Sources, Conversions and Matrix Solutions

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  - **ICP – Solving simultaneous equations using the inverse matrix method**

Dependent Sources - Introduction

- The term <u>independent</u> specifies that the magnitude of the source is independent of the network to which it is applied and that the source displays its terminal characteristics even if completely isolated.
- A <u>dependent</u> or <u>controlled source</u> is one whose magnitude is determined (or controlled) by a current or voltage of the system in which it appears.

Dependent Sources - Introduction

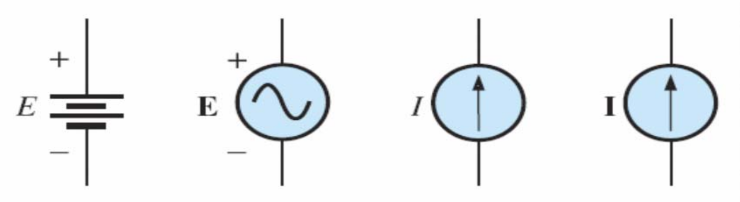


FIG. 18.1 *Independent sources.*

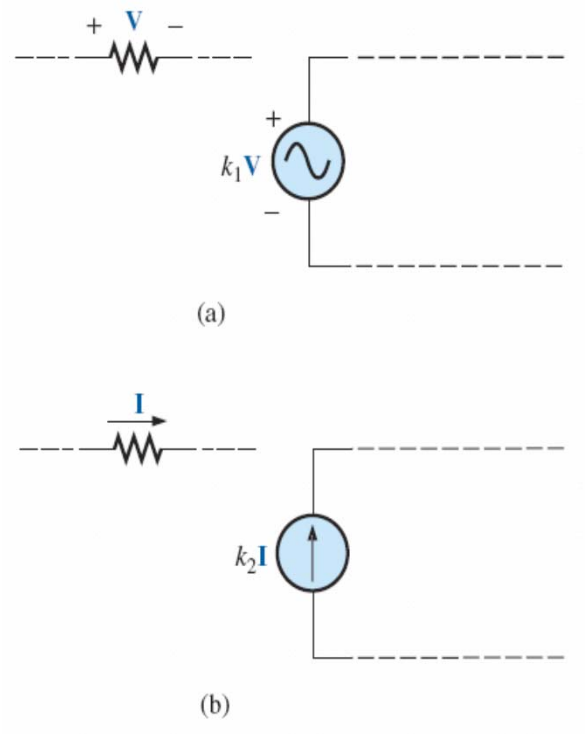


FIG. 18.2 *Controlled or dependent sources.*

Note that the value of the voltage source is determined by the voltage drop across a resistor elsewhere in the circuit. Just one possibility of many for a dependent voltage source.

Note that the value of the current source is determined by the current through a resistor elsewhere in the circuit. Just one possibility of many for a dependent current source.

Dependent Sources - Notation

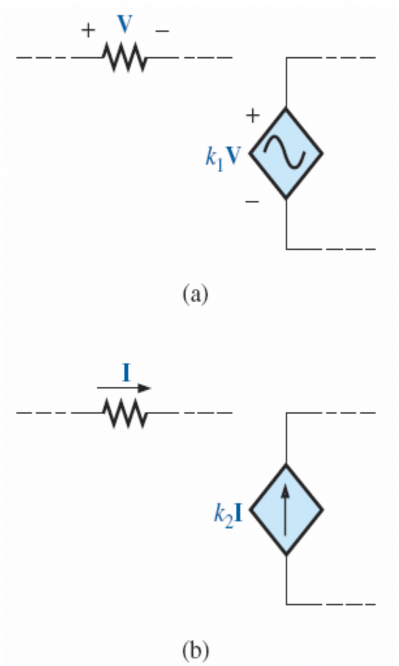


FIG. 18.3 *Special notation for controlled or dependent sources.*

The diamond notation helps us quickly determine that it's an dependent source.

Just like independent sources, a dependent voltage source with V=0 volts means a short circuit and a dependent current source with I=0 amps means an open circuit.

