Series Resonance

- Objectives
- Introduction
- Mechanical Example
- Series Circuit Resonance
  - Circuit properties
  - Quality factor
  - **Z_T**
  - Selectivity (bandwidth)
- In Class Problem
  - Similar to problems 1 and 3 (HW)

Series Resonance - Objectives

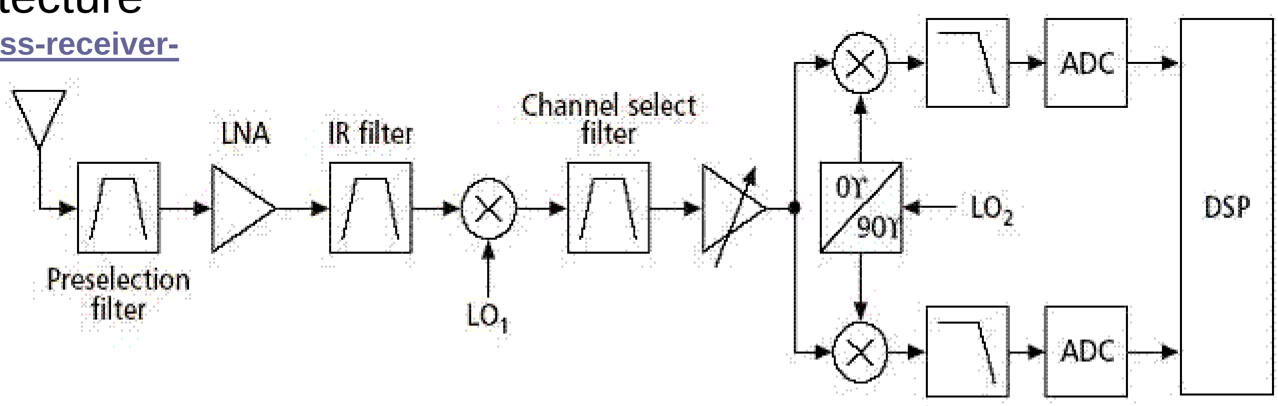
- Become familiar with the frequency response of a series resonant circuit and how to calculate the resonant frequency (<u>f_s</u>) and cutoff frequencies (<u>f_1, f_2</u>).
- Be able to calculate a tuned network's quality factor (<u>Q</u>), bandwidth (<u>BW</u>), and power levels at important frequency levels.
- Understand the impact of the quality factor on the frequency response of a series or parallel resonant network.

<u>Some new vocabulary today</u>

Series Resonance - Introduction

- Resonant (or tuned) circuits are fundamental to the operation of a wide variety of electrical and electronic systems in use today.
- The resonant circuit is a combination of R, L, and C elements having a frequency response characteristic.
- A resonant electrical circuit must have both inductance and capacitance. In addition, resistance will always be present due either to the lack of ideal elements or to control the shape of the resonance curve.
- When resonance occurs due to the application of the proper frequency (fr), the energy absorbed by one reactive element is the same as that released by another reactive element within the system.

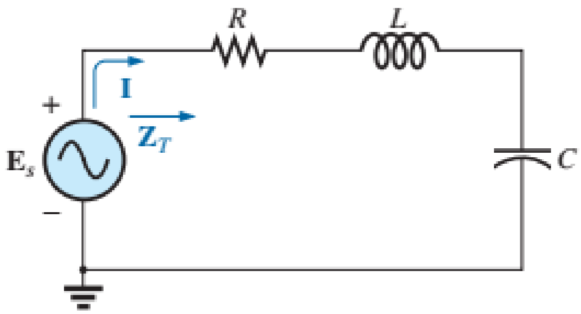
One example receiver architecture
(from http://analog.intgckts.com/wireless-receiver-architectures/)



