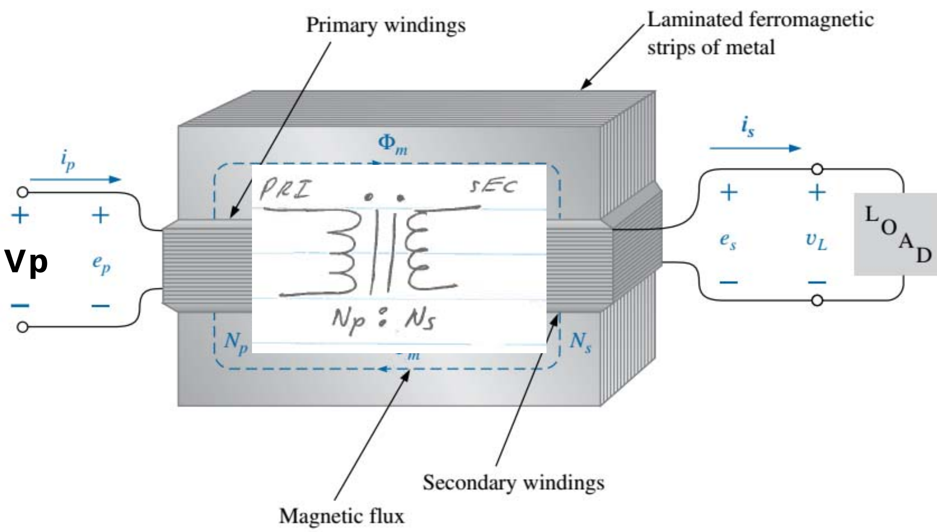
Transformers – Impedance Matching and Simulation

- Impedance Transformation
  - Analysis
  - Example
  - Pmax
- Transformer Simulation
  - Turns ratio and model
  - Interpretation and Analysis

Transformers – Impedance Transformation

Consider the following transformer
(ideal model, k=1):



Recall:

$$\frac{\vec{V}_p}{\vec{V}_s} = \frac{N_p}{N_s} = a \qquad \textbf{(1)}$$

$$\frac{\vec{I}_s}{\vec{I}_p} = \frac{N_p}{N_s} = a \qquad \textbf{(2)}$$

(1) MULTIPLIED BY (2)

$$\frac{\vec{V}_p}{\vec{V}_s} \cdot \frac{\vec{I}_s}{\vec{I}_p} = a^2$$

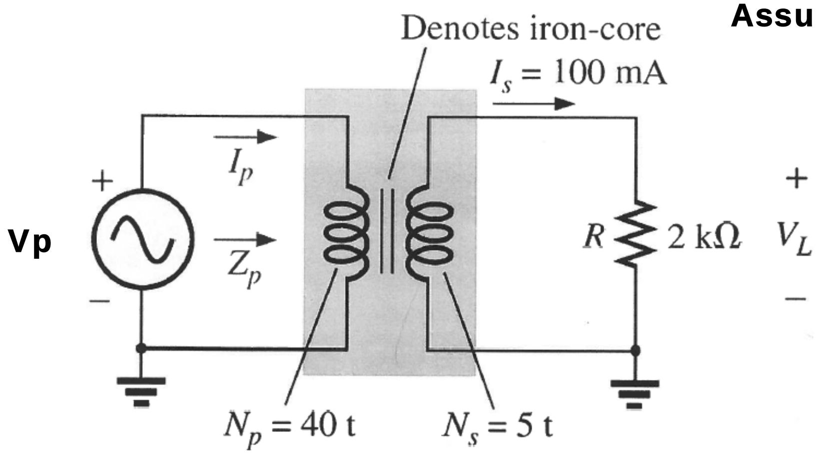
$$\therefore \frac{\vec{V}_p}{\vec{I}_p} = a^2 \frac{\vec{V}_s}{\vec{I}_s}$$

OR $\boxed{\vec{Z}_p = a^2 \vec{Z}_L}$

REFLECTED IMPEDANCE

Transformers – Impedance Transformation (example)

