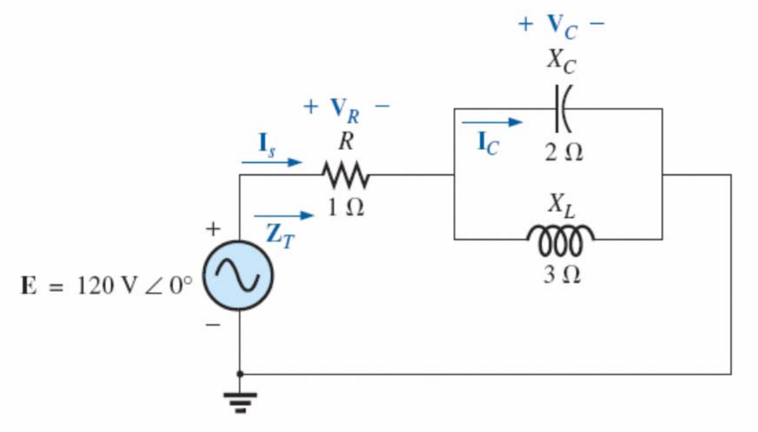
Series-Parallel AC Networks

- Introduction and Approach
  - Introduction
  - **Studying the problem**
  - Complete example (Work as we go in your calculator)
  - **ICP – Partial HW#6 problem**
- Ladder Networks
  - Discussion (specific case)
  - Development of the approach
  - Current divider – Development of the two-impedance special case

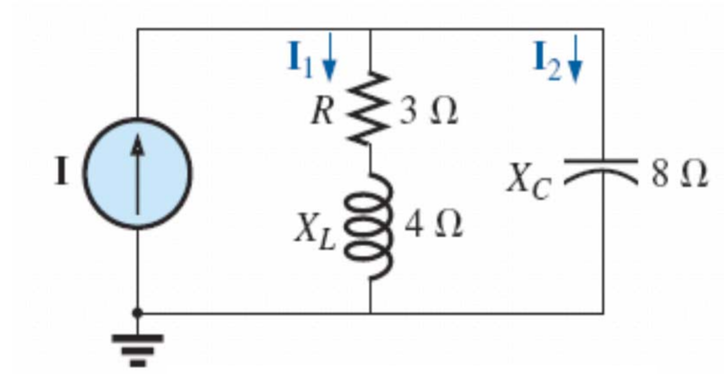
Introduction - Approach

- **In general**, when working with series-parallel ac networks, consider the following approach:
  - Redraw the network, using block impedances to combine obvious series and parallel elements, which will reduce the network to one that clearly reveals the fundamental structure of the system.
  - **1st**: Study the problem and make a brief mental sketch of the overall approach you plan to use. Doing this may result in time and energy-saving shortcuts.
  - After the overall approach has been determined, it is usually best to consider each branch involved in your method independently before tying them together in series-parallel combinations..
  - When you have arrived at a solution, check to see that it is reasonable by considering the magnitudes of the source voltage or current and the elements in the circuit.

