Electrical Engineering Technology

Transformers

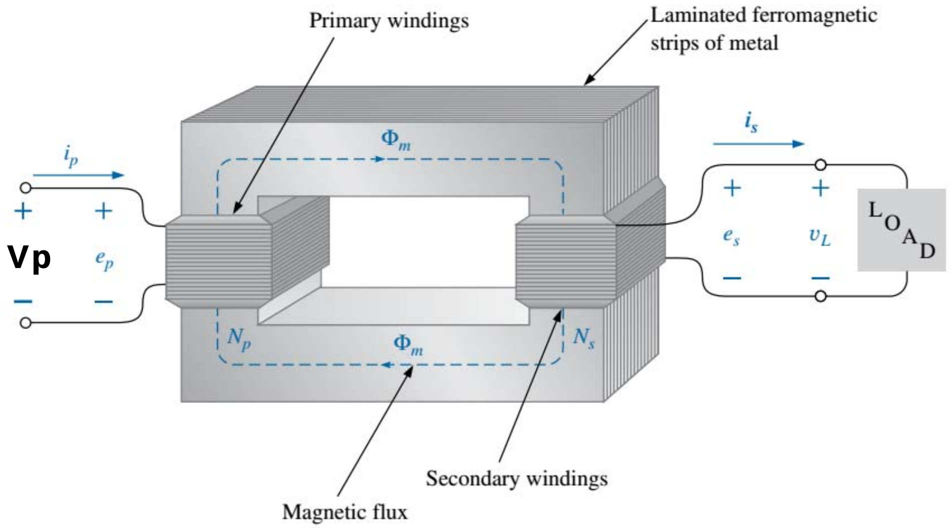
Spring 2019 (2185)

Transformers

- Analysis
  - Voltage ratio
  - Current ratio
  - Turns ratio
  - Dot convention
- Transformer Example Problem
- Transformer In Class Problem

Transformer Analysis (voltage-ratio)

Consider the following transformer
(ideal model, k=1):



LET $I_p(t) = I_m \sin(\omega t)$

"Ohm's Law" for magnetic circuits
(from Chapter 12):

Effect = cause/opposition

$$\underset{\substack{\text{FLUX}\\\text{(WEBERS)}}}{\phi} = \frac{\mathcal{F}}{R} \quad (12.5)$$

$\mathcal{F}$ ← MAGNETOMOTIVE FORCE (A·t)

R ← RELUCTANCE (A·t/WEBER)

∴ THE FLUX GENERATED BY $\vec{V}_p$, $\vec{I}_p$ WILL HAVE THE FORM:

$$\phi = \frac{N_p \cdot I_p(t)}{R}$$

OR $\phi = \Big(\underbrace{\frac{N_p}{R} I_m}_{\Phi_m}\Big)\sin(\omega t)$ (WEBERS)

∴ $e_p = N_p \dfrac{d\phi_p}{dt}$ BECOMES:

$e_p(t) = N_p \dfrac{d}{dt}\left[\Phi_m \sin(\omega t)\right]$ VOLTS

OR $e_p(t) = N_p \Phi_m \omega \cos(\omega t)$ VOLTS

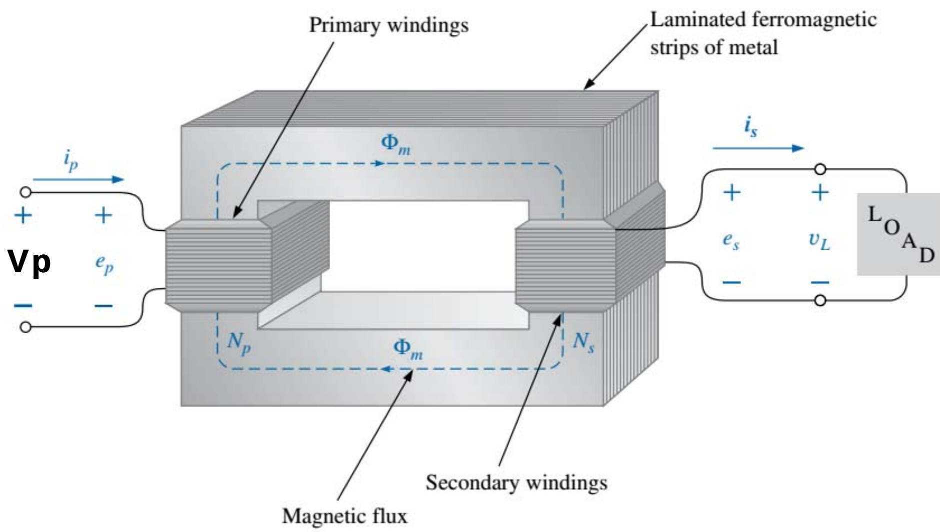
BUT $\cos(\omega t) = \sin(\omega t + 90°)$

