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Series Configuration and Voltage Divider

Spring 2019 (2185)

Series Configuration and Voltage Divider

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  - Comparison to DC Series Circuits
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Series Circuit Configuration

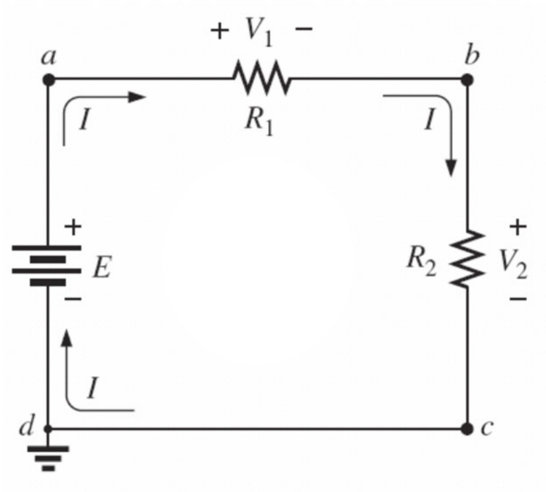
- Overall properties of AC series circuits are the same as those of DC series circuits if
  - The source is converted to phasor form

$$V_m \sin(\omega t + \theta°) \rightarrow \begin{cases} V_m\angle\theta° \ or \ V_{pk}\angle\theta° \\ \frac{V_m}{\sqrt{2}}\angle\theta° \rightarrow \ V_{rms}\angle\theta° \end{cases}$$

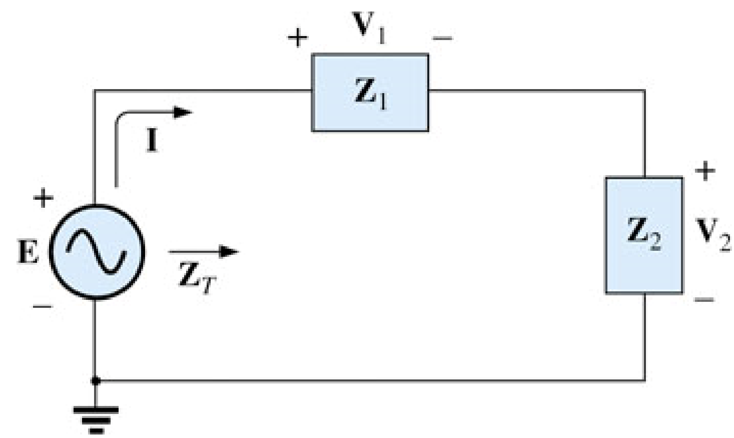
  - The circuit loads ($R, L \ or \ C$) are converted to their respective impedances ($Z_R, Z_L \ or \ Z_C$)

$$\boldsymbol{Z_R} = R\angle 0° \qquad \boldsymbol{Z_L} = X_L\angle 90° \qquad \boldsymbol{Z_C} = X_C\angle -90°$$

Series DC Circuit



Series AC Circuit



- The applied voltage of a series dc circuit will equal the algebraic sum of the voltage drops of the circuit (aka KVL)

$$E = V_1 + V_2 \qquad\qquad \boldsymbol{E = V_1 + V_2}$$

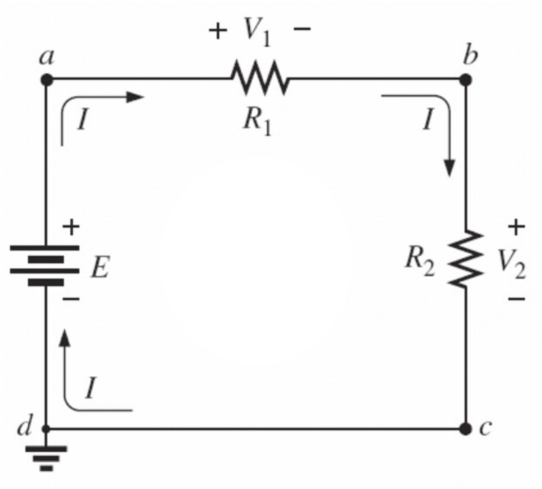
- The current is the same at every point in a series circuit

