Discharge

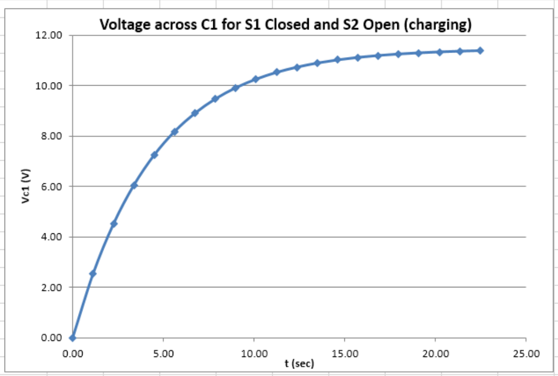


Charge Simulation

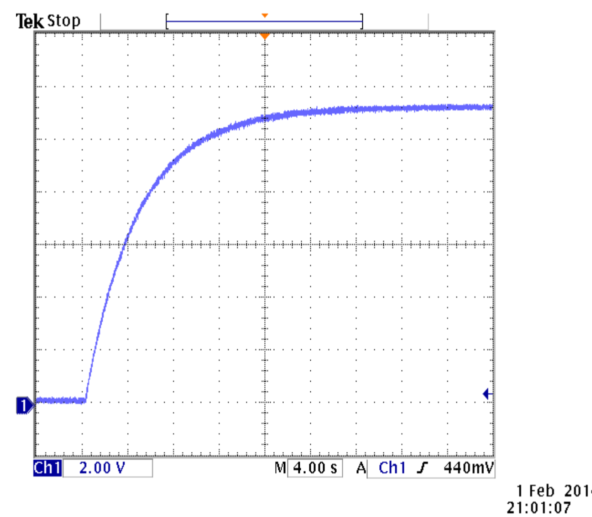


Charge in Excel

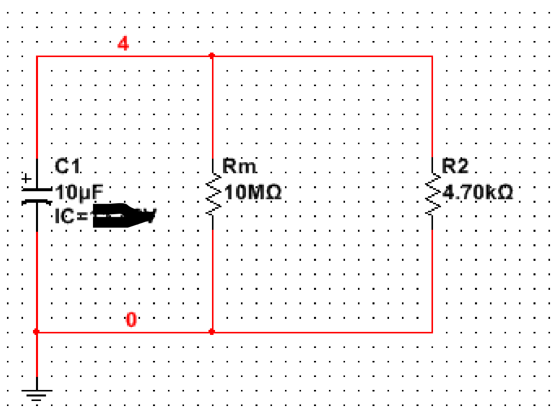
Charge Phase			
Vth (V)	[redacted]	t(sec)	Vc1(V)
Tau (sec)	[redacted]	0.00	0.00
Step (5*Tau/20)	1.1225	1.12	2.53
		2.25	4.51
		3.37	6.05
		4.49	7.24
		5.61	8.18
		6.74	8.90
		7.86	9.47
		8.98	9.91
		10.10	10.25
		11.23	10.52
		12.35	10.73
		13.47	10.89
		14.59	11.02
		15.72	11.11
		16.84	11.19
		17.96	11.25
		19.08	11.30
		20.21	11.33
		21.33	11.36
		22.45	11.38



