Norton’s Theorem and Max Power Transfer

- Norton’s Theorem
  - Introduction and general approach
  - Source conversion approach
  - Example
    - Only independent sources
    - Use your calculator
  - Example 2
    - Includes a dependent source controlled in-network
    - Requires a different method to determine **Z_N** (test source)
- Maximum Power Transfer Theorem
  - Statement and discussion
  - **In class problem**

Norton's Theorem

- Norton's theorem allows us to replace any two-terminal linear bilateral ac network with an equivalent circuit consisting of a current source and an impedance.
- The Norton equivalent circuit, like the Thévenin equivalent circuit, is applicable at only one frequency since the reactances are frequency dependent.

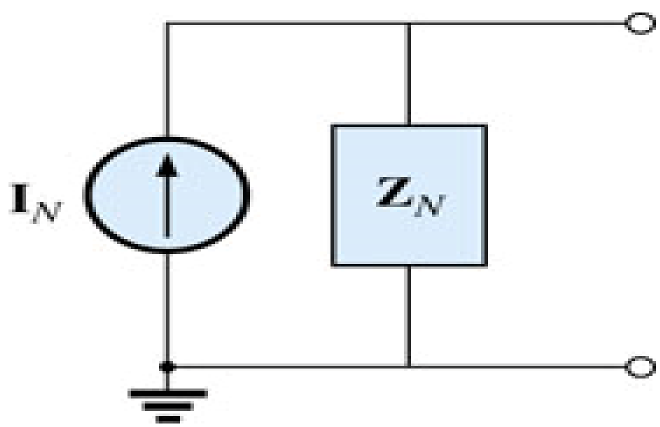


FIG. 19.60 *The Norton equivalent circuit for ac networks.*

Norton's Theorem - Approach

1. *Remove that portion of the network across which the Norton equivalent circuit is to be found.*
2. *Mark* **A and B or a-a'** *the terminals of the remaining two-terminal network.*
3. *Calculate Z_N by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the two marked terminals.*
4. *Calculate I_N by first replacing the voltage and current sources and then finding the short-circuit current between the marked terminals.*
5. *Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Norton equivalent circuit.*

For dependent sources – Must use an alternate method to find **Z_N** if the controlling variable is IN the network under investigation.

Norton's Theorem – Source Conversion

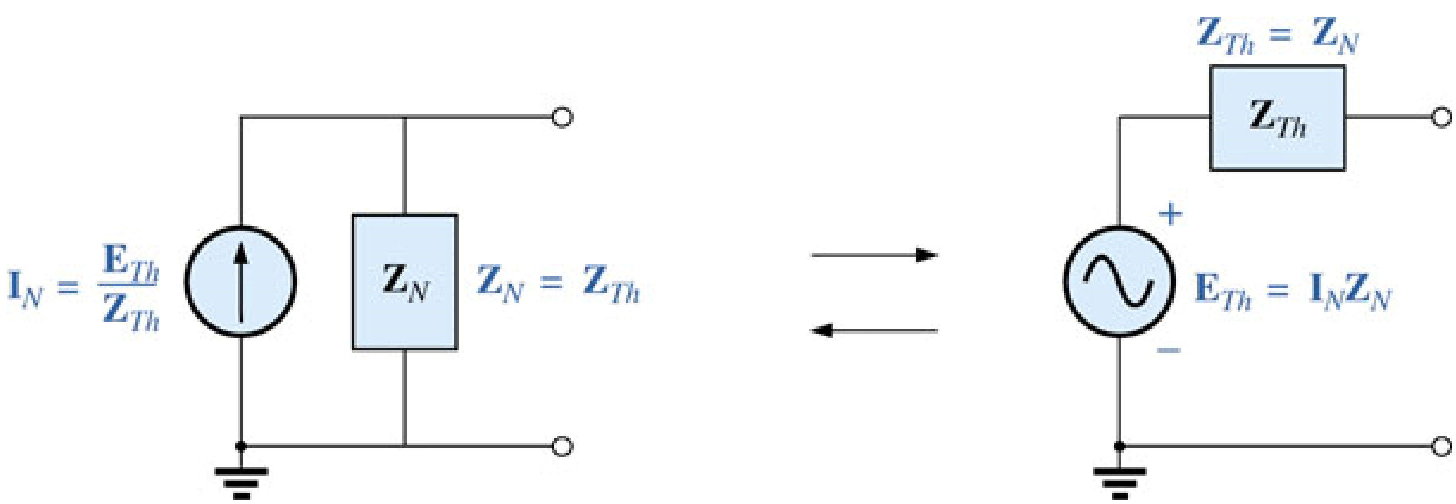
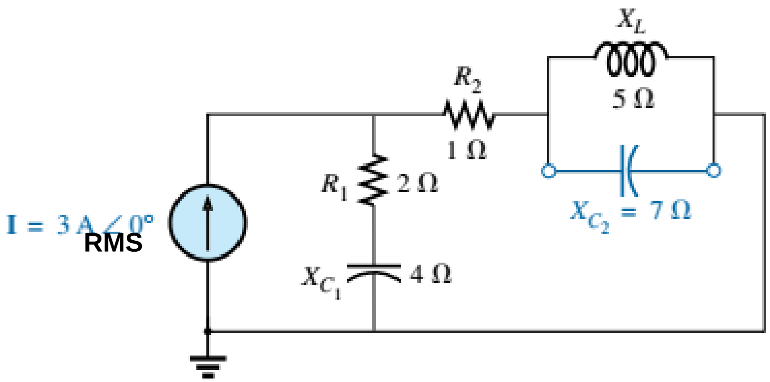