$$I = \frac{E}{R_T} \qquad\qquad \boldsymbol{I = \frac{E}{Z_T}}$$

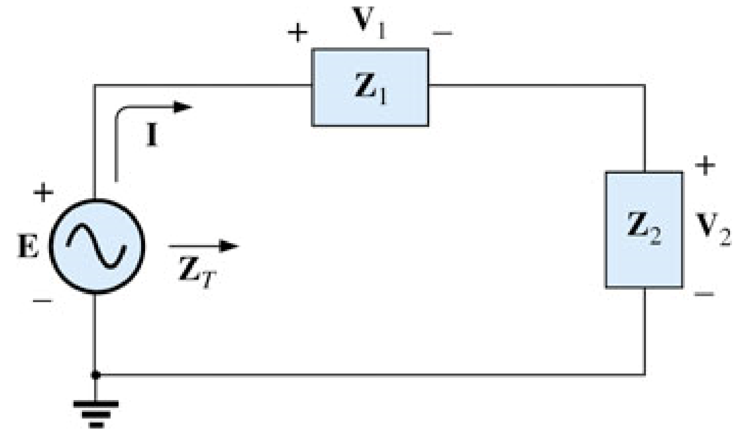
- The voltage across each element can be found by Ohm's law

$$V_1 = IR_1 \qquad V_2 = IR_2 \qquad\qquad \boldsymbol{V_1 = IZ_1} \qquad \boldsymbol{V_2 = IZ_2}$$

Series DC Circuit



Series AC Circuit



- The total resistance (impedance for AC) of a series configuration is the sum of the resistors (impedances for AC)

$$R_T = R_1 + R_2$$

$$\boldsymbol{Z_T} = \boldsymbol{Z_1} + \boldsymbol{Z_2}$$

- The total power delivered to the load is

$$P = EI$$

$$P = EI\cos(\theta_T)$$

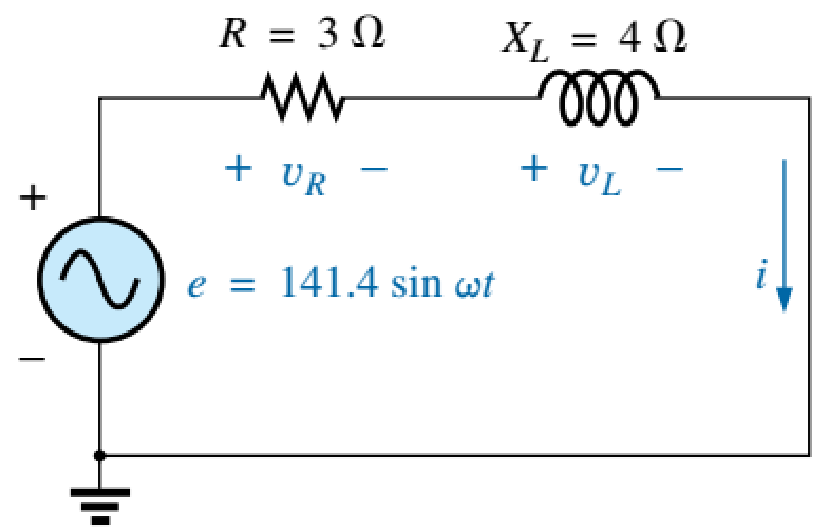
E and I are the effective (RMS) values

Where θ_T is the phase angle between voltage and current

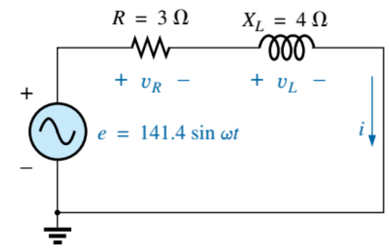
$$\theta_T = \theta_E - \theta_I$$

Series RL Circuit Example

- Consider the series RL circuit below
  1. Write the voltage E in phasor form.
  2. Calculate the total impedance Z_T of the circuit
  3. Find the current I and the voltages V_R and V_L
  4. Find the power delivered to the loads and the total power