Assume voltages and currents in RMS unless otherwise noted



Find: **|Ip|**, **|Vg|**, **Zp**

$a = 40/5 = \underline{8}$

$I_p = I_s / a = 100\,mA / 8$

$= \boxed{12.5\,mA}$

$V_L = I_s \cdot R = (100\,mA)(2k\Omega)$

$= \underline{200V}$

$V_p = (8)(V_s) = (8)(200V)$

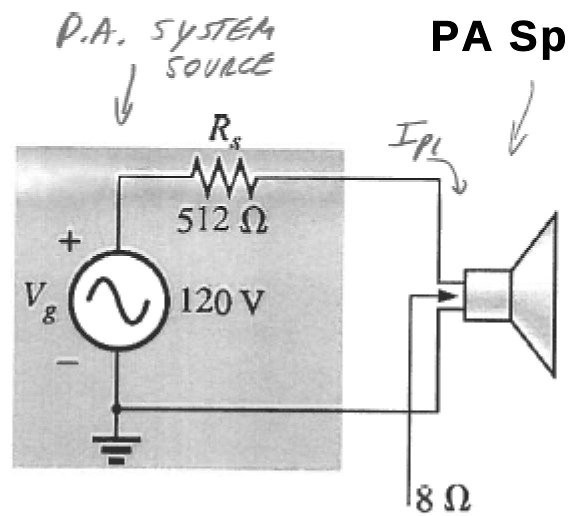
$= \boxed{1600V}$

$\vec{Z}_p = a^2 \vec{Z}_L$

$Z_p = 64(2k\Omega) = \boxed{128\,k\Omega}$

Transformers – Impedance matching for Pmax (example)

PA Speaker

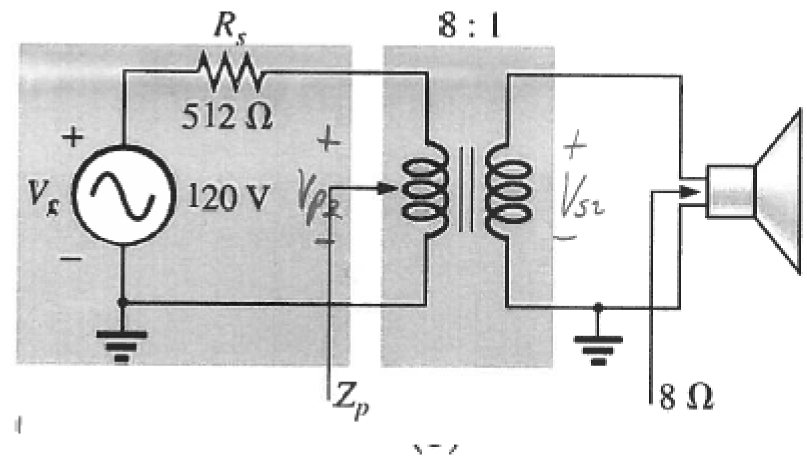


$$P_{8\Omega} = (I_{P1})^2(8\Omega)$$

$$I_{P1} = \frac{120V}{520\Omega} = \underline{231mA}$$

$$\therefore \boxed{P_{8\Omega} = 426mW}$$

Only 426mW into the speaker, 27.3W “lost” in Rs



$$Z_p = 8^2(8\Omega) = \underline{512\Omega}$$

$$\therefore V_{P2} = 120V\left(\frac{512\Omega}{1024\Omega}\right) = \underline{60V}$$

