# MESH Analysis

- General Approach Only (will not cover the format approach)
  - Introduction
  - Example (work along)
  - Special cases
    - Dependent voltage sources
    - Independent current sources
    - Dependent current sources
  - **ICP with a dependent voltage source**

# General Approach

1. *Assign a distinct current in the clockwise direction to each independent closed loop of the network.*

2. *Indicate the polarities within each loop for each impedance as determined by the assumed direction of loop current for that loop.*

3. *Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and to prepare us for the format approach to follow.*

   a. *If an impedance has two or more assumed currents through it, the total current through the impedance is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents passing through in the opposite direction.*

   b. *The polarity of a voltage source is unaffected by the direction of the assigned loop currents.*

4. *Solve the resulting simultaneous linear equations for the assumed loop currents.*

In other words:

**- Do what we did in DC Circuits**

# Example 8.5 (text) – Work Along Using Your Calculator

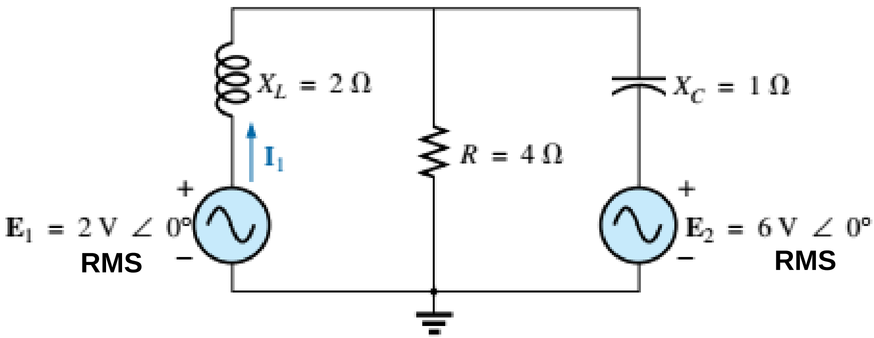



**FIG. 18.10**
*Example 18.5.*

Convert to impedance boxes, draw the mesh currents:

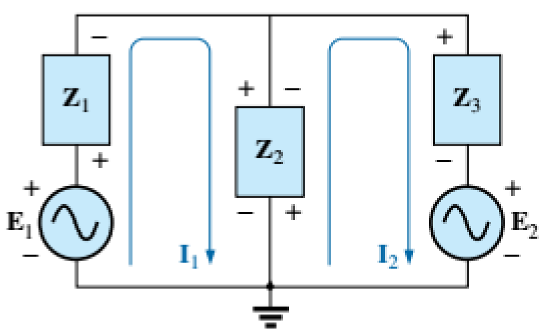



**FIG. 18.11**
*Assigning the mesh currents and subscripted impedances for the network in Fig. 18.10.*

$$\mathbf{Z}_1 = +jX_L = +j\,2\ \Omega \qquad \mathbf{E}_1 = 2\text{ V} \angle 0° \text{ RMS}$$
$$\mathbf{Z}_2 = R = 4\ \Omega \qquad \mathbf{E}_2 = 6\text{ V} \angle 0° \text{ RMS}$$
$$\mathbf{Z}_3 = -jX_C = -j\,1\ \Omega$$

Write the KVL equations:

$$+\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{Z}_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$-\mathbf{Z}_2(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{I}_2\mathbf{Z}_3 - \mathbf{E}_2 = 0$$

Reorganize the equations:

$$\mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 = \mathbf{E}_1$$
$$-\mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) = -\mathbf{E}_2$$

**Text**: Determinants to solve

**Us**: Inverse Matrix Method:

# Example 8.5 (text) – Work Along Using Your Calculator

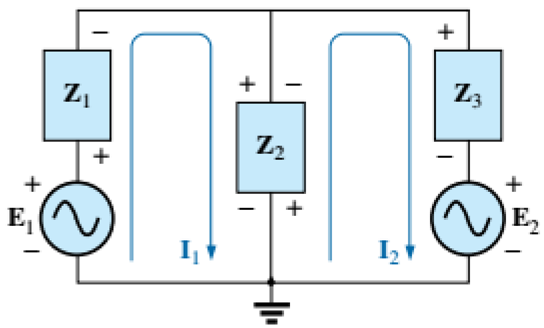


**FIG. 18.11**
*Assigning the mesh currents and subscripted impedances for the network in Fig. 18.10.*

$\mathbf{Z}_1 = +jX_L = +j\,2\,\Omega \qquad \mathbf{E}_1 = 2\text{ V}\angle 0°$ RMS