Resonance – Mechanical Example (from the text)

https://youtu.be/nFzu6CNtqec

Series Resonance – Circuit Properties at Resonance



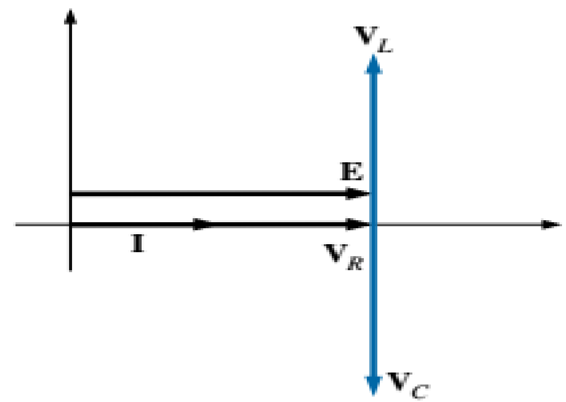


FIG. 21.4
Phasor diagram for the series resonant circuit at resonance.

The resonant frequency (w for XL = XC) is:

$$\boxed{\omega_s = \frac{1}{\sqrt{LC}}}$$ r/s

$$\boxed{f_s = \frac{1}{2\pi\sqrt{LC}}}$$

f = hertz (Hz)
L = henries (H)
C = farads (F)

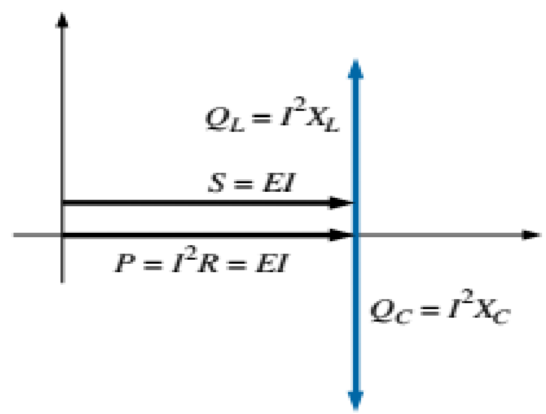


FIG. 21.5
Power triangle for the series resonant circuit at resonance.

<u>At Resonance (f = fs)</u>

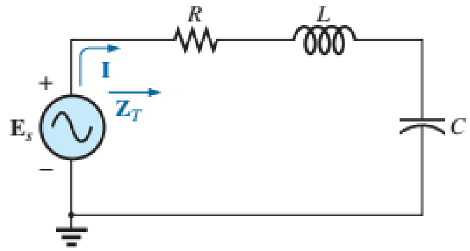
$$\boxed{\mathbf{Z}_{T_s} = R}$$

$$\boxed{F_{p_s} = 1}$$

$$\mathbf{I} = \frac{E \angle 0^\circ}{R \angle 0^\circ} = \frac{E}{R} \angle 0^\circ$$

$$\left.\begin{aligned} \mathbf{V}_L &= (I \angle 0^\circ)(X_L \angle 90^\circ) = IX_L \angle 90^\circ \\ \mathbf{V}_C &= (I \angle 0^\circ)(X_C \angle -90^\circ) = IX_C \angle -90^\circ \end{aligned}\right\} 180^\circ \text{ out of phase}$$

Series Resonance –Quality Factor (Q)



$$\boxed{Q_s = \frac{\text{reactive power}}{\text{average power}}}$$

Or (since XL and XC are equal at fs)

$$Q_s = \frac{I^2X_L}{I^2R}$$

Hence: $$\boxed{Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R}}$$

For a coil (with a series resistance of Rl):

$$\boxed{Q_s = Q_{\text{coil}} = Q_l = \frac{X_L}{R_l}} \qquad R = R_l$$

At higher frequencies Q drops off due to the skin effect (see chapter 20) and winding capacitance