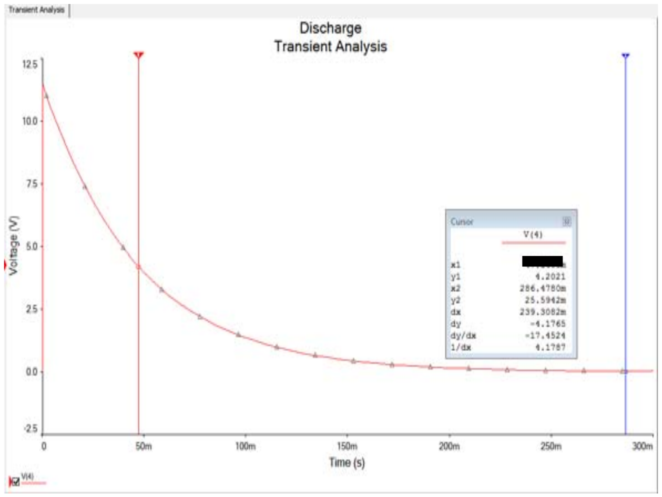
Charge in Lab



Lab #2 – Capacitor Charge and Discharge

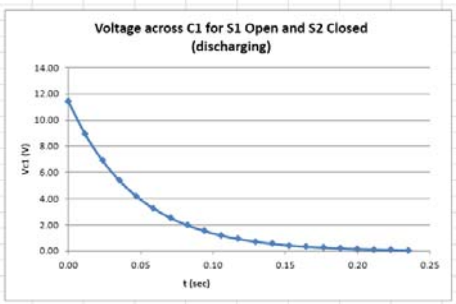


Discharge in Simulation

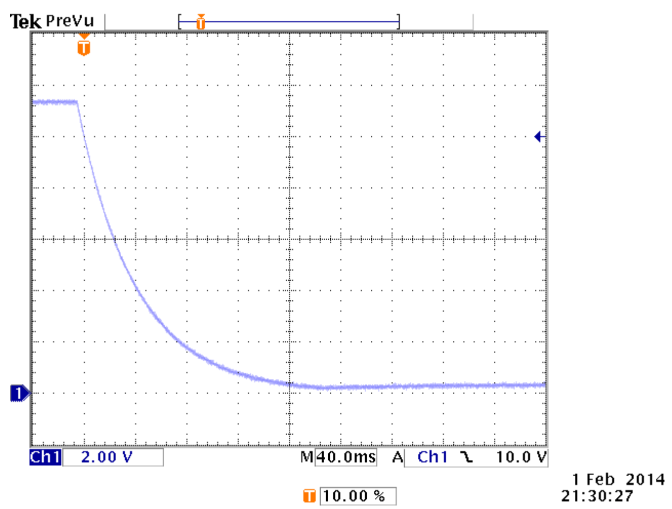


Discharge in Excel

Discharge Phase			
Vth (V)	[redacted]	t(sec)	Vc1(V)
Tau (sec)	[redacted]	0.00	11.46
Step (5*Tau/20)	11.7E-3	0.01	8.93
		0.02	6.95
		0.04	5.41
		0.05	4.22
		0.06	3.28
		0.07	2.56
		0.08	1.99
		0.09	1.55
		0.11	1.21
		0.12	0.94
		0.13	0.73
		0.14	0.57
		0.15	0.44
		0.16	0.35
		0.18	0.27
		0.19	0.21
		0.20	0.16
		0.21	0.13
		0.22	0.10
		0.23	0.08



Discharge in Lab



Complex Numbers – Definition and Intro

DEFINE: $j = +\sqrt{-1}$

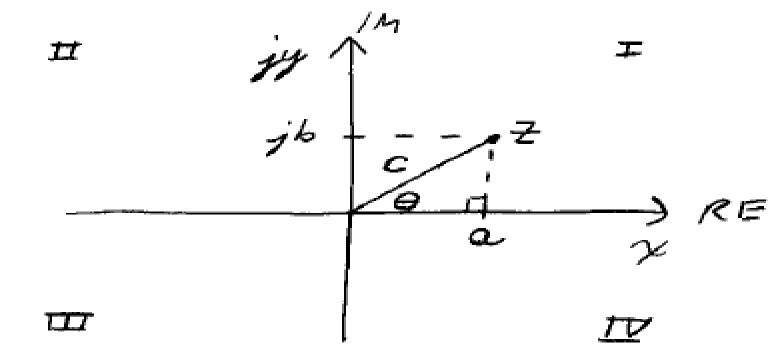
$\therefore \; j^2 = -1$

$j^3 = -j$

$j^4 = 1$

$j^5 = j$

COMPLEX NUMBER PLANE



POSITIVE ∡ MADE WITH THE RE AXIS (COUNTERCLOCKWISE)