Introduction - Approach



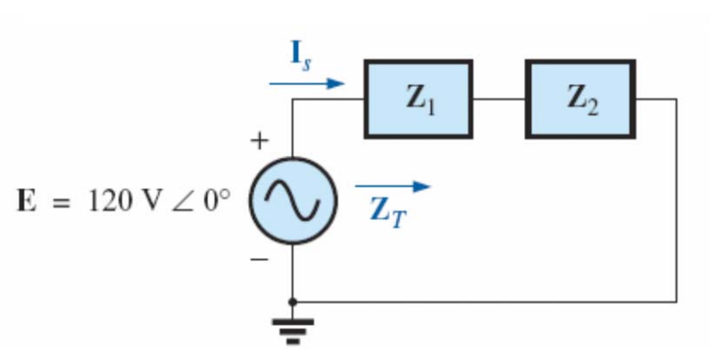
Find: Z_T, I_S, V_R, V_C, I_C

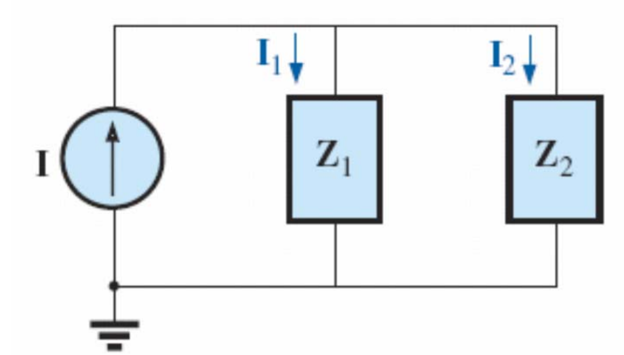


Find: I_1, I_2

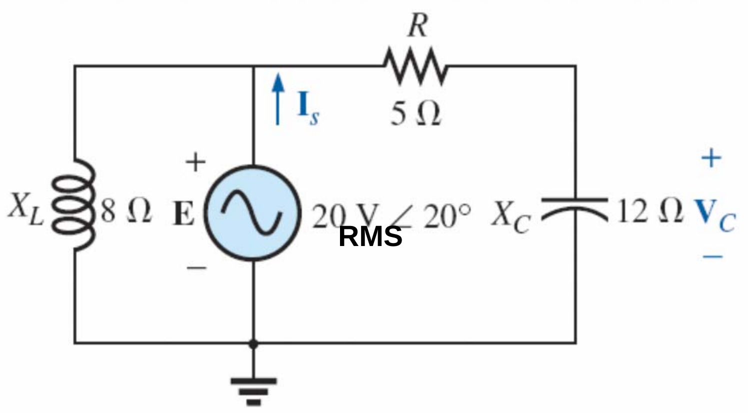
Strategy/Plan 1st
1) Combine L and C
2) Analyze the *series* circuit
   - Z_T, then I_S
   - I_C, then V_C

Strategy/Plan 1st
1) Combine R and L
2) Analyze the *parallel* circuit
   - Use current divider

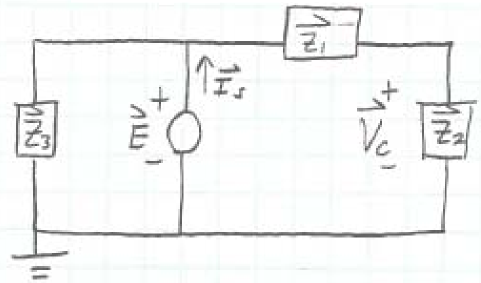




Example – Work in Your Calculator as We Go



Redrawing the circuit with the knowns:



$\vec{E} = 20V_{RMS} \angle 20°$

$\vec{Z}_1 = 5\Omega$

$\vec{Z}_2 = -j12\Omega = 12_\Omega \angle -90°$

$\vec{Z}_3 = +j8\Omega = 8\Omega \angle 90°$

Find: **Is, Vc**, FP and the average power delivered to the network

Strategy/Plan 1st

1) Keep each element
2) Analyze the *series* branch
   - ☐ **VC** (voltage divider)
   - ☐ Combine R and C
3) Analyze the parallel circuit
   - ☐ **IS**, FP, Pave

Check:

- ❑ |**Vc**| on the order of 15VRMS?
- ❑ **E** lead **Is**?

Finding **Vc:**

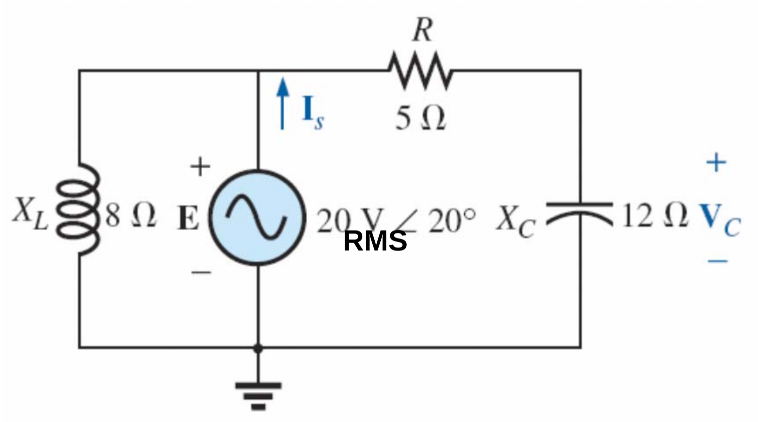
$$\vec{V}_c = \left(\frac{\vec{Z}_2}{\vec{Z}_2 + \vec{Z}_1}\right)\vec{E}$$