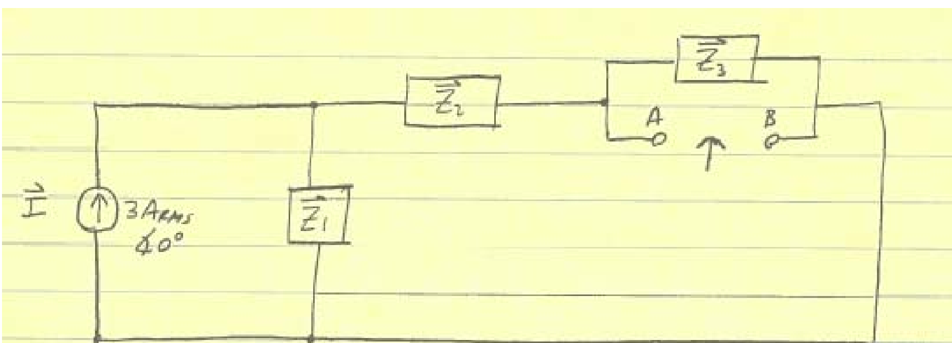
FIG. 19.61 *Conversion between the Thévenin and Norton equivalent circuits.*

Norton's Theorem – Example (use your calculator)



Find: the Norton equivalent circuit external to capacitor, C_2

Steps 1&2: Redraw, removing C_2 (external to the network terminals), keep track of A-B

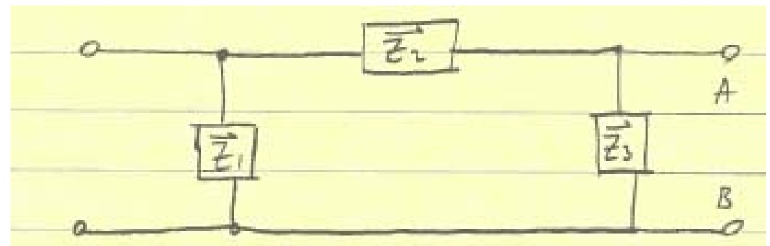


$\vec{Z}_1 = (2 - j4)\ \Omega$

$\vec{Z}_2 = 1\ \Omega$

$\vec{Z}_3 = j5\ \Omega$

Step 3: Find **Z_N**
(Remove independent sources,
Z_N = **Z_{TH}** = **Zab o/c**)

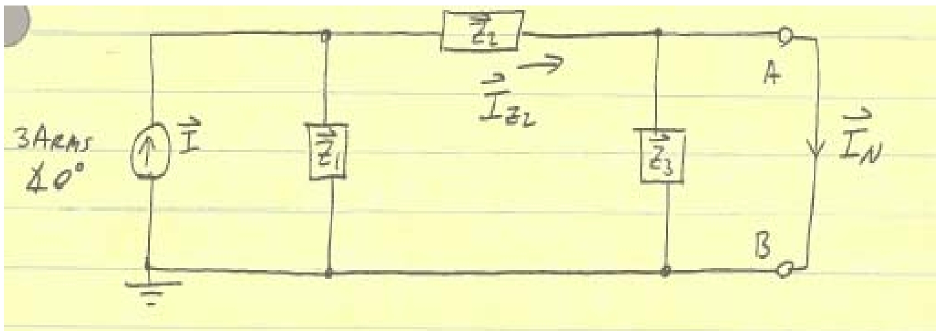


$$\vec{Z}_N = (\vec{Z}_1 + \vec{Z}_2) \parallel \vec{Z}_3$$
$$= (3 - j4)\ \Omega \parallel (j5\ \Omega)$$