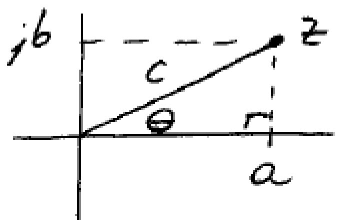
$z = x + jy = C\angle\theta$

RECTANGULAR FORM

POLAR FORM

1.2 Polar / Rectangular Conversions

QUADRANT I



R → P

$z = a + jb$, a & b ARE POSITIVE

$C = \sqrt{a^2 + b^2}$, MAGNITUDE

$\tan(\theta) = o/adj = b/a$

$\therefore \; \theta = \tan^{-1}(b/a)$, ANGLE

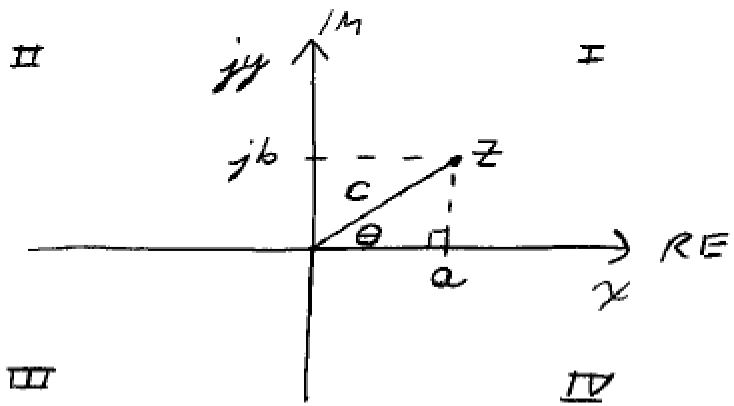
$z = C\angle\theta$

Complex Numbers – Definition and Intro

DEFINE: $j = +\sqrt{-1}$

$$\therefore \quad j^2 = -1$$
$$j^3 = -j$$
$$j^4 = 1$$
$$j^5 = j$$

COMPLEX NUMBER PLANE



POSITIVE ∠ MADE WITH THE RE AXIS (COUNTERCLOCKWISE)

$$Z = x + jy = C \angle \theta$$

RECTANGULAR FORM

POLAR FORM

P → R

$$Z = C \angle \theta$$

$$\cos(\theta) = \frac{ADJ}{hyp} = \frac{a}{C}$$

$$\therefore \quad \underline{a = C \cos\theta}$$

$$\sin(\theta) = \frac{opp}{hyp} = \frac{b}{c}$$

$$\therefore \quad \underline{b = C \sin\theta}$$

$\therefore$ HAVE $\underline{\underline{Z = a + jb}}$

Complex Numbers – On the Sharp EL-516

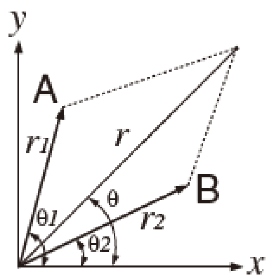
Mathematical operations (rectangular mode)

25 [MODE] **(CPLX)**

$(12 - 6i) + (7 + 15i) - (11 + 4i) =$	[MODE] [3] 12 [−] 6 [i] [+] 7 [+] 15 [i] [−] [(] 11 [+] 4 [i] [)] [=]	8. +5.i
$6 \times (7 - 9i) \times (-5 + 8i) =$	6 [×] [(] 7 [−] 9 [i] [)] [×] [(] [(−)] 5 [+] 8 [i] [)] [=]	222. +606.i

Complex Numbers – On the Sharp EL-516

Mathematical operations (polar form – think VECTORS)



$r1 = 8$, $\theta 1 = 70°$ $r2 = 12$, $\theta 2 = 25°$ $\rightarrow r = ?$, $\theta = ?°$	[2ndF] [→rθ] 8 [∠] 70 [+] 12 [∠] 25 [=]	18.5408873 ∠42.76427608
$1 + i$ $\rightarrow r = ?$, $\theta = ?°$	[2ndF] [→xy] 1 [+] [i] [=]	1. +1.i
	[2ndF] [→rθ]	1.414213562 ∠45.

R->P Conversion

See our text for more examples, addition/subtraction are is easier in rectangular form and multiplication/division is easier in polar form