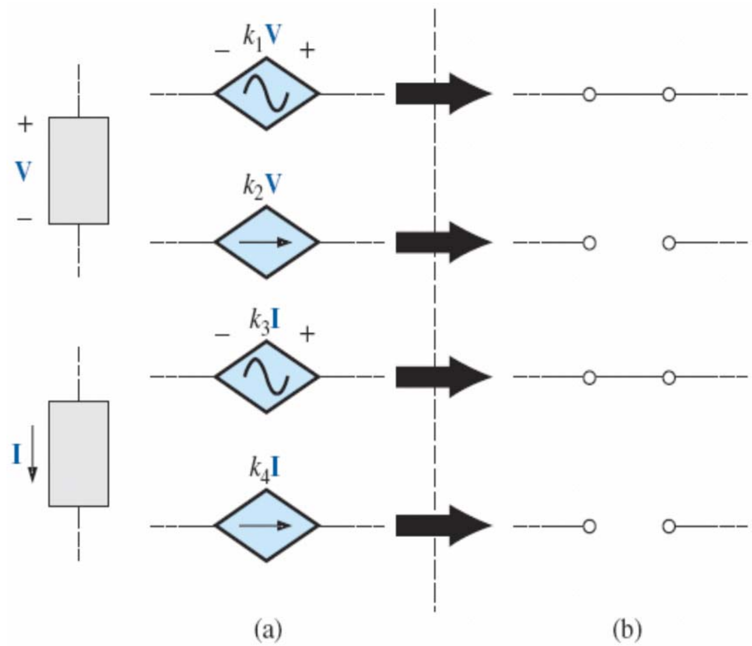


FIG. 18.4 *Conditions of* **V =** *0 V and* **I =** *0 A for a controlled source.*

Source Conversions

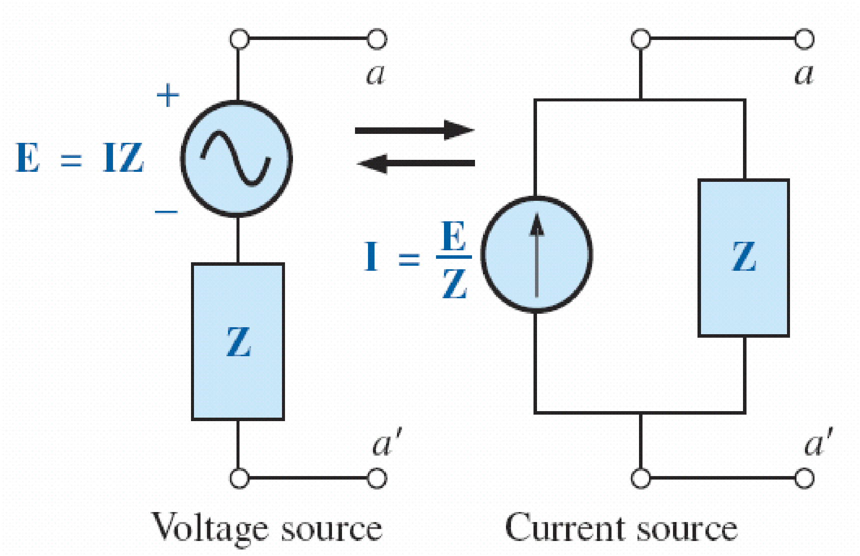
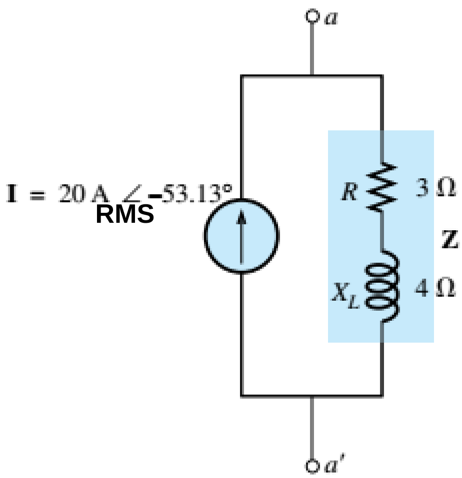


FIG. 18.5 *Source Conversion.*

We use the same approach to source conversions in AC that we used in DC BUT we are working with complex numbers so the magnitude AND angle must be accounted for.

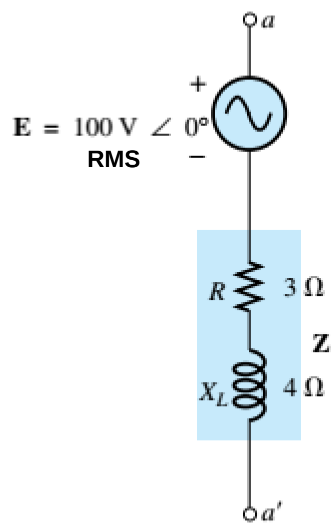
Source Conversions – Independent Source Example



$$\vec{E} = \vec{I} \cdot \vec{Z}$$
$$= (20A_{RMS} \angle -53.13^\circ) \cdot (3 + j4) \,\Omega$$

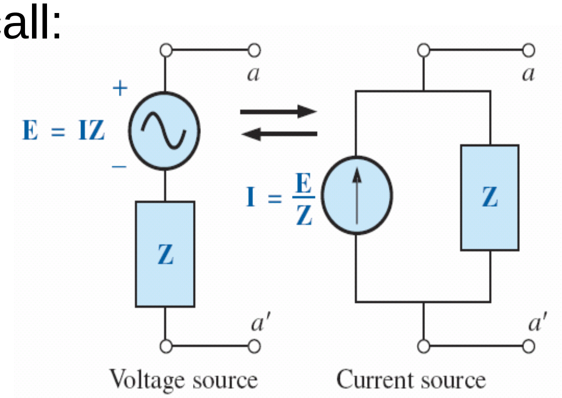
$$\boxed{\vec{E} = 100V_{RMS} \angle 0^\circ}$$

Convert to a voltage source



Note that the impedance stays the same, just as in DC

Recall:



Source Conversions – Dependent Sources

We follow the same procedure with dependent source conversions. The voltage or current that the original source depends on remains unchanged however.

