Ladder Networks – ICP

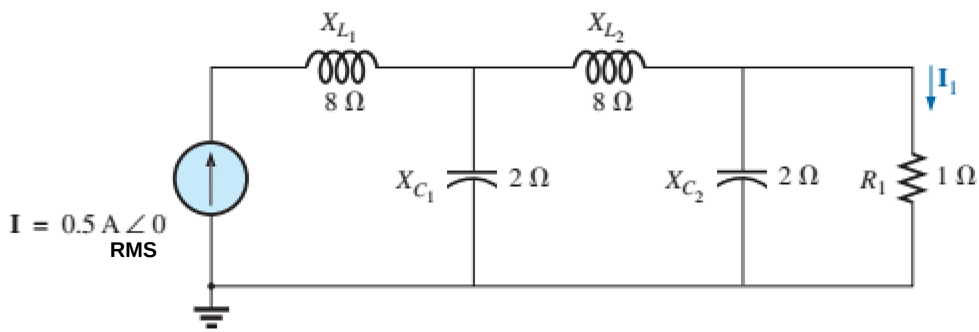

FIG. 17.54

Find: The voltage across L_2, the current through C_2 (not $\mathbf{I_1}$) and the average power delivered to the circuit by the source

One Strategy:
- Combine individual elements to find $\mathbf{Z_T}$ and then **Vin** and Pave
- Use voltage divider to find $\mathbf{V_{C1}}$ and $\mathbf{V_{C2}}$ and hence $\mathbf{V_{L2}}$
- Use Ohms Law to find $\mathbf{I_{C2}}$

Check(s):
- Vin on the order of $0.5A_{RMS}$ * 8 Ohms or 4V
- Pave = $|\mathbf{V_{C2}}|^2/R_1$, check against V_{inRMS}*I_{inRMS}*Cos(theta)

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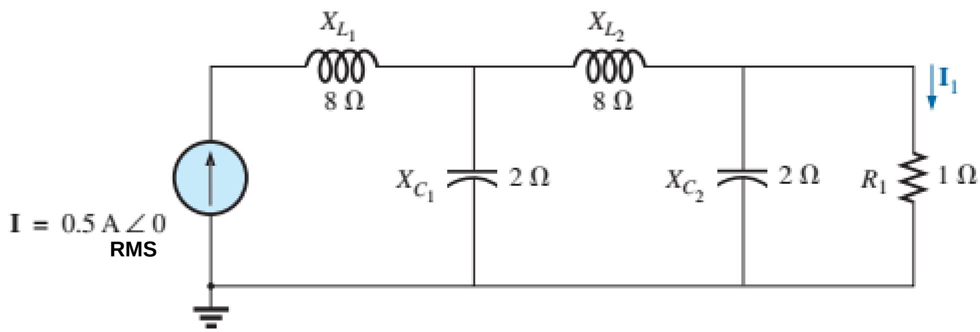
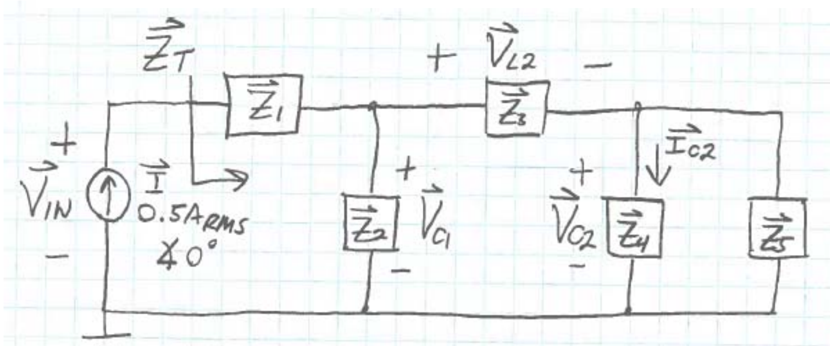


FIG. 17.54

Finding **ZT**

$$\vec{Z}_T = \Bigg(\overbrace{\Big[\underbrace{\underbrace{\left(\vec{Z}_4 /\!/ \vec{Z}_5\right)}_{\vec{Z}_{4,5}} + \vec{Z}_3}_{\vec{Z}_{3,4,5}}\Big] /\!/ \vec{Z}_2}^{\vec{Z}_{2,3,4,5}}\Bigg) + \vec{Z}_1$$