$$= 20V_{RMS} \angle 20° \left(\frac{12\Omega \angle -90°}{(5-j12)\Omega}\right)$$

$$\boxed{\vec{V}_c = 18.46V_{RMS} \angle -2.62°}$$

Does this make sense?

Example – Work in Your Calculator as We Go



<u>Find</u>: **Is, Vc,** FP and the average power delivered to the network

<u>Strategy/Plan 1st</u>

1) Keep each element
2) Analyze the *series* branch
   - ☐ **VC** (voltage divider)
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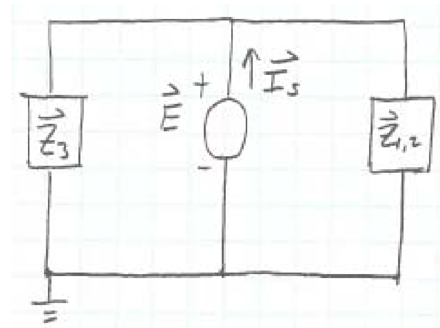
3) Analyze the parallel circuit
   - ☐ **IS,** FP, Pave

<u>Check</u>:

- ❑ **|Vc| on the order of 15VRMS?**
- ❑ **E** lead **Is**?

Redrawing the circuit (per the plan):



Finding **Is:**

$$\vec{I}_s = \frac{\vec{E}}{\vec{Z}_{1,2} // \vec{Z}_3} = \frac{20V_{RMS}\angle 20^\circ}{16.24\Omega\angle 61.3^\circ}$$

$$\boxed{\vec{I}_s = 1.23A_{RMS}\angle -41.28^\circ}$$

Finding Fp**:**

$$F_p = Cos(\theta) = Cos(\theta_V - \theta_I)$$

$$F_p = Cos(20^\circ - (-41.28^\circ))$$

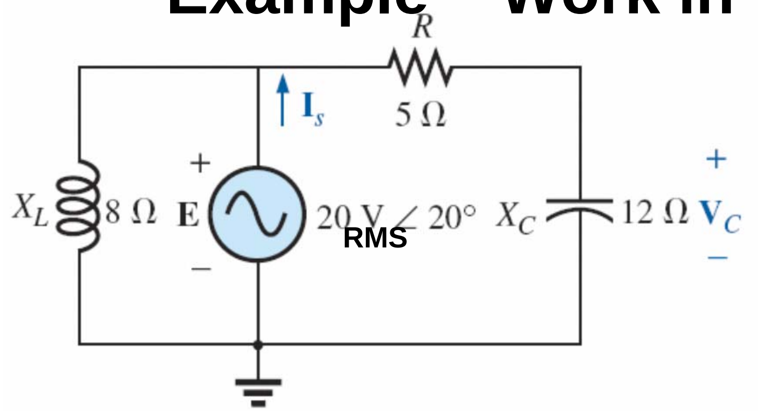
$$F_p = Cos(20^\circ + 41.28^\circ)$$

$$= Cos(61.28^\circ) = 0.48$$

Is Fp leading or lagging? Does it make sense?