Series RL Circuit Example



- Source voltage $e = 141.4_{pk}\sin(\omega t)$
  - Sinusoidal $\boldsymbol{E} = \frac{141.4}{\sqrt{2}}{}_{rms}\angle 0°$ → $\boldsymbol{E} = 100V_{rms}\angle 0°$
  - Polar & Rectangular $\boldsymbol{E} = (100 + j0)Vrms$
- Impedance
  - Total impedance $\boldsymbol{Z_T} = \boldsymbol{Z_R} + \boldsymbol{Z_L}$

$\boldsymbol{Z_R} = R\angle 0°$ $\quad \boldsymbol{Z_R} = 3\Omega\angle 0° = (3 + j0)\Omega$

$\boldsymbol{Z_L} = X_L\angle 90°$ $\quad \boldsymbol{Z_L} = 4\Omega\angle 90° = (0 + j4\ )\ \Omega$

$$\boldsymbol{Z_T} = \boldsymbol{Z_R} + \boldsymbol{Z_L} = (3 + j0) + (0 + j4\ ) = (3 + j4)\Omega$$

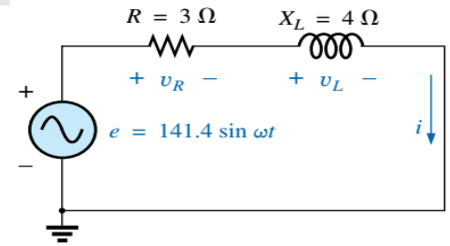
$$\boldsymbol{Z_T} = \sqrt{(3^2 + 4^2)}\angle\tan^{-1}\left(\tfrac{4}{3}\right) \ = (5\Omega\angle 53.13°)$$

- Current

$$\boldsymbol{I} = \boldsymbol{I_R} = \boldsymbol{I_L} = \frac{\boldsymbol{E}}{\boldsymbol{Z_T}} = \frac{100V_{rms}\angle 0°}{5\Omega\angle 53.13°} = (20A_{rms}\angle -53.13°\ )$$

$$\boldsymbol{I} = 20\cos(-53.13°) + j20\sin(-53.13°) = (12\ \ - j16)A_{rms}$$

Series RL Circuit Example



- Total Power Delivered

Where θ_T is the phase angle between E and I

$$P_T = EI\cos\theta_T \qquad P_T = (E)(I)\cos(\theta_E - \theta_I)$$

$$P_T = (100V_{rms})(20A_{rms})\cos\big(0 - (-53.13°)\big)$$

$$P_T = (100V_{rms})(20A_{rms})\cos(53.13°) = (2000\ W)(0.6) = 1200\ W$$

- Power Factor

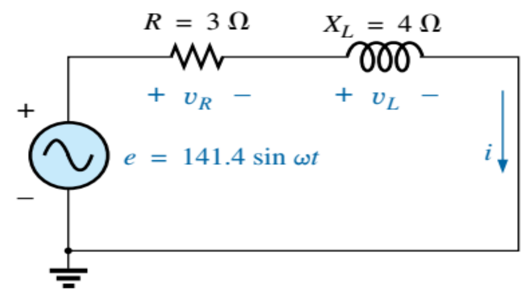
$$F_P = \cos\theta_T = \frac{R}{Z_T}$$

$$F_P = \cos(\theta_E - \theta_I) = \frac{R}{Z_T}$$

$F_P = \cos\big(0 - (-53.13°)\big) = \frac{3}{5} = 0.6$ lagging (current lags voltage)

❑ Find **VR**, **VL** and sum to check the input voltage

The results are summarized in the table below.



