Phasors and Impedance

- Phasor Representations
  - Introduction for voltage and current
  - Phasor arithmetic (addition)
  - Peak vs RMS Values
  - **ICP 1 – Phasors (conversion and arithmetic)**
- Impedance for the Basic Elements
  - Resistors
  - Inductors
  - Capacitors
  - **ICP 2 – Impedance (Find Z, find the component given Z)**
- Lab #3 – Prelab Discussion and Demo
  - Finding the impedance of and current through a series circuit
  - Using Multisim to verify your results

PHASOR REPRESENTATION FOR $v(t)$, $i(t)$

RECALL: $v(t) = V_m \sin(\omega t + \theta)$
$i(t) = I_m \sin(\omega t + \theta)$ } SAME FREQUENCY

- WE NEED A CONVENIENT WAY TO REPRESENT VOLTAGE, CURRENT + IMPEDANCE
  > ALL HAVE MAGNITUDE + PHASE
  > SAME FREQUENCY FOR A GIVEN CIRCUIT / PROBLEM

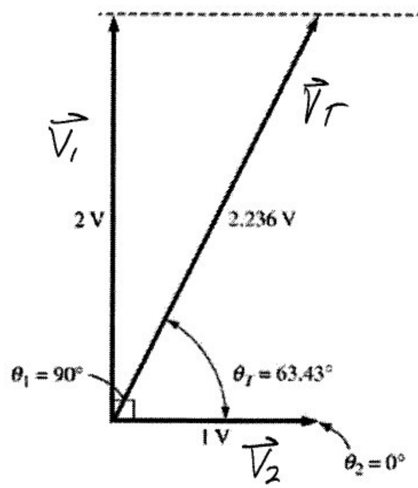
- SINCE ONLY MAGNITUDE + PHASE ARE PARAMETERS, WE DEFINE VOLTAGE, CURRENT + IMPEDANCE PHASORS:

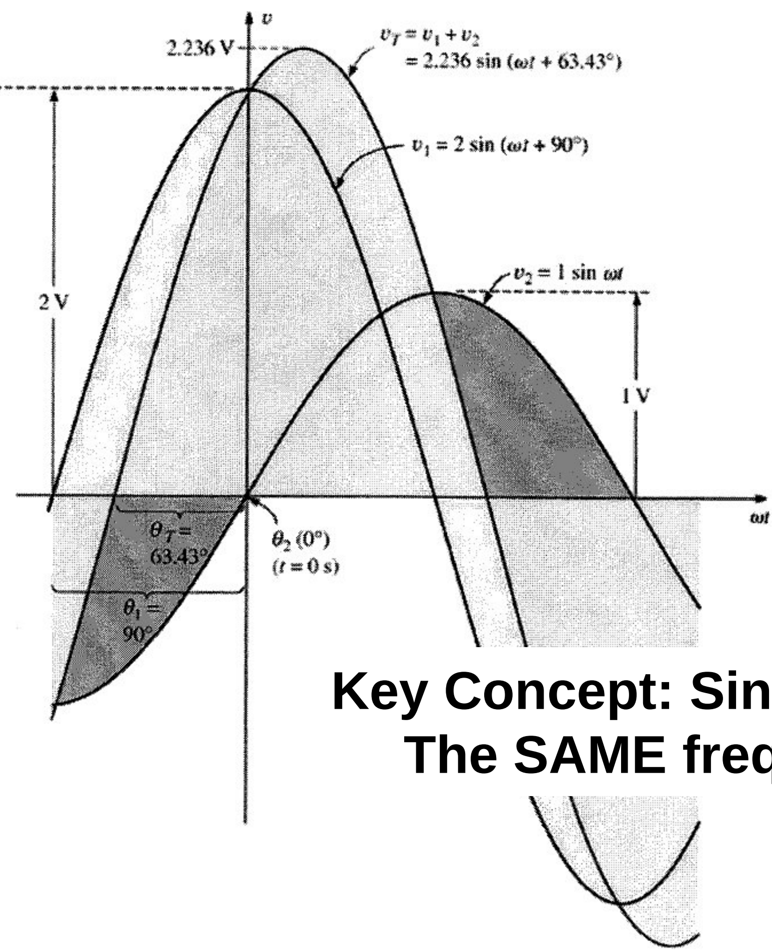
$A \sin(\omega t + \theta) \longrightarrow A \angle \theta$, FOR VOLTAGE OR CURRENT

$V_m \sin(\omega t + \theta) \longrightarrow \vec{V} = V_m \angle \theta$

$I_m \sin(\omega t + \theta) \longrightarrow \vec{I} = I_m \angle \theta$

Using Phasor Representation to Add Sinusoids





(b)

SNAPSHOT OF THE WAVEFORMS AT $t = 0$

$$\vec{V}_T = \vec{V}_1 + \vec{V}_2$$

$$= 2\angle 90° + 1\angle 0°$$

$$\boxed{\vec{V}_T = 2.236V \angle 63.44°}$$

Key Concept: Sinusoids of The SAME frequency

Robert L. Boylestad
Introductory Circuit Analysis, 10ed.