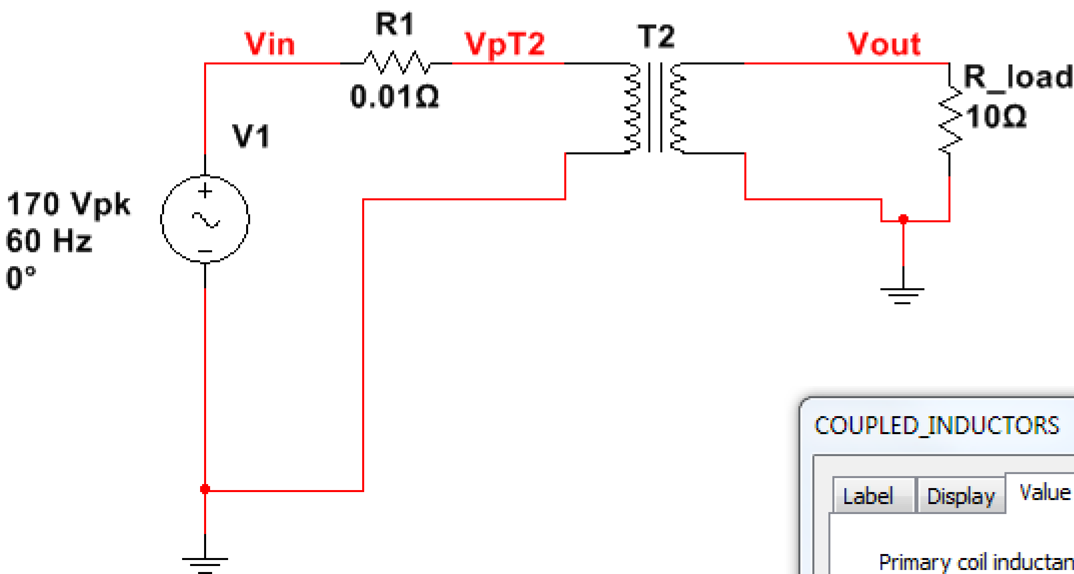
$$\therefore P_{IN} = (60V)^2/512\Omega = \boxed{7.03W}$$

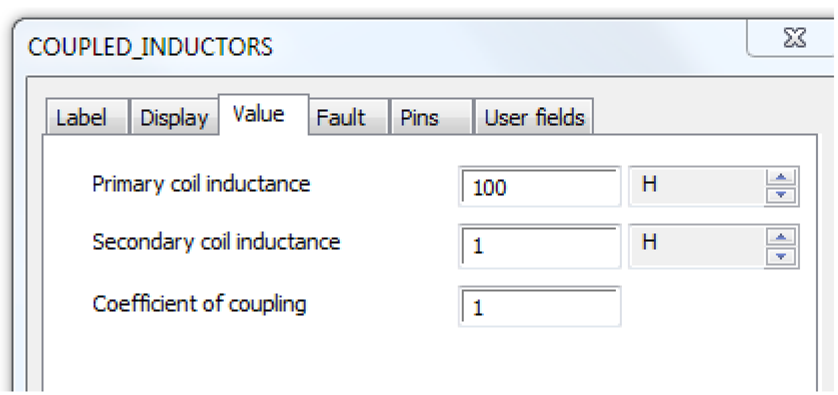
CHECK: $V_{S2} = 60V/8 = \underline{7.5V}$

$$\therefore P_{8\Omega} = \frac{(7.5V)^2}{8\Omega} = \underline{7.03W}$$

½ of the 14.06W available to the speaker ☺

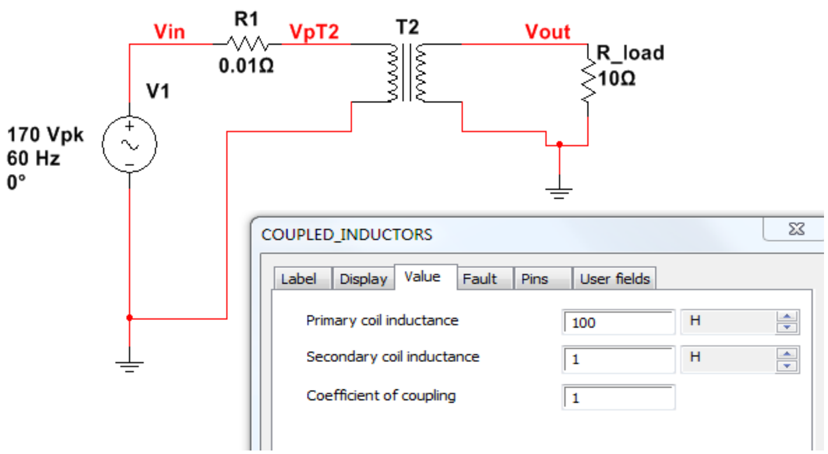
Transformers - Simulation





What is the turns ratio?

Transformers – Simulation (turns ratio)



What is the turns ratio?