∴ $e_p(t) = N_p(2\pi f)\Phi_m \sin(\omega t + 90°)$ VOLTS

$e_p(t) \cong (6.283)(f N_p)\Phi_m \sin(\omega t + 90°)$ V

Transformer Analysis (voltage-ratio)

Consider the following transformer (ideal model, k=1):



$$I_p(t) = I_m \sin(\omega t)$$

$$e_p(t) \cong (6.283)(f N_p)\Phi_m \sin(\omega t + 90^\circ)\ V$$

So, $\boxed{\vec{V}_p \cong (4.44\, f N_p \Phi_m) \angle 90^\circ\ V_{RMS}}$

$$\boxed{\vec{I}_p = \frac{I_m}{\sqrt{2}} \angle 0^\circ\ A_{RMS}}$$

Note:

$\vec{V}_p$ LEADS $\vec{I}_p$ BY 90° (ELI)

$|\vec{V}_p|$ IS DIRECTLY RELATED TO f, N_p + Φ_m

Looking at the secondary:

$e_s = N_s \dfrac{d\phi_m}{dt}$ BECOMES:

$$e_s(t) = N_s(2\pi f)\Phi_m \sin(\omega t + 90^\circ)\ V$$

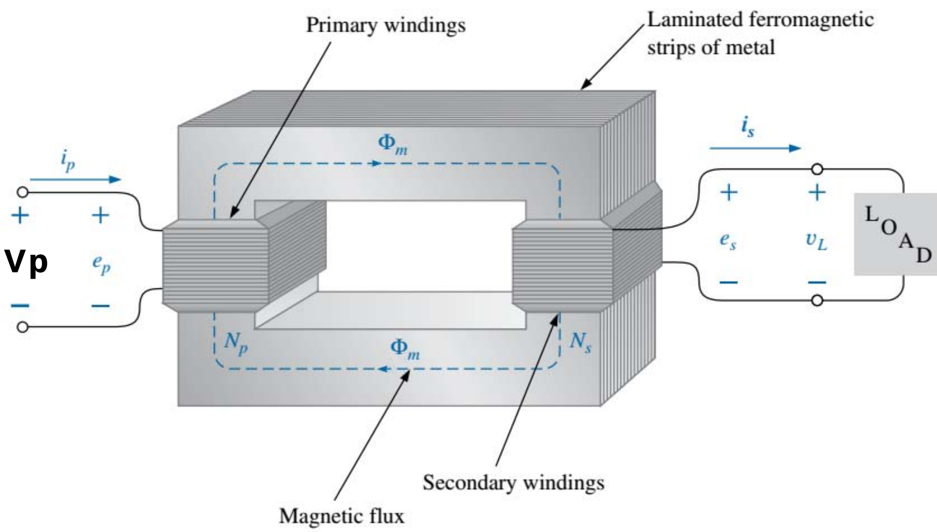
+ $\boxed{\vec{V}_s \cong 4.44\, f N_s \Phi_m \angle 90^\circ\ V_{RMS}}$

So we have the relationship:

$$\boxed{\frac{\vec{V}_p}{\vec{V}_s} = \frac{N_p}{N_s}}$$

Transformer Analysis (turns ratio, dot convention)

Consider the following transformer
(ideal model, k=1):



$$\boxed{\frac{\vec{V}_p}{\vec{V}_s} = \frac{N_p}{N_s}}$$

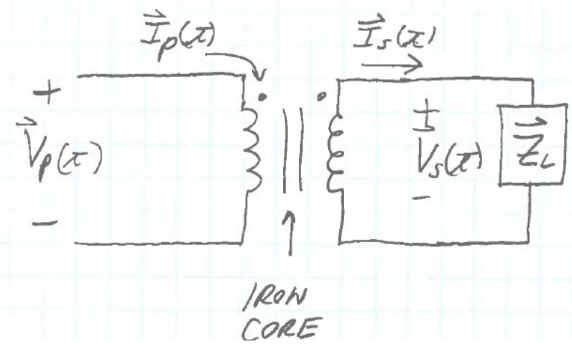
LET $a = \dfrac{N_p}{N_s}$, THE TURNS RATIO

$$\left.\begin{array}{l} \vec{V}_p = a\vec{V}_s \\ V_s = \dfrac{\vec{V}_p}{a} \end{array}\right\}$$

$a<1: |\vec{V}_s| > |\vec{V}_p|$, STEP UP

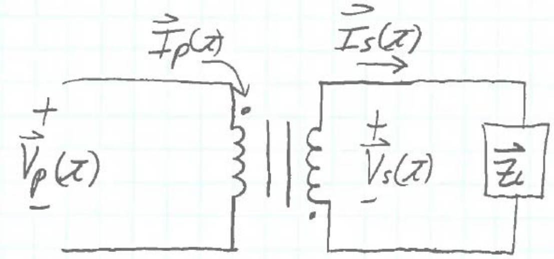
$a>1: |\vec{V}_s| < |\vec{V}_p|$, STEP DOWN

