Transformer Intro

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Transformer Intro - Objectives

- Become familiar with the flux linkages that exist between the coils of a transformer and how the voltages across the primary and secondary are established (and related).
- Understand the operation of an iron-core and air-core transformer and know how to calculate the currents and voltages of the primary and secondary circuits.
- Be aware of how the transformer is used for impedance matching purposes to ensure a high level of power transfer.
- Become aware of all the components that make up the equivalent circuit of a transformer and how they affect its performance and frequency response. <- Machines and Transformers

Transformer Introduction

- Mutual inductance is a phenomenon basic to the operation of the transformer, an electrical device used today in almost every field of electrical engineering.
- Transformers play an integral part in power distribution systems and can be found in many electronic circuits and measuring instruments.
- In this chapter, we discuss three of the basic applications of a transformer: to build up or step down the voltage or current, to act as an impedance matching device, and to isolate (no physical connection) one portion of a circuit from another.

Transformer Analysis/Equations

Review from Chapter 11 (inductors)

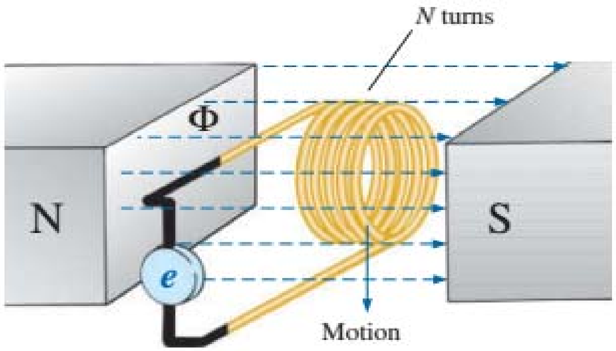
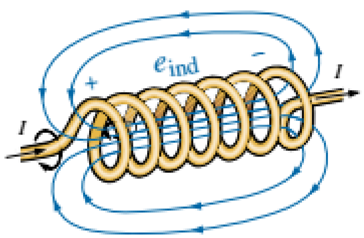


FIG. 11.29
Demonstrating Faraday's law.

$$\boxed{e = N\frac{d\phi}{dt}} \quad \text{(volts, V)}$$

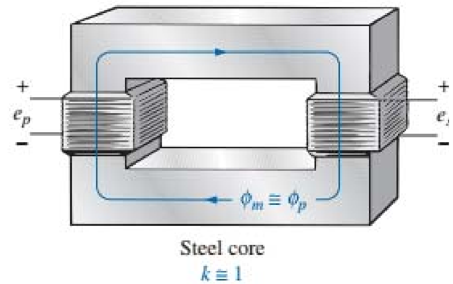


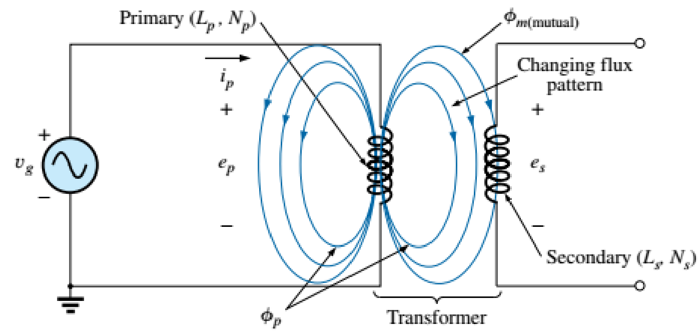
$$\boxed{L = N\frac{d\phi}{di_L}} \quad \text{(henries, H)}$$

$$e = N\frac{d\phi}{dt} = \left(N\frac{d\phi}{di_L}\right)\left(\frac{di_L}{dt}\right)$$

$$\boxed{e_L = L\frac{di_L}{dt}} \quad \text{(volts, V)}$$

Applying to a transformer





$$\boxed{e_p = N_p\frac{d\phi_p}{dt}} \quad \text{(volts, V)}$$

$$\boxed{e_p = L_p\frac{di_p}{dt}} \quad \text{(volts, V)}$$

$$\boxed{e_s = N_s\frac{d\phi_m}{dt}} \quad \text{(volts, V)}$$

If: $\phi_m = \phi_p$

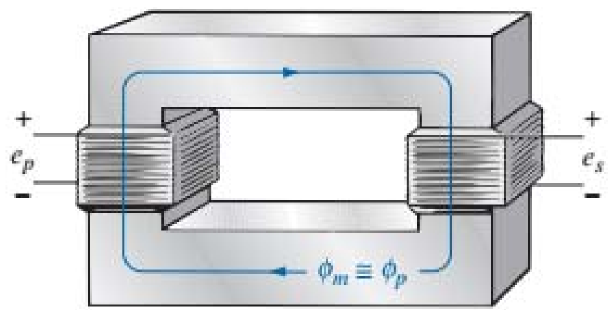
$$\boxed{e_s = N_s\frac{d\phi_p}{dt}} \quad \text{(volts, V)}$$