$$\boxed{\vec{Z}_N = (7.5 + j2.5)\ \Omega}$$

Norton's Theorem – Example (use your calculator)

Step 4: Find **I_N (I_{AB} s/c)**

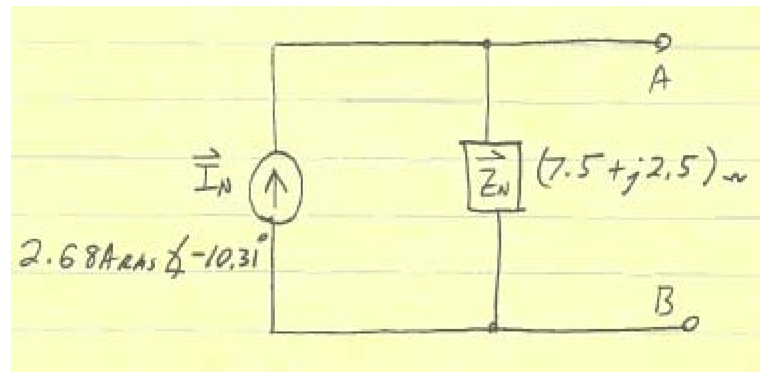


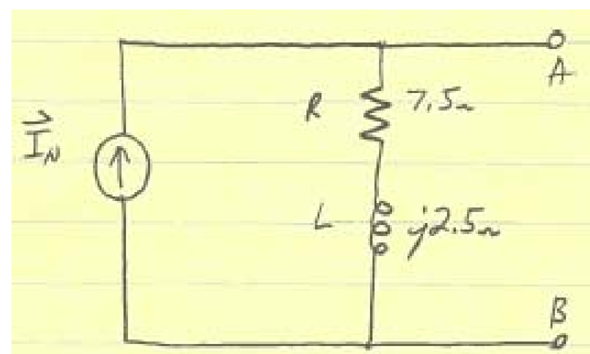
$$\vec{I}_N = \vec{I}_{Z_2} = \vec{I}\left(\frac{\vec{Z}_1}{\vec{Z}_1+\vec{Z}_2}\right)$$

$$= 3A_{RMS}\angle 0^\circ \ \frac{(2-j4)\,\Omega}{(3-j4)\,\Omega}$$

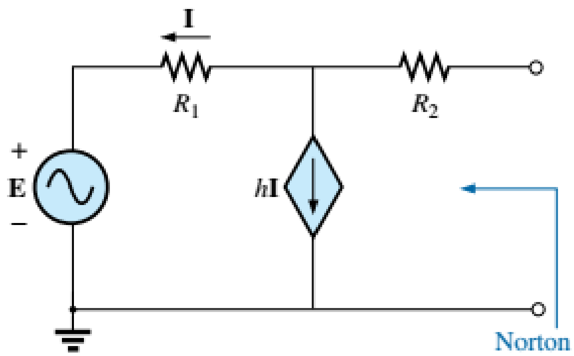
$$\boxed{\vec{I}_N = 2.68A_{RMS}\angle -10.31^\circ}$$

Step 5: Draw the Norton Equivalent Circuit





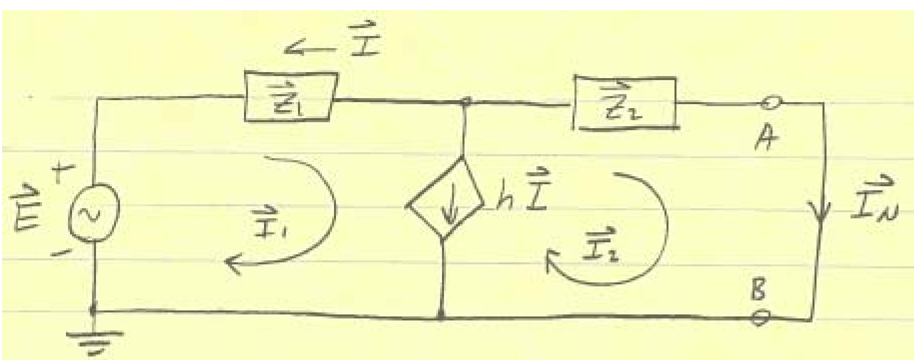
Norton's Theorem – Example 2 (dependent source controlled within network)