Recall:

$$L = N^2\left(\frac{\mu A}{\ell}\right)$$

Therefore:

$$\frac{L_p}{L_s} = \frac{N_p^2\left(\frac{\mu A}{\ell}\right)}{N_s^2\left(\frac{\mu A}{\ell}\right)}$$

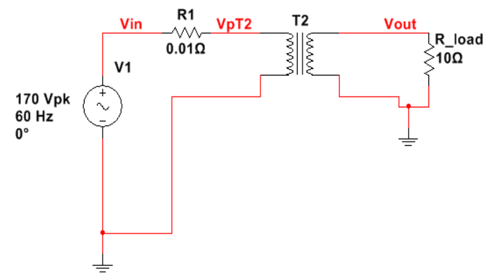
or $\frac{L_p}{L_s} = \left(\frac{N_p}{N_s}\right)^2$

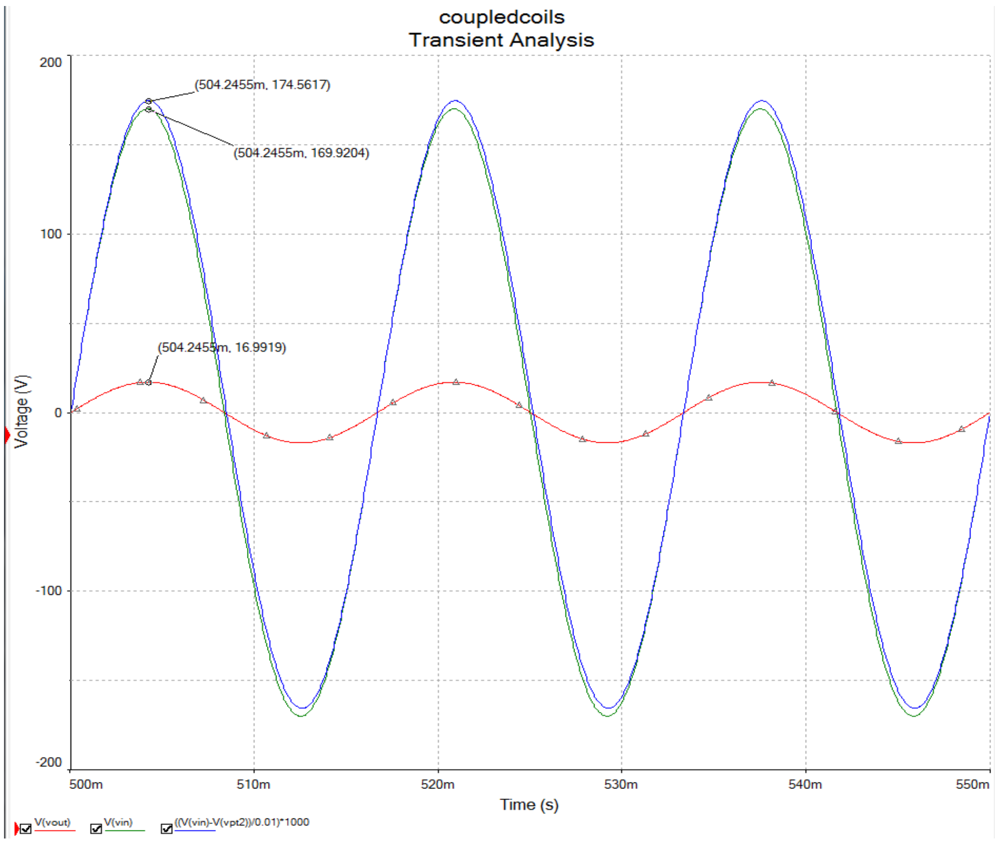
Which yields:

$$\therefore \frac{N_p}{N_s} = \sqrt{L_p/L_s}$$

$$= \boxed{10}$$

Transformers – Turns Ratio (interpretation/analysis)





Notes: Voltages and currents all in-phase, R1 -> Sample resistor and for simulation

Vin = 170Vpk ~ 120VRMS

Vout = 17Vpk ~ 12VRMS

So Np/Ns = Vin/Vout = 10

Iin = 174.6pk/1000= 174.6mApk ~ 123mARMS

Iout = Vout/ 10 Ohms = 1.2ARMS

Pin = Vin*Iin = 14.8W

Pout = Vout*Iout = 14.4W

Zin_exp = a^2*ZL = 1000 Ohms

Zin_sim = Vin/Iin = 976 Ohms

Minor errors due to winding inductance here (1H -> j377 Ohms at 60 Hz)