LAGGING (I LAGS V)

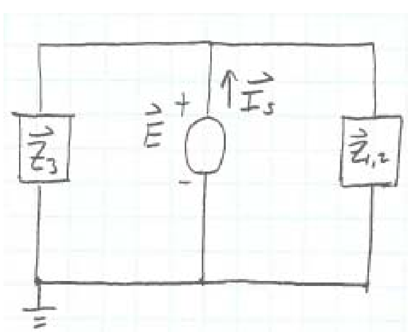
Example – Work in Your Calculator as We Go



Find: **Is, Vc,** FP and the average power delivered to the network

Strategy/Plan 1st

1) Keep each element
2) Analyze the *series* branch
   - ☐ **VC** (voltage divider)
   - ☐ Combine R and C
3) Analyze the parallel circuit
   - ☐ **IS**, FP, Pave



$$P_{DELIV} = V_{RMS} I_{RMS} \cos(\theta)$$

$$= (20\,V_{RMS})(1.23\,A_{RMS})(0.48)$$

$|\vec{E}|$ $\quad$ $|\vec{I}_s|$

$$\boxed{P_{DELIV} = 11.8\,W}$$

Is there an easy check for this number?

$$P_{DELIV} = P_R$$

$$P_R = |\vec{V}_{R_{RMS}}|^2 / R$$

$$\text{But } \vec{V}_R = \vec{E} - \vec{V}_C$$

$$\vec{V}_R = 20\,V_{RMS} \angle 20^\circ - 18.46\,V_{RMS} \angle -2.62^\circ$$

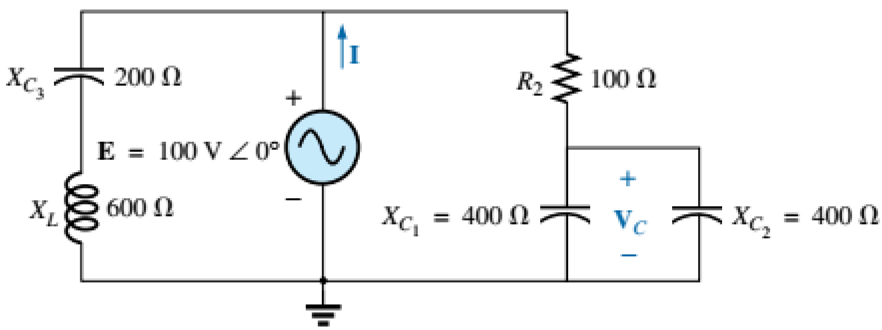
$$\vec{V}_R = 7.69\,V_{RMS} \angle 87.4^\circ$$

$$\therefore P_R = \frac{(7.69\,V_{RMS})^2}{5\,\Omega}$$

$$\boxed{P_R = 11.8\,W}$$

CHECK ☺

In Class Problem (partial HW problem)

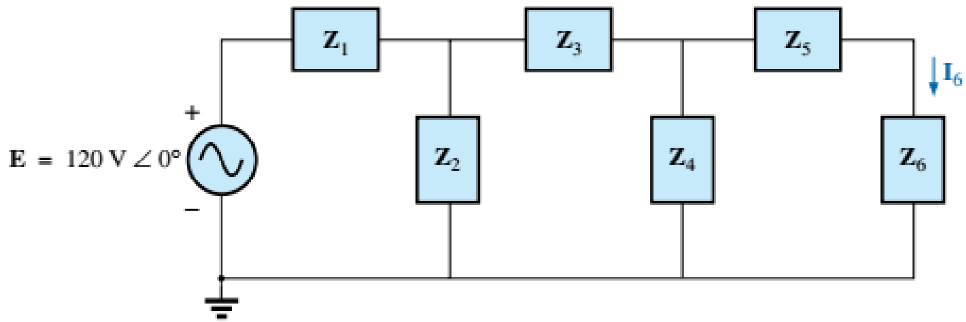


Find: **I,** P_{R2}

(one possible) Strategy/Plan 1st

1) Combine R_2, C_1, C_2 -> $\mathbf{Z_1}$
2) Combine C_3, L -> $\mathbf{Z_2}$
3) Analyze the parallel circuit
   - **I**
   - P_{R2}