Using Phasor Representation to Add Sinusoids

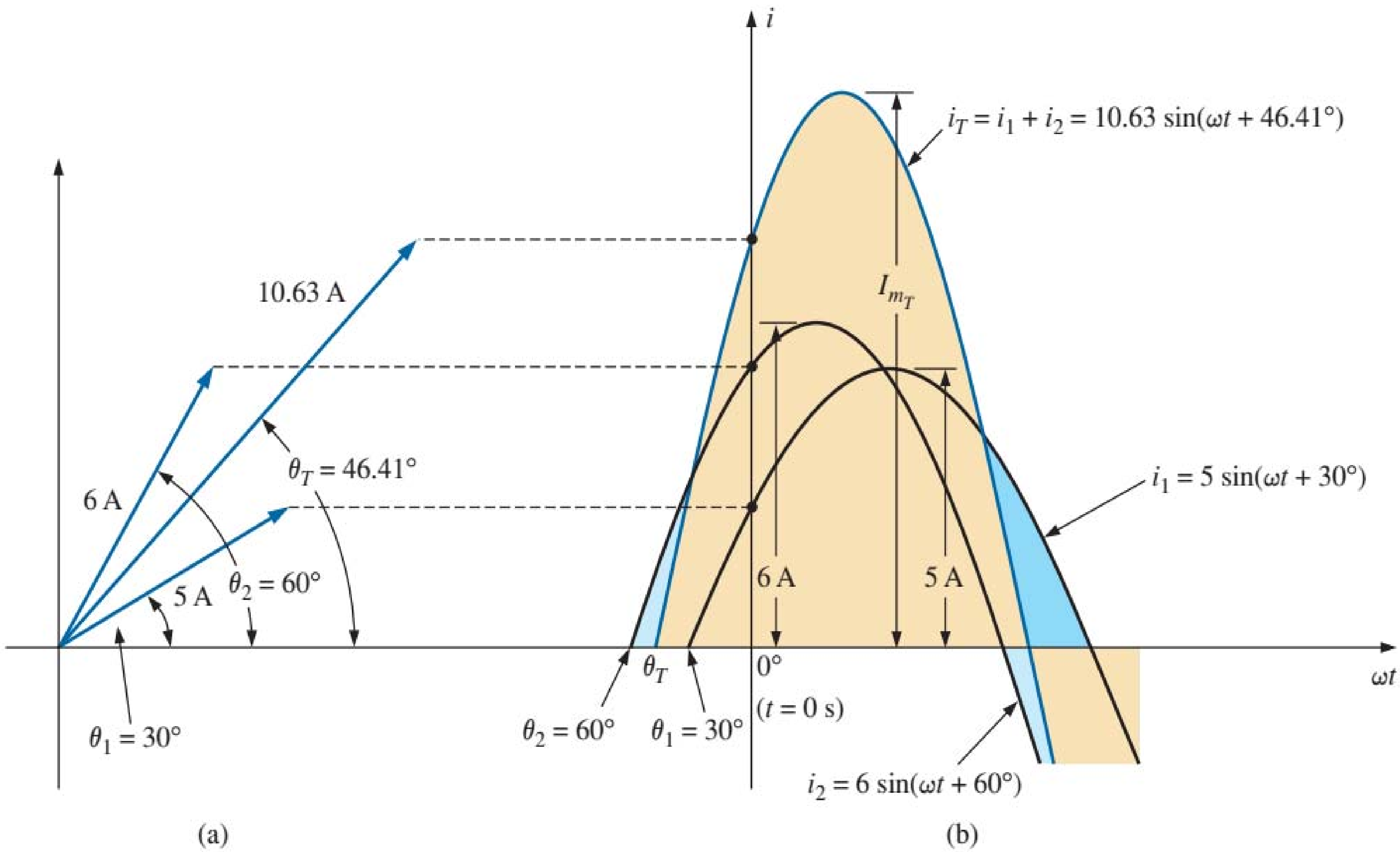


FIG. 14.68
Example 14.27

Phasor Representation for Voltage and Current

FOR THE TEXT:

$V_m \sin(\omega t + \theta) \longrightarrow \vec{V} = (V_m/\sqrt{2}) \angle \theta$