Redrawing the circuit with the knowns



$\vec{Z}_1 = +j8\,\Omega$ $\quad$ $\vec{Z}_4 = -j2\,\Omega$

$\vec{Z}_2 = -j2\,\Omega$ $\quad$ $\vec{Z}_5 = 1\,\Omega$

$\vec{Z}_3 = +j8\,\Omega$

$$\vec{Z}_T = \Bigg(\overbrace{\Big[\underbrace{\underbrace{\left(-j2\,\Omega /\!/ 1\,\Omega\right)}_{(0.8 - j0.4)\,\Omega} + j8\,\Omega}_{(0.8 + j7.6)\,\Omega}\Big] /\!/ -j2\,\Omega}^{(0.1 - j2.7\,\Omega)}\Bigg) + j8\,\Omega$$

$$\underline{\vec{Z}_T = (0.1 + j5.3)\,\Omega}$$

Finding **Vin**

$$\therefore \vec{V}_{IN} = \vec{I} \cdot \vec{Z}_T = (0.5A_{RMS} \measuredangle 0°)(0.1 + j5.3\,\Omega)$$

$$= \underline{2.65\,V_{RMS} \measuredangle 88.92°} \quad \checkmark$$

Check: Vin is on the order of 4V

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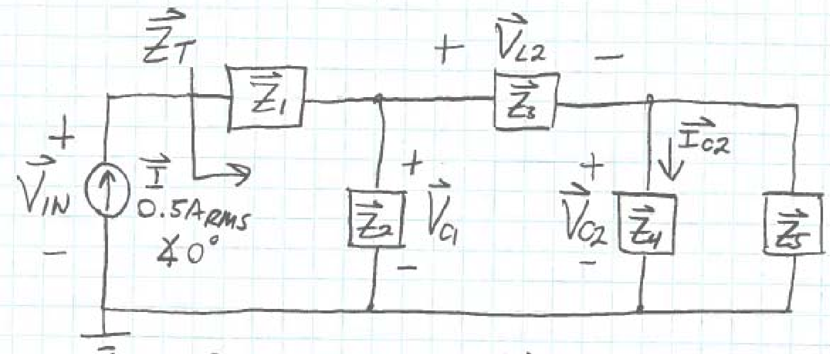


$\vec{Z}_1 = +j8\,\Omega$ $\quad$ $\vec{Z}_4 = -j2\,\Omega$

$Z_2 = -j2\,\Omega$ $\quad$ $\vec{Z}_5 = 1\,\Omega$

$\vec{Z}_3 = +j8\,\Omega$

Finding **Vc1**

$$\vec{V}_{C1} = \vec{V}_{IN}\left(\frac{\vec{Z}_{2,3,4,5}}{\vec{Z}_{2,3,4,5} + \vec{Z}_1}\right)$$

$$= 2.65\,V_{RMS}\angle 88.92^\circ \left(\frac{(0.1 - j2.7)\,\Omega}{(0.1 + j5.3)\,\Omega}\right)$$