DOT CONVENTION



$\vec{V}_p, \vec{V}_s$ IN PHASE

$\vec{I}_p, \vec{I}_s$ IN PHASE

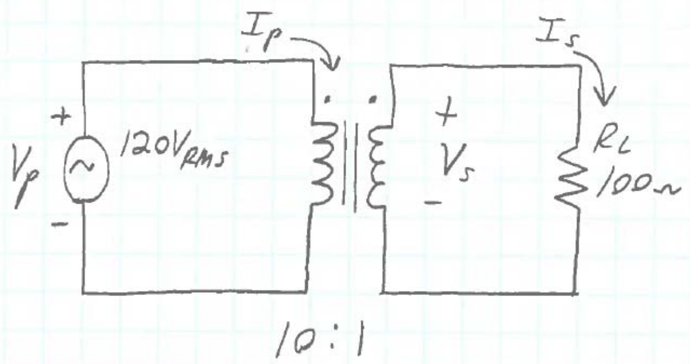


$\vec{V}_p, \vec{V}_s$ 180° OUT OF PHASE

$\vec{I}_p, \vec{I}_s$ 180° OUT OF PHASE

Transformer Analysis (current ratio)

EXAMPLE



$$a = \frac{N_P}{N_S} = 10, \quad \text{STEP DOWN}$$

FIND: V_S, I_S, I_P (RMS VALUES)

$$V_S = \frac{V_P}{a} = \frac{120V_{RMS}}{10} = \boxed{12V_{RMS}}$$

$$I_S = \frac{V_S}{R_L} = \frac{12V_{RMS}}{100\Omega} = \boxed{120\,mA_{RMS}}$$

What about Ip?

An iron-core, **lossless** transformer (k=1)

Therefore, **Pin = Pout**

$$P_{RL} = I_S^2 \cdot R_L$$
$$= (120mA)^2 \cdot 100\Omega = \underline{\underline{1.44W}}$$

$$\therefore P_{PRIMARY} = 1.44W$$
$$= (120V_{RMS})(I_P)$$

$$\therefore \boxed{I_P = 12\,mA_{RMS}}$$

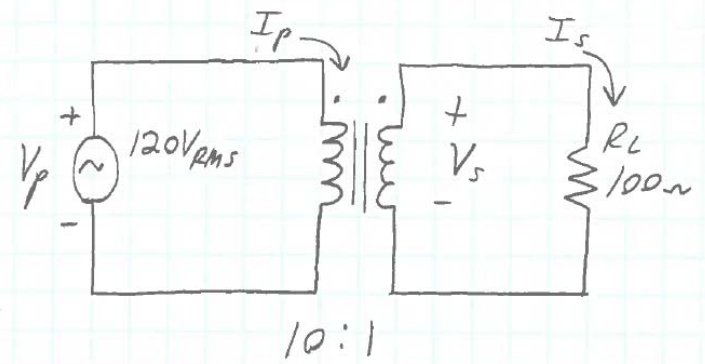
$$\text{OR} \quad I_P = \frac{I_S}{a}$$

So, we have established:

$$\boxed{\frac{V_P}{V_S} = \frac{N_P}{N_S} = a = \frac{I_S}{I_P}}$$

Transformer Analysis (example)

EXAMPLE



$\therefore \quad \boxed{I_p = 12\,mA_{RMS}}$

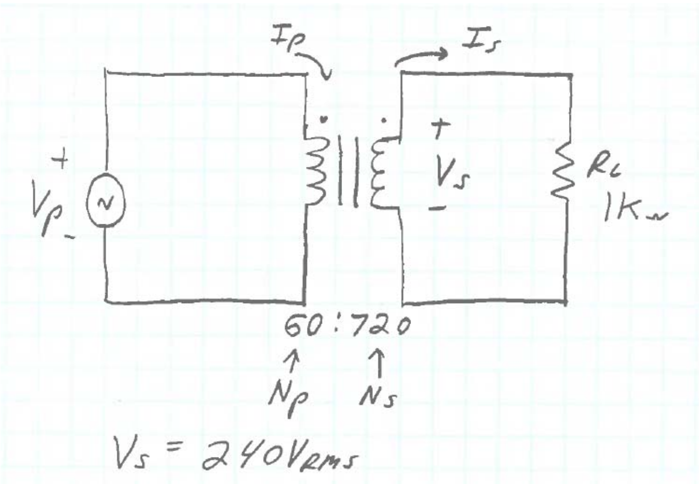
$$\boxed{\frac{V_p}{V_s} = \frac{N_p}{N_s} = a = \frac{I_s}{I_p}}$$

What’s the magnitude of the input impedance?

$$\frac{V_p}{I_p} = \frac{120V_{RMS}}{12\,mA_{RMS}} = \boxed{10{,}000\ \Omega}$$

$$= a^2 \cdot R_L$$

Transformer Analysis (In Class Problem)



Find:

a) Vp
b) Is
c) Ip
d) Psupplied by Vp