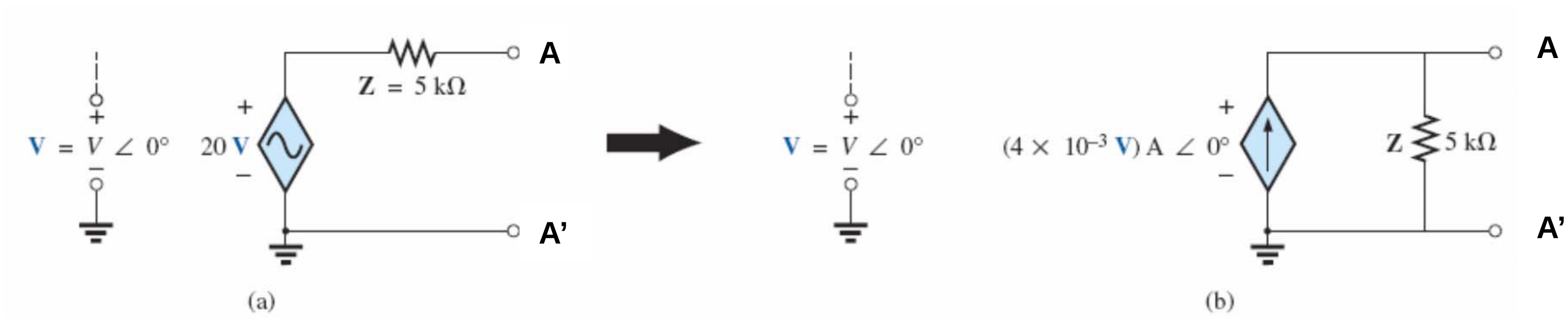


FIG. 18.8 *Source conversion with a voltage-controlled voltage source.*

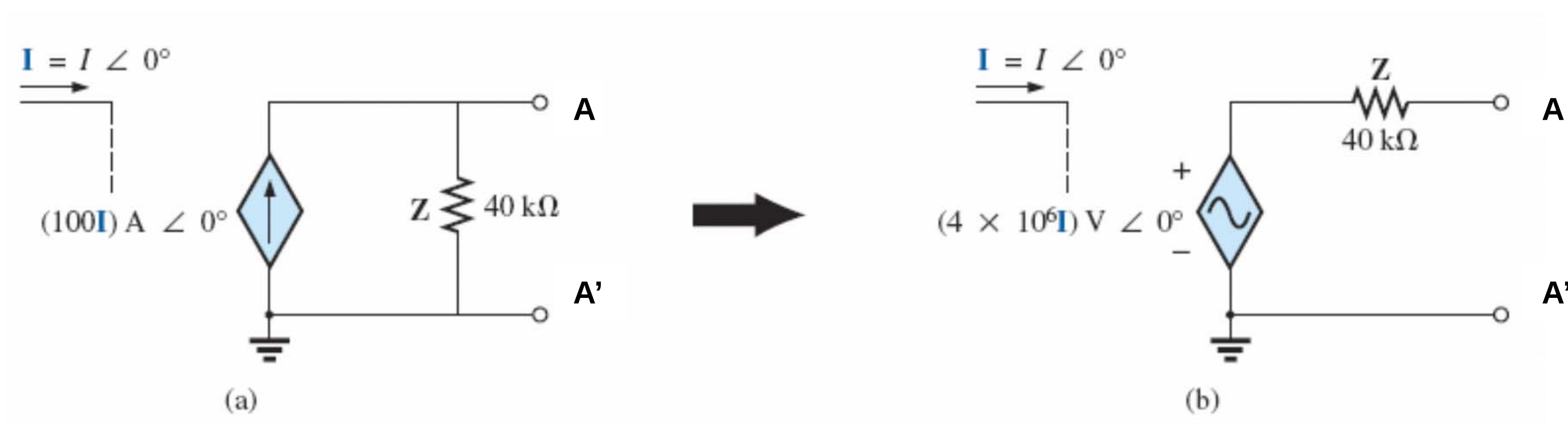
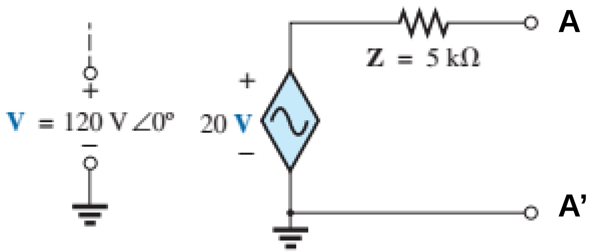