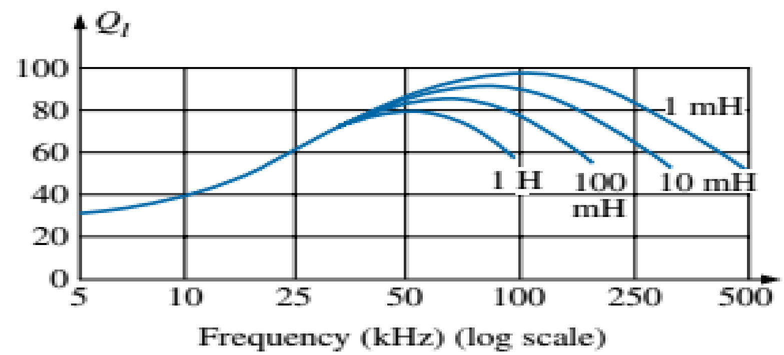


FIG. 21.7

The inductors used in Lab Project #2:

Features

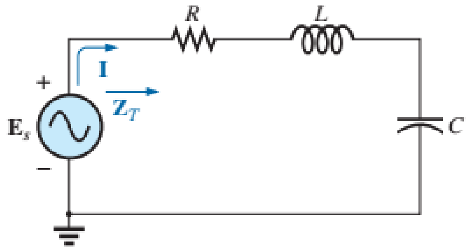
- Current rating up to 1.2 A
- Dielectric strength: 500 VRMS
- RoHS compliant*

78F Series Conformal Coate

Electrical Specifications (@ 25 °C)

Part Number	Value (µH)	Q Min.	Test Frequency (MHz)	SRF (MHz) Min.	DCR (Ω) Max.	IDC (mA) Max.
78FR10K-RC	0.10 ±10 %	20	25	400	0.060	1200
78FR12K-RC	0.12 ±10 %	20	25	400	0.070	1200
78FR15K-RC	0.15 ±10 %	20	25	380	0.070	1200
78FR18K-RC	0.18 ±10 %	20	25	380	0.073	1150

Series Resonance – ZT(f)



$$\mathbf{Z}_T = R + jX_L - jX_C$$

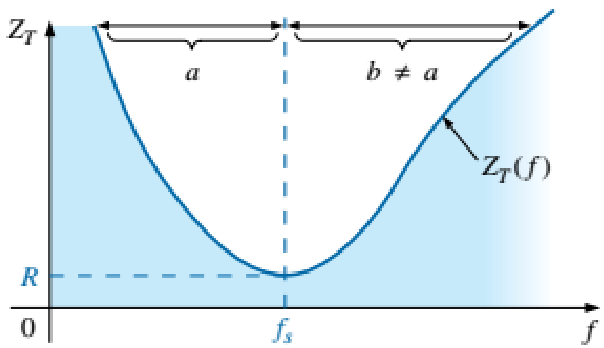
$$\mathbf{Z}_T = R + j(X_L - X_C)$$

So the <u>magnitude of **ZT** </u>is:

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

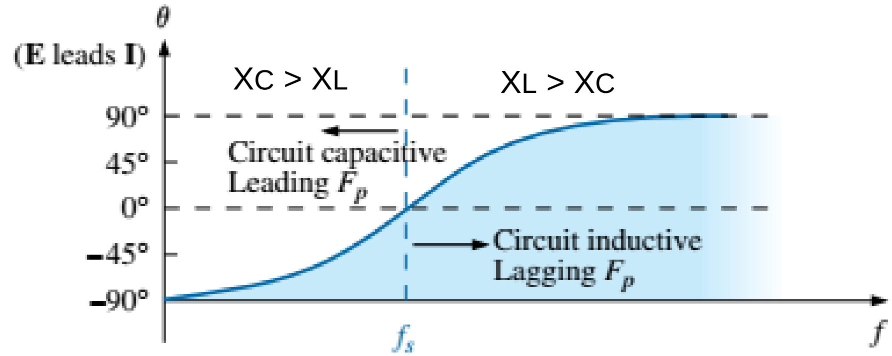
Or, as a function of frequency:

$$\boxed{Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2}}$$



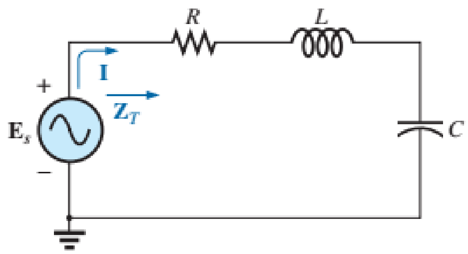
In general, we consider R to remain constant over frequency (depends on the material and frequency range)

$$\boxed{\theta_{ZT} = \tan^{-1}\frac{(X_L - X_C)}{R}}$$



$f < f_s$: network capactive; **I** leads **E**
$f > f_s$: network inductive; **E** leads **I**
$f = f_s$: network resistive; **E** and **I** are in phase

Series Resonance – Selectivity



Looking at the current as a function of frequency:

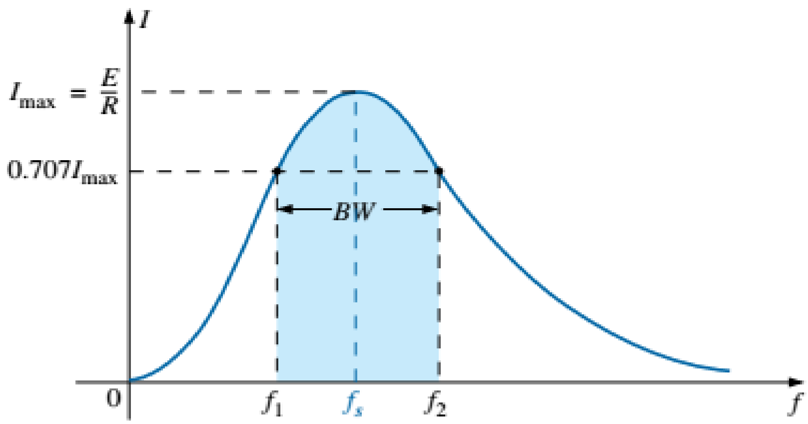
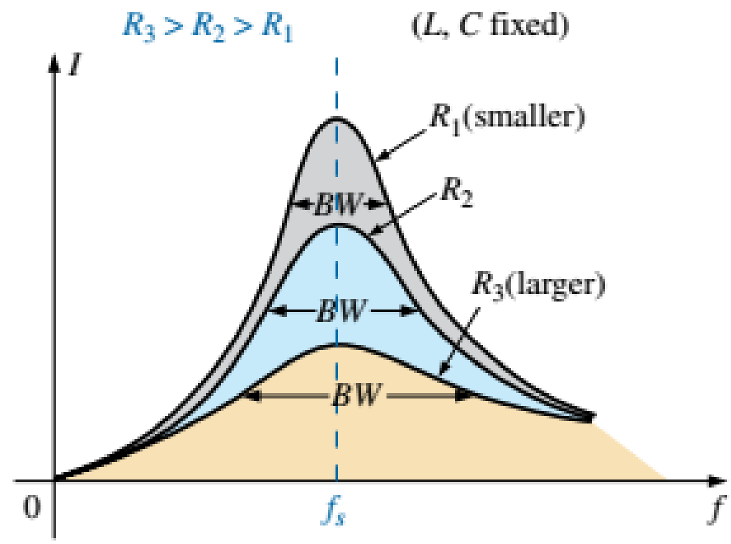


FIG. 21.15
I versus frequency for the series resonant circuit.