		R	L	Total
E	**Polar**	$60\,V_{rms}\angle -53.13°$	$80V_{rms}\angle 36.87°$	$100V_{rms}\,\angle 0°$
	Rectangular	$(36 - j48)V_{rms}$	$(64 + j48)V_{rms}$	$(100 + j0)V_{rms}$
I	**Polar**	$20A_{rms}\angle -53.13°$	$20A_{rms}\angle -53.13°$	$20A_{rms}\angle -53.13°$
	Rectangular	$(12\;\; - j16)A_{rms}$	$(12\;\; - j16)A_{rms}$	$(12\;\; - j16)A_{rms}$
Z	**Polar**	$3\Omega\angle 0°$	$4\Omega\angle 90°$	$5\Omega\angle 53.13°$
	Rectangular	$(3 + j0)\;\Omega$	$(0 + j4)\;\Omega$	$(3 + j4)\Omega$
P		$\mathbf{1200\;W}$	0	$\mathbf{1200\;W}$

ICP 1 - Series RC Circuit

- Consider the series RC circuit in Fig. 15.33, analyze the circuit similar to the RL circuit.
    1. Write the current **I** in phasor form.
    2. Calculate the total impedance $\boldsymbol{Z_T}$ of the circuit
    3. Find the voltage across the current source, $\mathbf{Vin}$ and the voltages $\boldsymbol{V_R}$ and $\boldsymbol{V_C}$
    4. Find the power delivered to the loads and the total power delivered

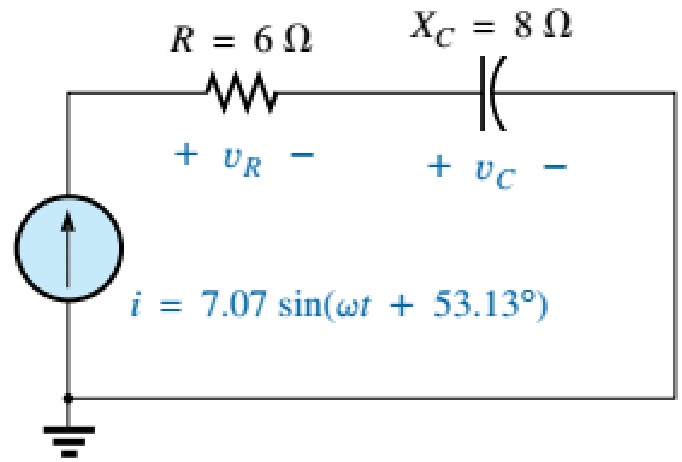


FIG. 15.33
Series R-C ac circuit.

Voltage Divider Rule (VDR)

- The basic format for the voltage divider rule in AC circuits is exactly the same for DC circuits (covered in Chap. 5.7)
  - VDR is used to determine the voltage across a series load without first having to determine the current through the circuit.

$$V_x = \frac{Z_x E}{Z_T}$$

where
V_x is the voltage across one or more elements
E is the source voltage
Z_x is the impedance with unknown voltage
Z_T is the total impedance of the series circuit

Voltage Divider Rule (VDR): Example

- Using the voltage divider rule, find the voltage across each element of the circuit shown below
  - Voltage across the capacitor V_C?
  - Voltage across the resistor V_R?

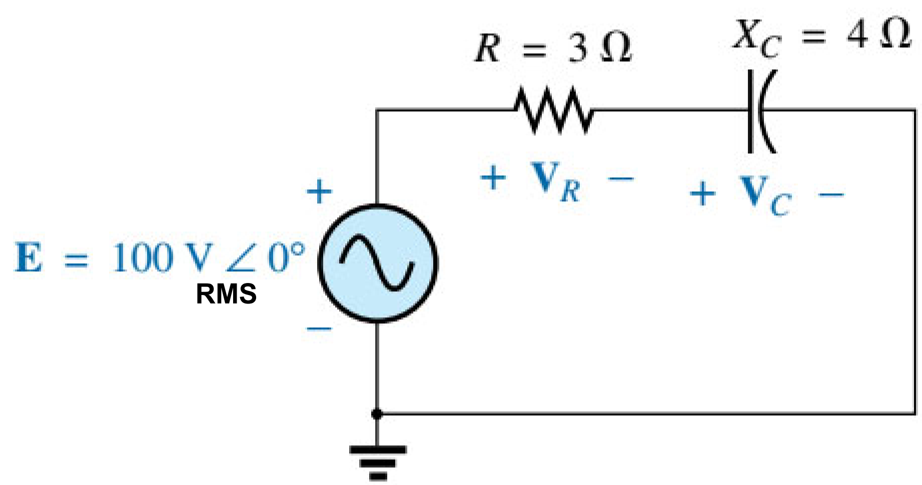


FIG. 15.43
Example 15.11.