k is defined:

$$\boxed{k \text{ (coefficient of coupling)} = \frac{\phi_m}{\phi_p}}$$

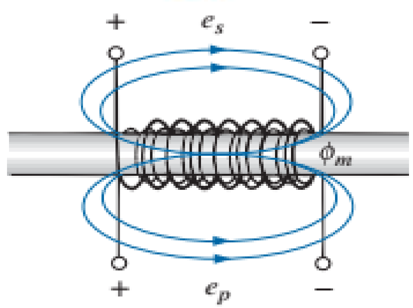
Transformer Intro – Coefficient of Coupling

$$k\ (\text{coefficient of coupling}) = \frac{\phi_m}{\phi_p}$$



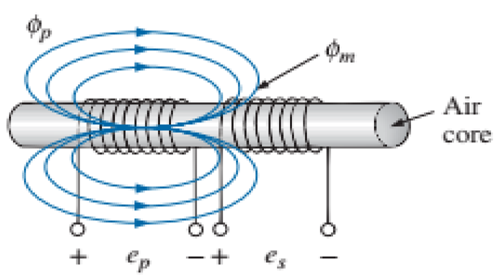
Steel core
$k \cong 1$

Steel core: k~1, most of the flux linking the primary also links the secondary



Any core
$k \cong 1$

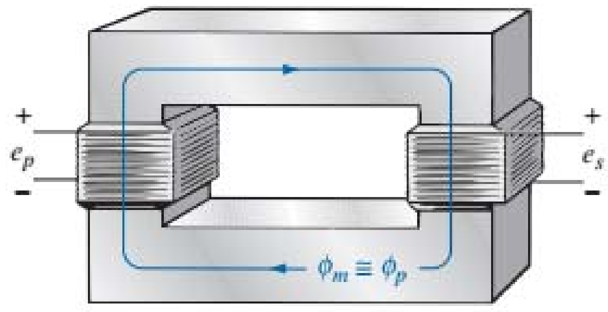
k~1, most of the flux linking the primary also links the secondary (same core, overlapping coils)



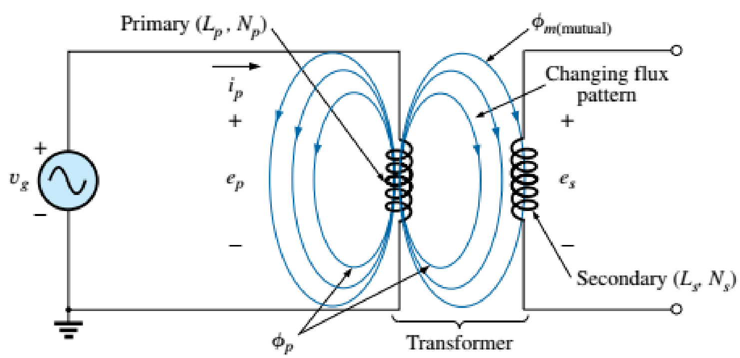
$$k = \frac{\phi_m}{\phi_p} << 1\ (0.01 \longrightarrow 0.3)$$

k<<1, loosely coupled (non-ferromagnetic core AND coils are not overlapping)

Transformer Intro – Mutual Inductance



Steel core
$k \cong 1$



Recall:

$$e_s = N_s \frac{d\phi_m}{dt} \quad \text{(volts, V)}$$

$$k \text{ (coefficient of coupling)} = \frac{\phi_m}{\phi_p}$$

Therefore, for any k:

$$e_s = kN_s \frac{d\phi_p}{dt} \quad \text{(volts, V)}$$

mutual inductance between two coils is proportional to the instantaneous change in flux linking one coil due to an instantaneous change in current through the other coil.

The mutual inductance between the primary and secondary coils is given as:

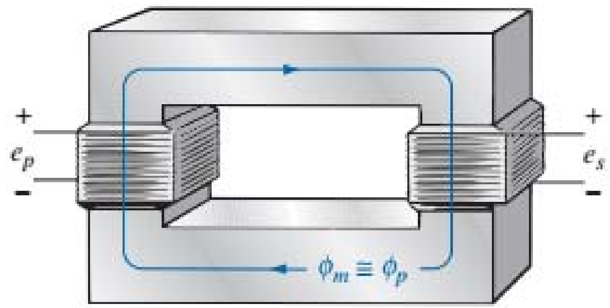
$$M = N_s \frac{d\phi_m}{di_p} \quad \text{(henries, H)}$$