FIG. 18.9 *Source conversion with a current-controlled current source.*

Source Conversions – Dependent Source Example



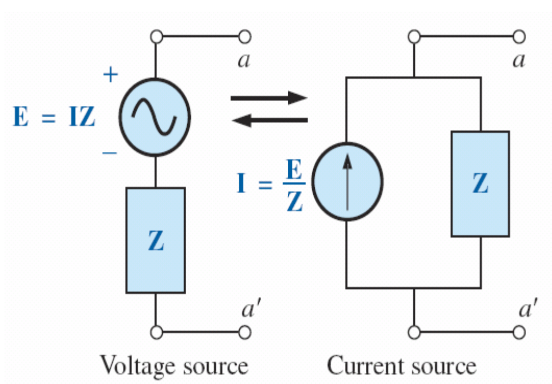
Convert to a current source

$$\vec{I} = \frac{\vec{E}}{\vec{Z}} = \frac{20 \cdot \vec{V}}{5000\,\Omega}$$

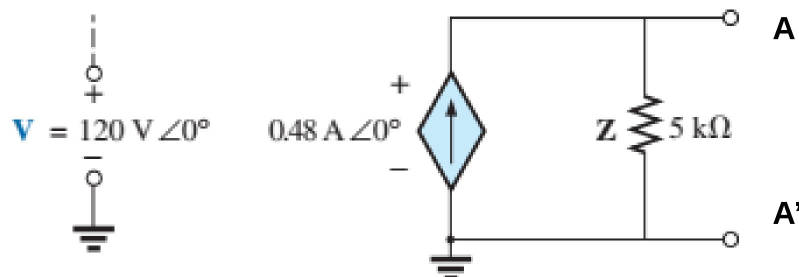
$$= \frac{20\,(120\,V_{RMS}\ \angle 0^\circ\,)}{5000\,\Omega}$$

$$\boxed{\vec{I} = 480\,mA_{RMS}\ \angle 0^\circ}$$

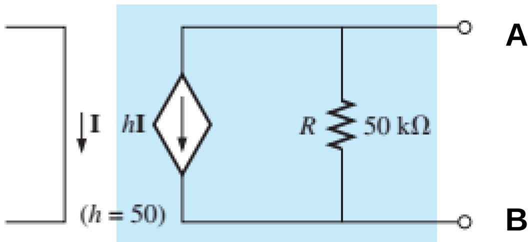
Recall:



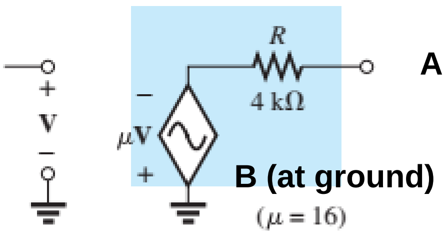
The equivalent source is:



In Class Problem Set – Source Conversions



Convert to a voltage source



Convert to a current source

Simultaneous Equations – Using the Inverse Matrix Method (and your calculator)

Consider the following 2x2 system of equations:

$$\begin{aligned} x + 2y &= 4 \\ 3x - 5y &= 1 \end{aligned}$$

The system can be described by the following matrices:

$$\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

So we can write:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}, \qquad X = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad \text{and} \qquad B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

And to solve, we use matrix algebra:

AX = B

Or $X = A^{-1}B$

Here:

$$A^{-1} = \begin{bmatrix} 454.5\times10^{-3} & 181.8\times10^{-3} \\ 272.7\times10^{-3} & -90.91\times10^{-3} \end{bmatrix}$$

$X = A^{-1}B$

And hence:

x = 2
y = 1

From: http://www.mathcentre.ac.uk/resources/uploaded/sigma-matrices8-2009-1.pdf

In Class Problem Set – Simultaneous Equations

Solve the following 3x3 system of equations (from last semester's mesh/nodal lab) using the inverse matrix method :

$7300\, I_1 - 2200\, I_2 - 3300\, I_3 = 15$ (E1)
$-2200\, I_1 + 3960\, I_2 - 560\, I_3 = 0$ (E2)
$-3300\, I_1 - 560\, I_2 + 4680\, I_3 = -5$ (E3)

Solve the following 2x2 system of **complex equations** (note I_1 and I_2 represent phasors and hence have magnitude and angle components) using the inverse matrix method:

$-1000\, \mathbf{I_1} + 12{,}000\, \mathbf{I_2} = 10$ (E1)

$0\, \mathbf{I_1} - (2000 - j4000)\, \mathbf{I_2} = -10$ (E2)