Set of selectivity curves for different R values)



Lower Q (higher R):
- Larger BW
- Decreased selectivity

Higher Q (lower R):
- Smaller BW
- Increased selectivity

At f_1 and f_2:

$$\boxed{P_{HPF} = \frac{1}{2} P_{max}}$$

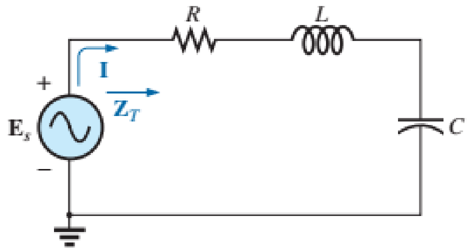
BW == $f_2 - f_1$

Check:

$$P_{max} = I_{max}^2 R$$

$$P_{HPF} = I^2 R = (0.707 I_{max})^2 R = (0.5)(I_{max}^2 R) = \frac{1}{2} P_{max}$$

Series Resonance – Selectivity



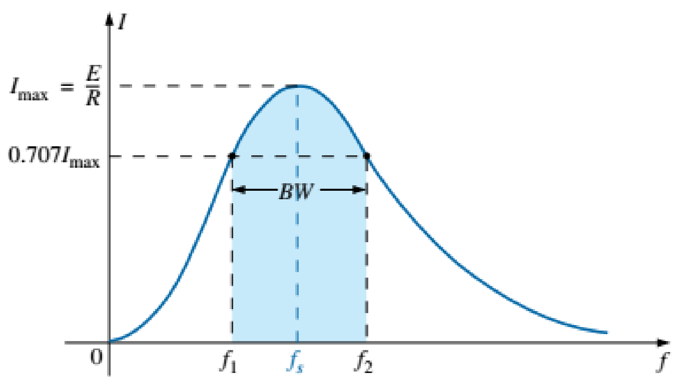


FIG. 21.15
I versus frequency for the series resonant circuit.

For Q >= 10, assume symmetry about fs
Text -> "High Q"

Values for f_1 and f_2 based on circuit elements (equations developed in the text):

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] \quad \text{(Hz)}$$

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right] \quad \text{(Hz)}$$

Subtracting the equations above yields:

$$BW = f_2 - f_1 = \frac{R}{2\pi L}$$

But, expanding and substituting yields:

$$BW = \frac{R}{2\pi L} = \left(\frac{1}{2\pi}\right)\left(\frac{R}{L}\right) = \left(\frac{f_s}{\omega_s}\right)\left(\frac{\omega_s}{Q_s}\right)$$

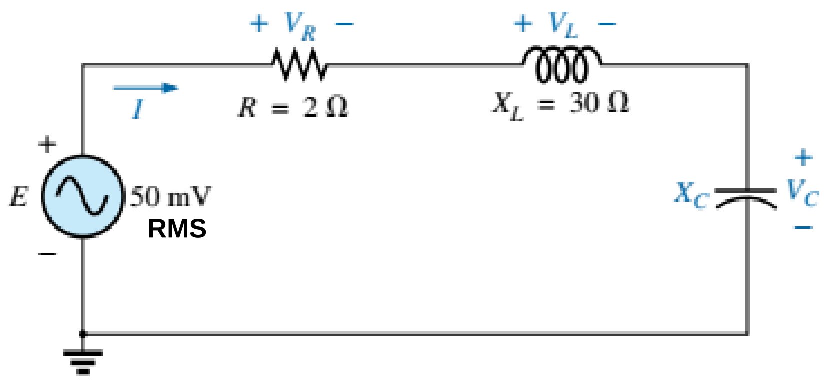
Hence:

$$BW = \frac{f_s}{Q_s}$$

$$\frac{f_2 - f_1}{f_s} = \frac{1}{Q_s}$$

So we can determine f_1 and f_2 based on knowing fs and Q or determine Q knowing (measuring) fs and f_1, f_2

Series Resonance – In Class Problem



Find:

a) Xc for resonance
b) **Z_T** at resonance
c) |**I**| at resonance
d) |**V_R**|, |**V_L**|, |**V_C**| at resonance
e) Q, the quality factor
f) The power <u>dissipated</u> by the circuit at resonance