Note: The network contains a dependent source that is controlled by a current in the network

- Find **I_N** as usual
- Find **Z_N** using an alternate method (test source)

Steps 1&2: Redraw using impedance boxes (keep track of terminals A-B)



Step 4: Find **I_N**

$$KVL\ (OUTER):\ \vec{E} + \vec{I}Z_1 - \vec{I}_N Z_2 = 0 \quad (1)$$

$$KCL \quad 0 = \vec{I} + h\vec{I} + \vec{I}_N \quad (2)$$

Solving (1) for **I**:

$$\vec{I}\vec{Z}_1 = \vec{I}_N\vec{Z}_2 - \vec{E}$$

$$\therefore \vec{I} = \frac{\vec{I}_N\vec{Z}_2 - \vec{E}}{\vec{Z}_1}$$

Into (2) yields:

$$\vec{I}_N = -\vec{I}(1+h)$$

$$\vec{I}_N = -\left(\frac{\vec{I}_N\vec{Z}_2 - \vec{E}}{\vec{Z}_1}\right)(1+h)$$

Norton's Theorem – Example 2 (dependent source controlled within network)

$$\vec{I}_N = -\left(\frac{\vec{I}_N \vec{Z}_2 - \vec{E}}{\vec{Z}_1}\right)(1+h)$$

Expanding to solve for **IN** in terms of the network values:

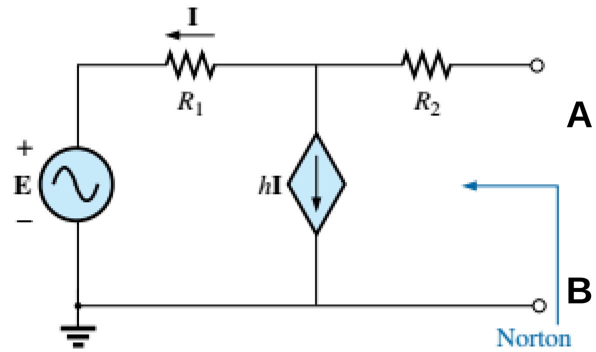
$$\vec{I}_N \vec{Z}_1 = -(1+h)(\vec{I}_N \vec{Z}_2 - \vec{E})$$
$$= -\vec{I}_N \vec{Z}_2 + \vec{E} - h\vec{I}_N \vec{Z}_2 + \vec{E}h$$

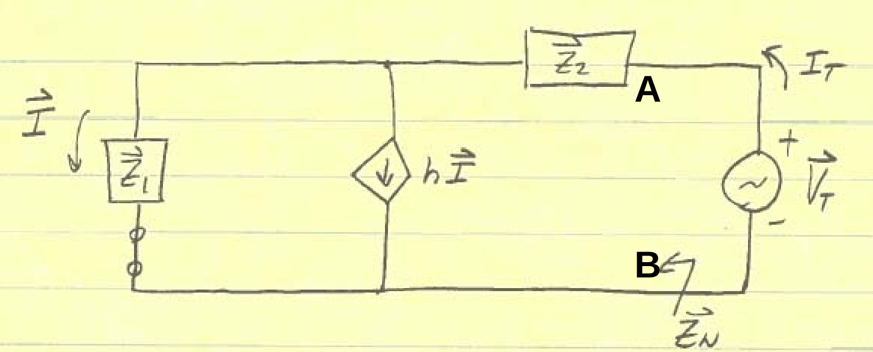
$$\vec{I}_N \vec{Z}_1 + \vec{I}_N \vec{Z}_2 + h\vec{I}_N \vec{Z}_2 = \vec{E}(1+h)$$