$I_m \sin(\omega t + \theta) \longrightarrow \vec{I} = (I_m/\sqrt{2}) \angle \theta$

RMS VALUES

EXPRESS IN PHASOR FORM

$v(t) = 42 \sin(377t + 0^\circ)$ V

Or 42Vpk < 0°

$\vec{V} = (42/\sqrt{2})\ V_{RMS} \angle 0^\circ$

* YOU MUST USE RMS DESIGNATOR $\angle$

$\boxed{\vec{V} = 29.7\ V_{RMS} \angle 0^\circ}$

$v(t) = 3.6\times10^{-6} \cos(754t - 20^\circ)$

Or 3.6E-6Vpk < 70°

$\vec{V} = \frac{3.6\times10^{-6}}{\sqrt{2}}\ V_{RMS} \angle 70^\circ$

$\boxed{\vec{V} = 2.55\mu V_{RMS} \angle 70^\circ}$

EXPRESS AS A SINE WAVE w/ f = 60Hz

If 120Vpk, we would have:
120 sin(377t) V

$\vec{V} = 120V \angle 0^\circ \rightarrow 120 V_{RMS} \angle 0^\circ$, TEXT

$\therefore v(t) = 120\sqrt{2} \sin(377t + 0^\circ)$

$\boxed{v(t) = 169.71 \sin(377t)\ V}$

ICP Set 1 – Phasors

Use PEAK Values for your voltage and current phasors and make sure you LABEL your phasors properly

1 - Express the following in phasor form:

a) $230 \sin(wt + 30^{O})$ V

b) $10 \sin(wt - 90^{O})$ A

2 - Express the following as sinusoids at f = 1kHz:

a) $\mathbf{I} = 10E\text{-}3A_{pk} < -80^{O}$

b) $\mathbf{V} = 169V_{pk} < 45^{O}$

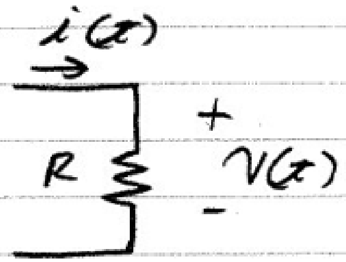
c) $\mathbf{V} = 10V_{RMS} < 23^{O}$

3 - Find v(t) = v1(t) + v2(t) if

$v1(t) = 10 \sin(wt + 10^{O})$ and $v2(t) = 5 \sin(wt - 10^{O})$

Impedance - Resistors

RECALL:



$$v(t) = V_m \sin(\omega t) \quad V$$

$$i(t) = I_m \sin(\omega t) \quad A$$

$$I_m = \frac{V_m}{R}$$

IN PHASOR FORM:

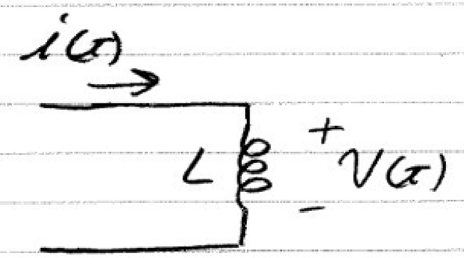
$$\vec{V} = V_m \angle 0^\circ$$

$$\vec{I} = I_m \angle 0^\circ = \frac{V_m}{R} \angle 0^\circ$$

$$\therefore \frac{\vec{V}}{\vec{I}} = \frac{V_m \angle 0^\circ}{\frac{V_m}{R} \angle 0^\circ} = \boxed{R \angle 0^\circ \text{ OR } R + j0}$$

$\nwarrow \vec{Z}$

Impedance - Inductors



$$i(t) = I_m \sin(\omega t) \quad A$$

$$V(t) = \omega L I_m \sin(\omega t + 90^\circ) \quad V$$

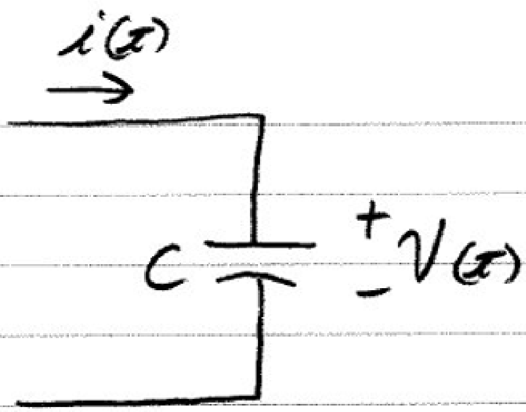
$$\vec{I} = I_m \angle 0^\circ$$

$$\vec{V} = \omega L I_m \angle 90^\circ$$

$$\therefore \frac{\vec{V}}{\vec{I}} = \frac{\omega L I_m \angle 90^\circ}{I_m \angle 0^\circ} = \boxed{\begin{array}{c} \omega L \angle 90^\circ \\ \text{OR} \\ j\omega L \\ \text{OR} \\ jX_L \end{array}}$$