Voltage Divider Rule (VDR): Example

- Using the voltage diver rule, find the voltage across each element of the circuit shown below
  - Voltage across the capacitor V_C?
  - Voltage across the resistor V_R?

$$V_c = \frac{Z_C E}{Z_T} = \frac{Z_C E}{Z_R + Z_C}$$

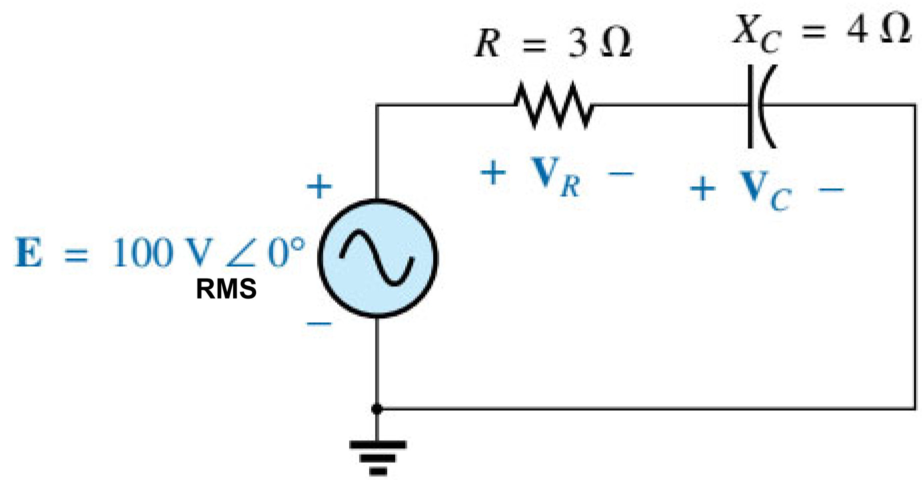


FIG. 15.43
Example 15.11.

Voltage Divider Rule (VDR): Example

- Using the voltage diver rule, find the voltage across each element of the circuit shown below
  - Voltage across the capacitor V_C?

$$V_c = \frac{Z_C E}{Z_T} = \frac{Z_C E}{Z_R + Z_C}$$

$$V_c = \frac{400\angle -90^\circ}{5\underset{\Omega}{\angle} - 53.13^\circ} = 80\underset{\text{RMS}}{V}\angle -36.87^\circ$$

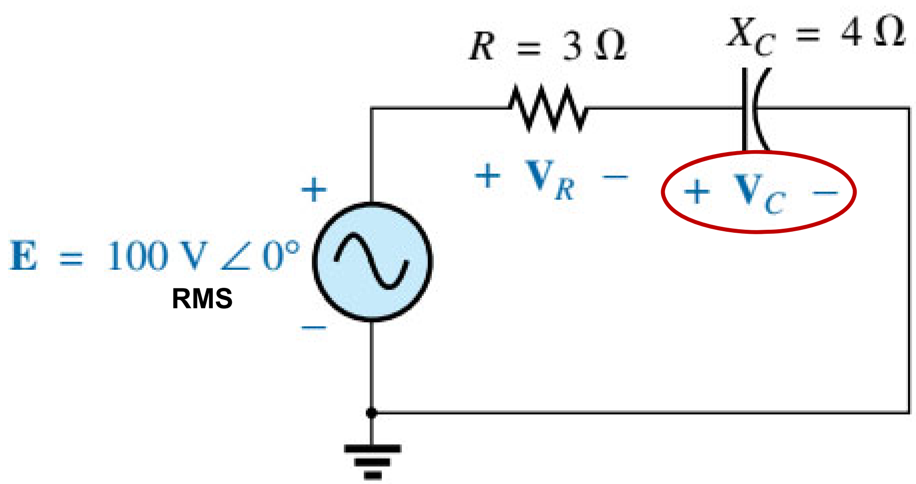


FIG. 15.43
Example 15.11.