$\mathbf{Z}_2 = R = 4\,\Omega \qquad \mathbf{E}_2 = 6\text{ V}\angle 0°$ RMS

$\mathbf{Z}_3 = -jX_C = -j\,1\,\Omega$

$$\begin{aligned} \mathbf{I}_1(\mathbf{Z}_1 + \mathbf{Z}_2) - \mathbf{I}_2\mathbf{Z}_2 &= \mathbf{E}_1 \\ -\mathbf{I}_1\mathbf{Z}_2 + \mathbf{I}_2(\mathbf{Z}_2 + \mathbf{Z}_3) &= -\mathbf{E}_2 \end{aligned}$$

Substituting values:

$$(4+j2)\,\vec{I}_1 - 4\,\vec{I}_2 = 2 \qquad (1)$$
$$-4\,\vec{I}_1 + (4-j)\,\vec{I}_2 = -6 \qquad (2)$$

AX = B form

$$A = \begin{bmatrix} (4+j2) & -4 \\ -4 & (4-j) \end{bmatrix} \qquad X = \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

Solving:

$$X = \begin{bmatrix} 0.2-j0.9 & 0.4-j0.8 \\ 0.4-j0.8 & 0.8-j0.6 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$X = \begin{bmatrix} -2+j3 \\ -4+j2 \end{bmatrix} \quad \text{OR} \quad \begin{bmatrix} 3.61 \angle 123.7 \\ 4.47 \angle 153.4 \end{bmatrix}$$

OR
$$\boxed{\begin{aligned} \vec{I}_1 &= 3.61\,A_{RMS} \angle 123.7° \\ \vec{I}_2 &= 4.47\,A_{RMS} \angle 153.4° \end{aligned}}$$

# Example 8.5 (text) – Work Along Using Your Calculator

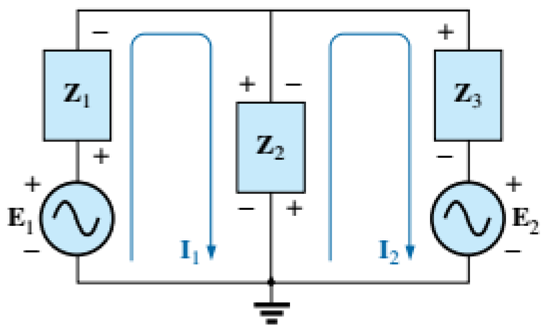



**FIG. 18.11**

*Assigning the mesh currents and subscripted impedances for the network in Fig. 18.10.*

$$\mathbf{Z}_1 = +jX_L = +j\,2\,\Omega \qquad \mathbf{E}_1 = 2\,\mathrm{V_{RMS}}\angle 0°$$
$$\mathbf{Z}_2 = R = 4\,\Omega \qquad \mathbf{E}_2 = 6\,\mathrm{V_{RMS}}\angle 0°$$
$$\mathbf{Z}_3 = -jX_C = -j\,1\,\Omega$$

OR
$$\boxed{\begin{aligned}\vec{I}_1 &= 3.61\,A_{RMS}\angle 123.7° \\ \vec{I}_2 &= 4.47\,A_{RMS}\angle 153.4°\end{aligned}}$$

CHECK

$$\vec{E}_1 \stackrel{?}{=} \vec{I}_1\vec{Z}_1 + (\vec{I}_1 - \vec{I}_2)\vec{Z}_2 \quad , \text{KVL}$$

$$2V_{RMS}\angle 0° \stackrel{?}{=} 7.22V_{RMS}\angle -146.3° + 8.93V_{RMS}\angle 26.7°$$

$$\underbrace{2V_{RMS}\angle 0° \stackrel{?}{=} 1.97V_{RMS}\angle 0.04°}_{}$$

YES, WITHIN ROUNDING ERROR

# Special Cases – Dependent Voltage Sources

Dependent Voltage Sources Present

2. Step 3 is modified as follows: Treat each dependent source like an independent source when Kirchhoff's voltage law is applied to each independent loop. However, once the equation is written, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen mesh currents.