$$\vec{I}_N = \vec{E}\,\frac{(1+h)}{\vec{Z}_1 + \vec{Z}_2 + h\vec{Z}_2}$$

$$\boxed{\vec{I}_N = \vec{E}\,\frac{(1+h)}{\vec{Z}_1 + \vec{Z}_2(1+h)}}$$

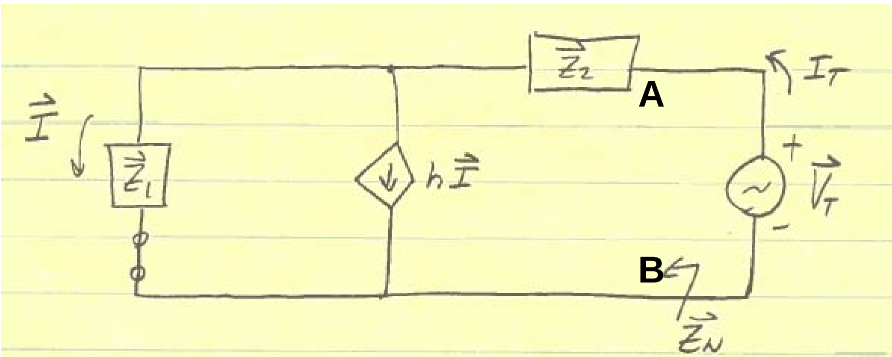
Step 3: Find **ZN** by
- Relaxing the independent sources (**E**)
- Applying a test-source across A-B





$$\vec{Z}_N = \frac{\vec{V}_T}{\vec{I}_T}$$

Norton's Theorem – Example 2 (dependent source controlled within network)



KVL:

$$\vec{V}_T = \vec{I}_T\vec{Z}_2 + \vec{I}\,\vec{Z} \quad \textbf{(3)}$$

KCL:

$$\vec{I}_T = \vec{I} + h\,\vec{I} \quad \textbf{(4)}$$

Solving (4) for **I** and substituting into (3):

$$\vec{I} = \frac{\vec{I}_T}{1+h}$$

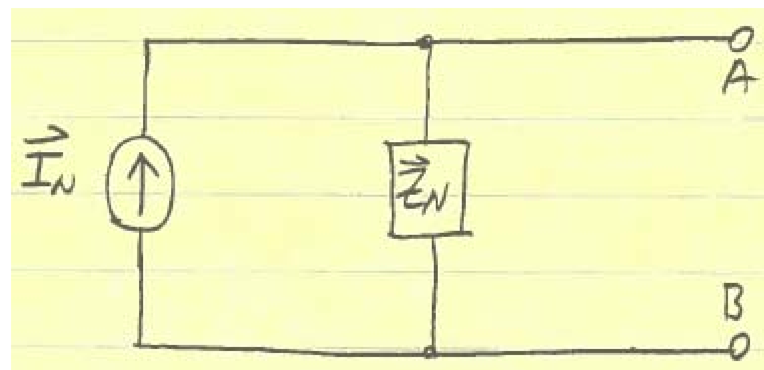
$$\vec{V}_T = \vec{I}_T\,\vec{Z}_2 + \vec{I}_T\left(\frac{1}{1+h}\right)\vec{Z}_1$$

Solving for **VT**/**IT** (**ZN**):

$$\frac{\vec{V}_T}{\vec{I}_T} = \vec{Z}_N = \vec{Z}_2 + \vec{Z}_1\left(\frac{1}{1+h}\right)$$

$$\boxed{\vec{Z}_N = \frac{\vec{Z}_2(1+h) + \vec{Z}_1}{1+h}}$$

Step 5: Draw the Norton Equivalent Circuit



$$\boxed{\vec{I}_N = \vec{E}\,\frac{(1+h)}{\vec{Z}_1 + \vec{Z}_2(1+h)}}$$

Maximum Power Transfer Theorem

- When applied to ac circuits, the maximum power transfer theorem states that maximum power will be delivered to a load when the load impedance is the conjugate of the Thévenin impedance across its terminals.