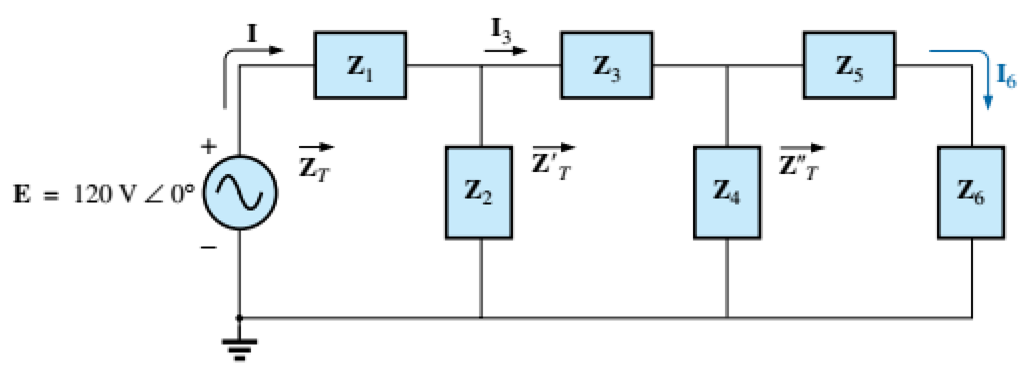
Ladder Networks – One Approach (same as in DC Circuits)



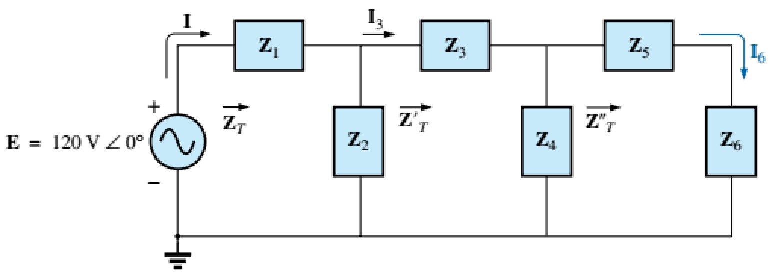
A specific series-parallel configuration

Analyze by:
- Collapse the circuit from the far end
- Find the source current
- Expand the circuit again, finding voltages or currents along the way
    - Successive application of voltage or current divider

Are there other approaches?



Ladder Networks – One Approach



Finding **ZT**

$$\mathbf{Z}''_T = \mathbf{Z}_5 + \mathbf{Z}_6$$

and $$\mathbf{Z}'_T = \mathbf{Z}_3 + \mathbf{Z}_4 \| \mathbf{Z}''_T$$

with $$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 \| \mathbf{Z}'_T$$

Finding **I**

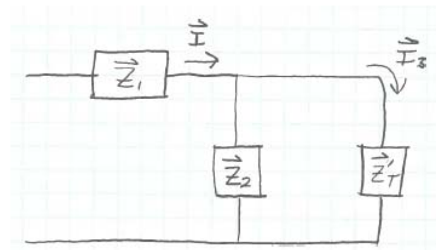
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T}$$

Current Divider (special case of 2 impedances)

$$\mathbf{I}_3 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}'_T}$$

$$\mathbf{I}_6 = \frac{\mathbf{Z}_4\mathbf{I}_3}{\mathbf{Z}_4 + \mathbf{Z}''_T}$$

Development of **I3** (special case equation)



GENERAL FORM:

$$\vec{I}_x = \vec{I}_T\left(\frac{\vec{Z}_T}{\vec{Z}_x}\right)$$

HERE:

$$\vec{I}_3 = \vec{I}\left(\frac{\vec{Z}'_T // \vec{Z}_2}{\vec{Z}'_T}\right)$$

BUT $$\vec{Z}'_T // \vec{Z}_2 = \frac{\vec{Z}'_T \cdot \vec{Z}_2}{\vec{Z}'_T + \vec{Z}_2}$$

$$\therefore \vec{I}_3 = \vec{I}\left(\frac{\vec{Z}'_T \cdot \vec{Z}_2}{\vec{Z}'_T + \vec{Z}_2}\right)\left(\frac{1}{\vec{Z}'_T}\right)$$

$$\vec{I}_3 = \vec{I}\,\frac{\vec{Z}_2}{\vec{Z}'_T + \vec{Z}_2}$$