$$\underline{\vec{V}_{C1} = 1.35\,V_{RMS}\angle -87.88^\circ}$$

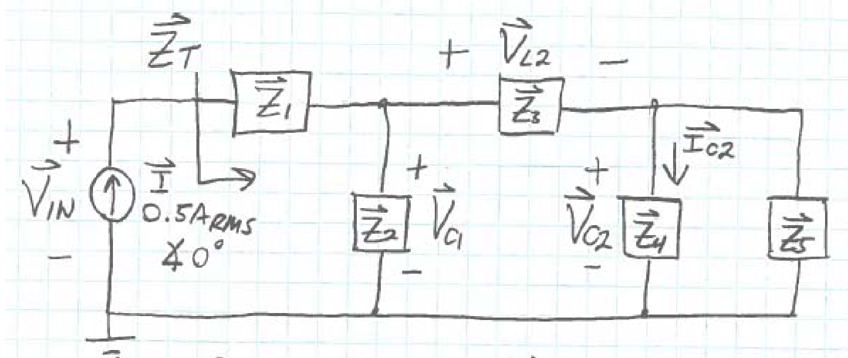
Finding Pave

$$P_{AVE} = V_{IN_{RMS}} \cdot I_{RMS} \cdot \cos(\theta_V - \theta_I)$$

$$= (2.65V)(0.5A)\ \cos(88.92^\circ - 0^\circ)$$

$$= (2.65V)(0.5A)(0.019) = \boxed{24.97\ mW}$$

Ladder Networks – ICP



$\vec{Z}_1 = +j8\,\Omega \qquad \vec{Z}_4 = -j2\,\Omega$

$\vec{Z}_2 = -j2\,\Omega \qquad \vec{Z}_5 = 1\,\Omega$

$\vec{Z}_3 = +j8\,\Omega$

Finding **Ic2**

$$\vec{I}_{C2} = \frac{\vec{V}_{C2}}{\vec{Z}_4} = \frac{158\,mV_{RMS}\,\angle 161.6^\circ}{-j2\,\Omega}$$

$$\boxed{\vec{I}_{C2} = 79\,mA\,\angle -108.4^\circ}$$

Finding **Vc2** and hence **VL2**

$$\vec{V}_{C2} = \vec{V}_{C1}\left(\frac{\vec{Z}_{4,5}}{\vec{Z}_{3,4,5}}\right)$$

$$= 1.35\,V_{RMS}\,\angle -87.88^\circ \left(\frac{(0.8 - j0.4)\,\Omega}{(0.8 + j7.6)\,\Omega}\right)$$

$$\vec{V}_{C2} = \underline{158\,mV_{RMS}\,\angle 161.6^\circ}$$

$$\therefore \ \vec{V}_{L2} = \vec{V}_{C1} - \vec{V}_{C2}$$

$$= \boxed{1.41\,V_{RMS}\,\angle -81.9^\circ}$$

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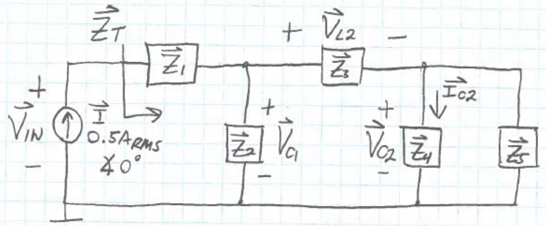
Ladder Networks – ICP



$\vec{Z}_1 = +j8\Omega \qquad \vec{Z}_4 = -j2\Omega$

$\vec{Z}_2 = -j2\Omega \qquad \vec{Z}_5 = 1\Omega$

$\vec{Z}_3 = +j8\Omega$

A couple more checks

$$\boxed{\vec{I}_{C2} = 79\ mA\ \angle -108.4°}$$

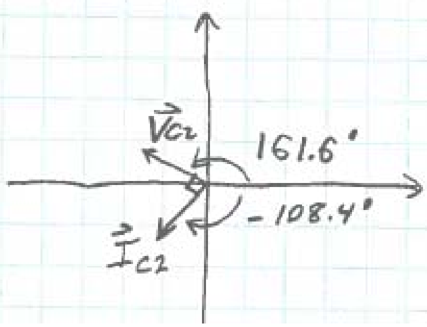
$$\vec{V}_{C2} = 158\ mV_{RMS}\ \angle 161.6°$$

CHECK

DOES $\vec{I}_{C2}$ LEAD $\vec{V}_{C2}$ BY $90°$?

$$-108.4° - (161.6°) = -53.2° \ ??$$

CONSIDER :



$$360° - 161.6° - 108.4°$$
$$= 90° \ \checkmark$$
$\therefore\ \vec{I}$ LEADS $\vec{V}$ $\checkmark$

CHECK

$P_{R1} = P_{AVE}$?

$$P_{R1} = \frac{|\vec{V}_{C2}|^2_{RMS}}{\vec{Z}_5}$$

$$\frac{(158\ mV)^2}{1\Omega}$$

$$= \boxed{24.96\ mW} \ \checkmark$$

Both check out ☺

NP