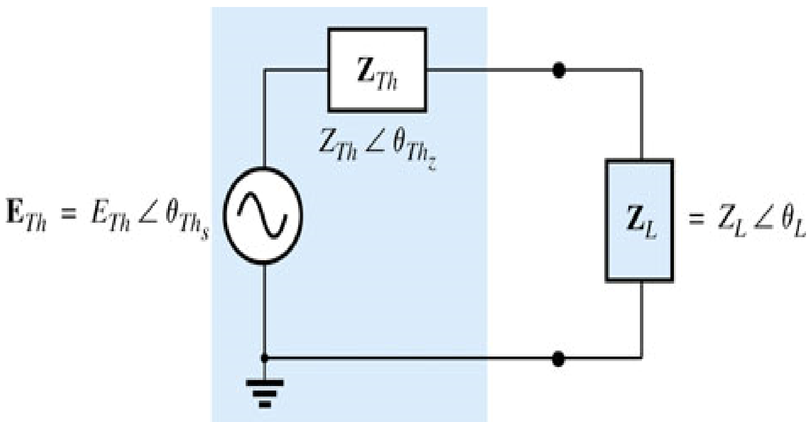


FIG. 19.81 *Defining the conditions for maximum power transfer to a load.*

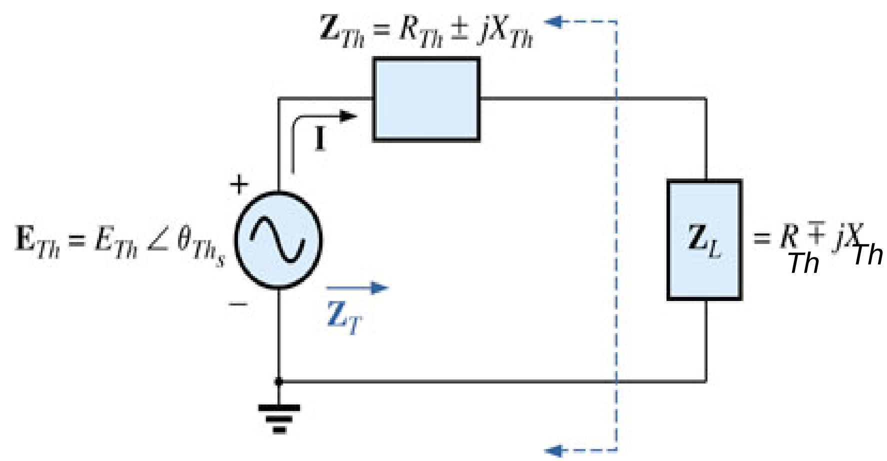
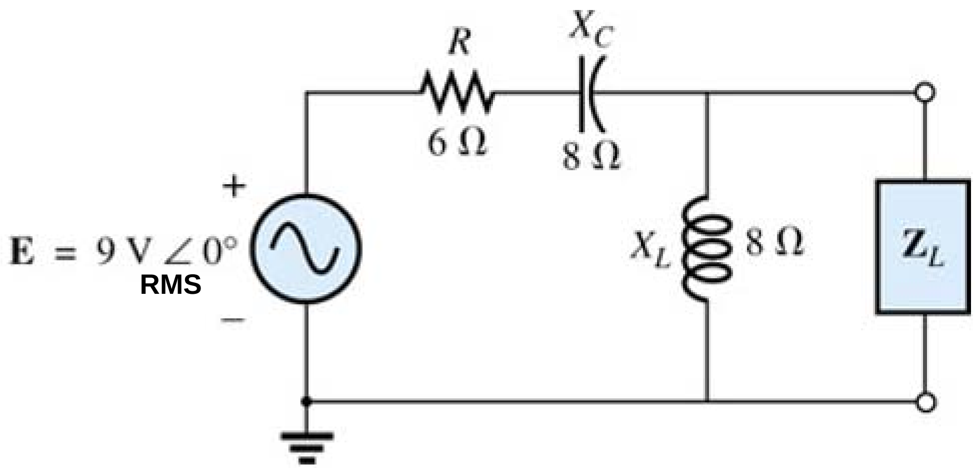


FIG. 19.82 *Conditions for maximum power transfer to* $\mathbf{Z}_L$*.*

NOTE:

When $\mathbf{Z}_L = \mathbf{Z}_{TH}^*$, the reactive components cancel and we have a series circuit with two equal value resistors… look familiar?

In Class Problem



Find:

- The Thevenin equivalent circuit for the network external to $\mathbf{Z}_L$
- The value of $\mathbf{Z}_L$ for maximum power transfer
- The average power dissipated by this load

Approach:

- Standard Thevenin approach
- Set $\mathbf{Z}_L = \mathbf{Z}_{TH}^*$
- $P_{ZL} = V_{RMS}\, I_{RMS} \cos(\Theta)$