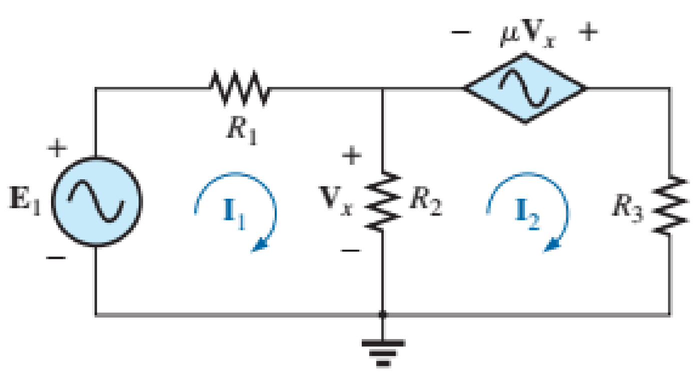



**FIG. 18.12**

*Applying mesh analysis to a network with a voltage-controlled voltage source.*

Write the KVL equations (note a few mistakes in the text):

*Step 3:*

$$\mathbf{E}_1 - \mathbf{I}_1R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$-R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu\mathbf{V}_x - \mathbf{I}_2R_3 = 0$$

Then substitute $\mathbf{V}_x = (\mathbf{I}_1 - \mathbf{I}_2)R_2$.

$$\mathbf{E}_1 - \mathbf{I}_1R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$
$$-R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu R_2(\mathbf{I}_1 - \mathbf{I}_2) - \mathbf{I}_2R_3 = 0$$

Rearranging yields the following 2x2 complex system:

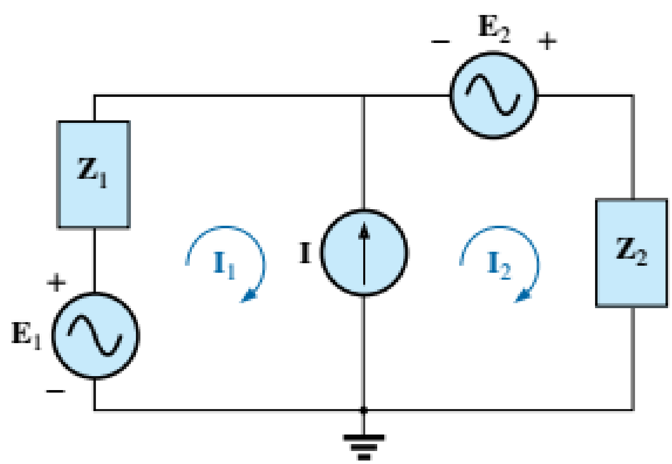
$$(R_1 + R_2)\vec{I}_1 - R_2\vec{I}_2 = \vec{E}_1 \quad (1)$$
$$(R_2 + \mu R_2)\vec{I}_1 - (R_2 + \mu R_2 + R_3)\vec{I}_2 = 0 \quad (2)$$

# Special Cases – Independent Current Sources

Independent Current Sources Present

2. Step 3 is modified as follows: Treat each current source as an open circuit (recall the *supermesh* designation in Chapter 8), and write the mesh equations for each remaining independent path. Then relate the chosen mesh currents to the dependent sources to ensure that the unknowns of the final equations are limited to the mesh currents.

**The same as in DC Circuits**




Only one remaining independent path, KVL:

$$\textit{Step 3:}\quad \mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 + \mathbf{E}_2 - \mathbf{I}_2\mathbf{Z}_2 = 0$$

But:

$$\mathbf{I}_1 + \mathbf{I} = \mathbf{I}_2$$

Rearranging yields the following 2x2 complex system:

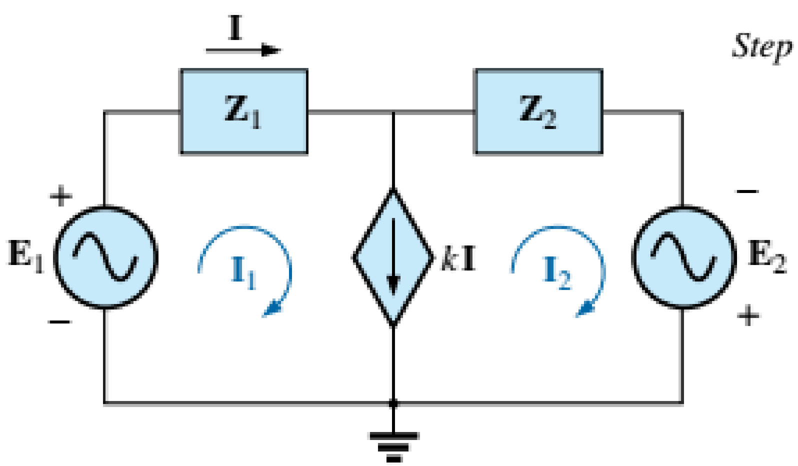
$$\vec{Z}_1 \vec{I}_1 + \vec{Z}_2 \vec{I}_2 = \vec{E}_1 + \vec{E}_2 \quad (1)$$

$$-\vec{I}_1 + \vec{I}_2 = \vec{I} \quad (2)$$

# Special Cases – Dependent Current Sources

Dependent Current Sources Present

2. Step 3 is modified as follows: The procedure is essentially the same as that applied for independent current sources, except now the dependent sources have to be defined in terms of the chosen mesh currents to ensure that the final equations have only mesh currents as the unknown quantities.




*Step 3:*

*Supermesh*, KVL yields:

$$\mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2 + \mathbf{E}_2 = 0$$

Relating k**I** to the mesh currents:

$$k\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2$$

$$\mathbf{I} = \mathbf{I}_1 \text{ so that } \qquad k\mathbf{I}_1 = \mathbf{I}_1 - \mathbf{I}_2$$

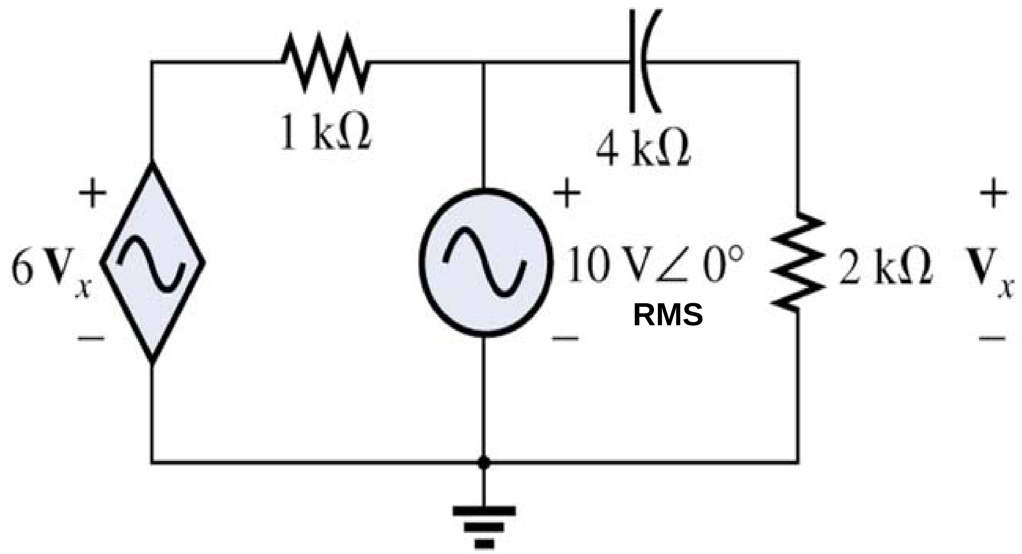
$$\mathbf{I}_2 = \mathbf{I}_1(1 - k)$$

Rearranging yields the following 2x2 complex system:

$$\vec{Z}_1\vec{I}_1 \quad + \quad \vec{Z}_2\vec{I}_2 \quad = \quad \vec{E}_1 + \vec{E}_2 \qquad (1)$$

$$(1-k)\,\vec{I}_1 \quad - \quad \vec{I}_2 \quad = \quad 0 \qquad (2)$$

# ICP – MESH Analysis w/Dependent Voltage Source




**Find:**

- The current through each resistor

**Check:**

- KVL on the LHS or RHS