$$\vec{Z} \rightarrow$$

Impedance - Capacitors



$$v(t) = V_m \sin(\omega t) \quad V$$

$$i(t) = \omega C V_m \sin(\omega t + 90^\circ) \quad A$$

$$\vec{V} = V_m \angle 0^\circ$$

$$\vec{I} = \omega C V_m \angle 90^\circ$$

$$\therefore \frac{\vec{V}}{\vec{I}} = \frac{V_m \angle 0^\circ}{\omega C V_m \angle 90^\circ} = \boxed{\begin{array}{c} \frac{1}{\omega C} \angle -90^\circ \\ \text{OR} \\ X_c \angle -90^\circ \\ \text{OR} \\ \frac{-j}{\omega C} \Rightarrow -jX_c \end{array}}$$

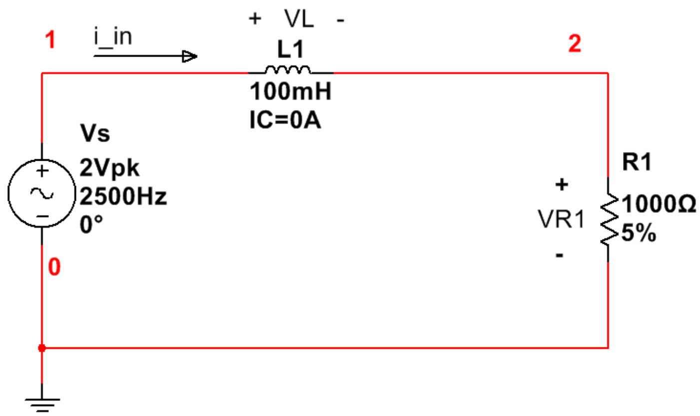
ICP Set 2 – Impedance

1 – Find the impedance of:

a) A 1000 Ohm resistor
b) A 100mH inductor at 60 Hz
c) A 100mH inductor at 2.5kHz
d) A 0.1uF capacitor at 60 Hz
e) A 0.1uF capacitor at 2.5kHz

2 - If a component has an impedance of $4\angle 90^{\circ}$ Ohms at f=400Hz, find the component type and value

Lab #3 Prelab (Partial)



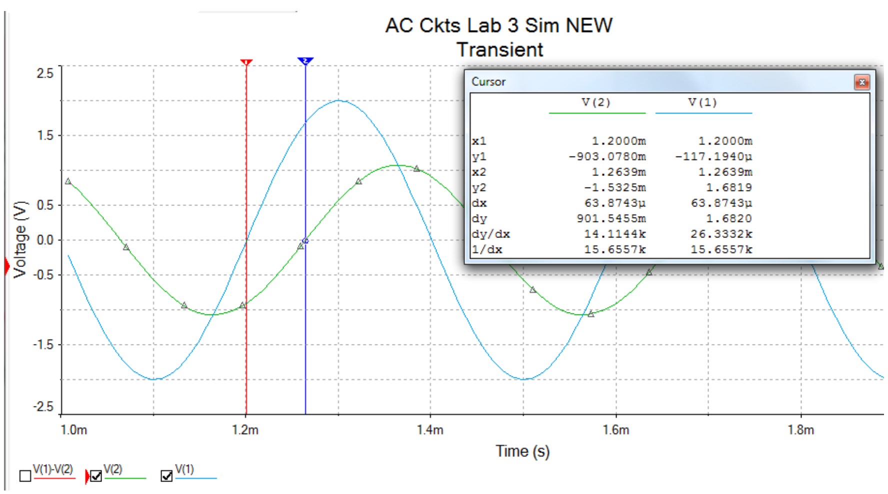
Z_{L1} = wL_1< 90°= 1,571 Ohms < 90°

Z_{R1} = R_1< 0 ° = 1000 Ohms < 0°

Z_T = Z_{L1}+Z_{R1} = 1862 Ohms < 57.5°

I_in = Vs/Z_T = (2Vpk<0) / (1862 Ohms < 57.5°)

1. **Find the impedance of each component**
2. **Use Ohm's Law for phasors to determine i_in(t), VL(t) and VR1(t)**
3. **Verify your results by simulating the circuit in Multisim**



I_in = V_{R1}/1000 = 1.07Vpk/1000 Ohms, lags **Vs** by 63.9uSec

I_in = 1.07mApk< -57.5°

i_in(t) = 1.07E-3 sin(15,708t-57.5°) A