Voltage Divider Rule (VDR): Example

- Using the voltage diver rule, find the voltage across each element of the circuit shown below
  - Voltage across the resistor V_R?

$$V_R = \frac{Z_R E}{Z_T} = \frac{Z_R E}{Z_R + Z_C}$$

$$V_R = \frac{300\angle 0^\circ}{\underset{\Omega}{5\angle} - 53.13^\circ} = \underset{\text{RMS}}{60V}\angle 53.13^\circ$$

- Check: Use VR and Vc to find E

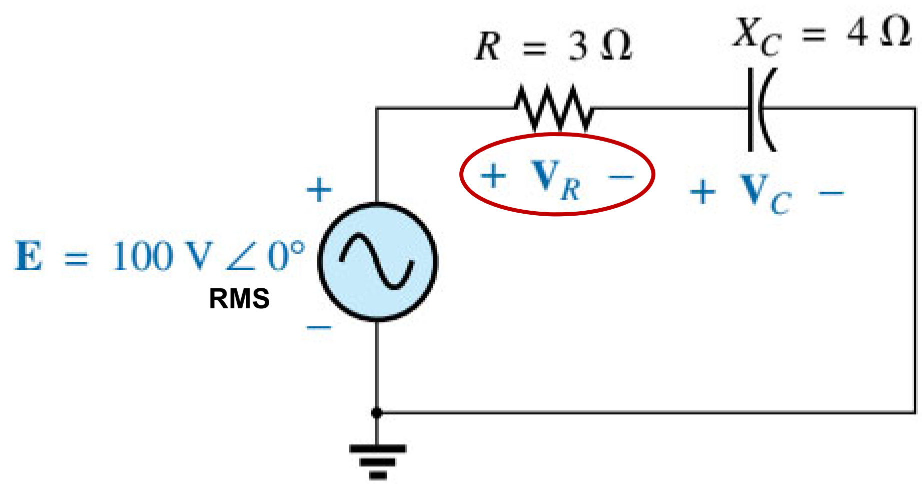


FIG. 15.43
Example 15.11.

ICP 2 – Voltage Divider Rule

- Using the voltage divider rule, find the unknown voltages V_R, V_L, V_C, and V_1 for the circuit shown below.

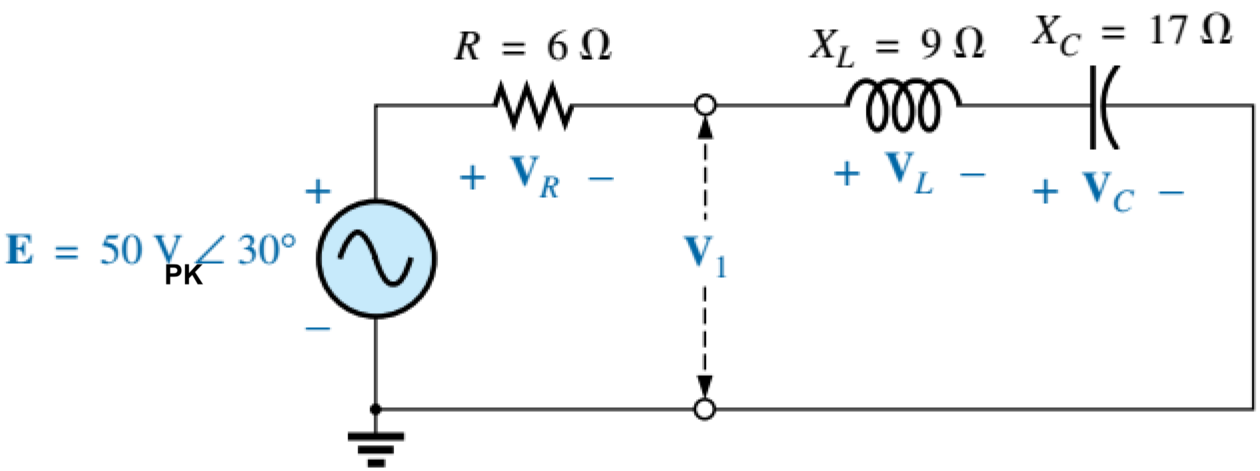


FIG. 15.44

Example 15.12.