$$M = N_p \frac{d\phi_m}{di_s} \quad \text{(henries, H)}$$

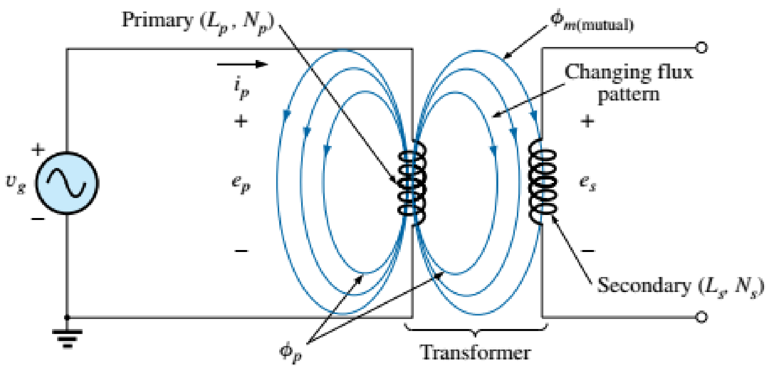
In terms of the primary and secondary inductances, it can be shown that:

$$M = k\sqrt{L_p L_s} \quad \text{(henries, H)}$$

Transformer Intro – Mutual Inductance



Steel core
$k \cong 1$



Recall:

$$\boxed{e_s = N_s \frac{d\phi_m}{dt}} \quad \text{(volts, V)}$$

Rewriting:

$$e_s = N_s\left(\frac{d\phi_m}{di_p}\right)\left(\frac{di_p}{dt}\right)$$

And recall:

$$\boxed{M = N_s \frac{d\phi_m}{di_p}} \quad \text{(henries, H)}$$

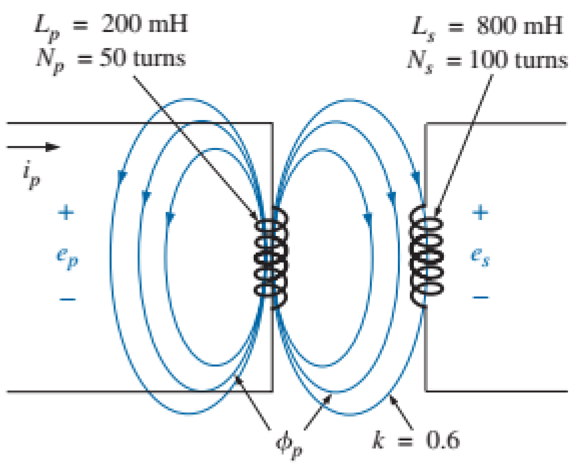
Combining, yields:

$$\boxed{e_s = M \frac{di_p}{dt}} \quad \text{(volts, V)}$$

And similarly:

$$\boxed{e_p = M \frac{di_s}{dt}} \quad \text{(volts, V)}$$

Transformer Intro – Example



Find:

a) e_p if the primary flux is changing at a rate of 450 mWb/s

b) e_s given the same rate of change in primary flux

c) M in Henries

d) e_p and e_s if i_p changes at the rate of 0.2A/ms

$$e_p = N_p \frac{d\phi_p}{dt} = (50)(450\text{ mWb/s}) = \mathbf{22.5\ V}$$

$$e_s = kN_s \frac{d\phi_p}{dt} = (0.6)(100)(450\text{ mWb/s}) = \mathbf{27\ V}$$

$$M = k\sqrt{L_p L_s} = 0.6\sqrt{(200\text{ mH})(800\text{ mH})}$$

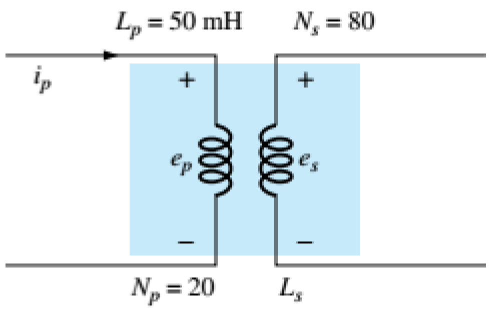
$$= 0.6\sqrt{16 \times 10^{-2}} = (0.6)(400 \times 10^{-3}) = \mathbf{240\ mH}$$

$$e_p = L_p \frac{di_p}{dt} = (200\text{ mH})(0.2\text{ A/ms})$$

$$= (200\text{ mH})(200\text{ A/s}) = \mathbf{40\ V}$$

$$e_s = M \frac{di_p}{dt} = (240\text{ mH})(200\text{ A/s}) = \mathbf{48\ V}$$

Transformer Intro – In Class Problem



k=1

Find:

a) Ls if M = 200 mH

b) e_p and e_s if the flux linking the primary coil changes at a rate of 0.08 Wb/s

c) e_p and e_s if i_p changes at a rate of 0.3 A/ms