Nodal Analysis

- General Approach Only (will not cover the format approach)
  - Introduction
  - Example (work along)
  - Special cases
    - Dependent current sources
    - Independent voltage sources
    - Dependent voltage sources
  - **ICP with a dependent current source and a voltage source**

General Approach

1. *Determine the number of nodes within the network.*

2. *Pick a reference node and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.*

3. *Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.*

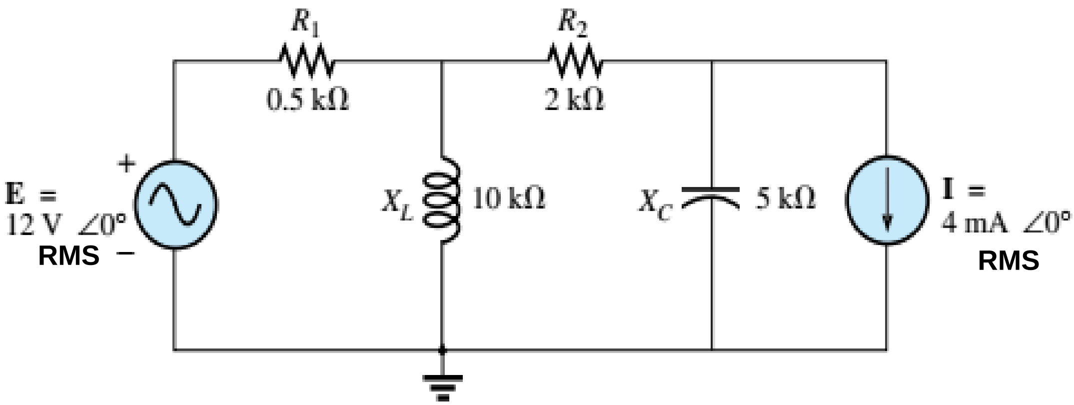
$$\sum \mathbf{I}_i = \sum \mathbf{I}_o$$

4. *Solve the resulting equations for the nodal voltages.*

Look familiar?

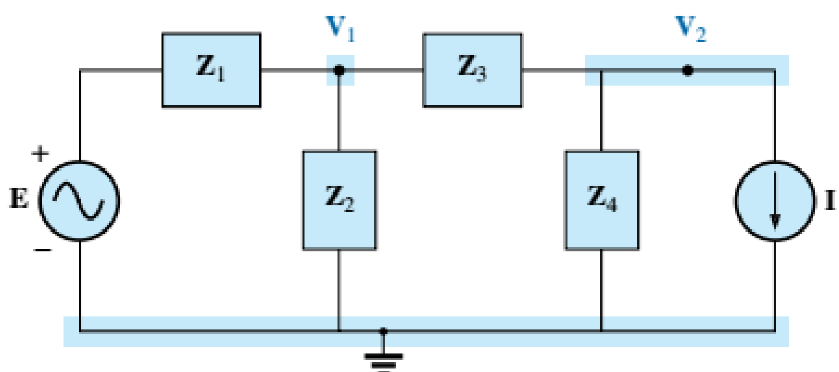
- **Do what we did in DC Circuits**

Example 18.13 (text) – Work Along Using Your Calculator



Find: the voltage across the inductor

Convert to impedance boxes, label the nodes:



E = 12 V_{RMS} < 0º
I = 4 mA_{RMS} < 0º

Z1 = 500 Ohms
Z2 = +j10,000 Ohms
Z3 = 2000 Ohms
Z4 = -j5000 Ohms

Apply KCL at each node:

$$\Sigma \mathbf{I}_i = \Sigma \mathbf{I}_o$$

At Node **V1**:

$$\frac{\mathbf{V}_1 - \mathbf{E}}{\mathbf{Z}_1} + \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_3} = 0$$

Collecting terms:

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right] = \frac{\mathbf{E}_1}{\mathbf{Z}_1} \qquad (18.1)$$

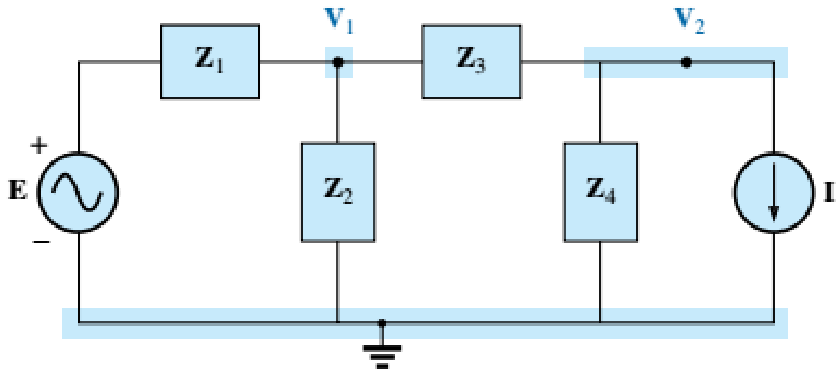
At Node **V2**:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_3} + \frac{\mathbf{V}_2}{\mathbf{Z}_4} + \mathbf{I} = 0$$

Collecting terms:

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4}\right] - \mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right] = -\mathbf{I} \qquad (18.2)$$

Example 18.13 (text) – Work Along Using Your Calculator



E = 12V_{RMS} < 0º
I = 4mA_{RMS} < 0º

$\mathbf{Z_1}$ = 500 Ohms
$\mathbf{Z_2}$ = +j10,000 Ohms
$\mathbf{Z_3}$ = 2000 Ohms
$\mathbf{Z_4}$ = -j5000 Ohms

Grouping 18.1 and 18.2:

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right]=\frac{\mathbf{E}}{\mathbf{Z}_1} \quad \textbf{(18.1)}$$

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}\right]=\mathbf{I} \quad \textbf{(18.2)}$$

Substituting values yields:

$$\mathbf{V}_1[2.5\text{ mS}\angle -2.29^\circ]-\mathbf{V}_2[0.5\text{ mS}\angle 0^\circ]=24\text{ mA}\angle 0^\circ \quad \textbf{(18.1)}$$

$$\mathbf{V}_1[0.5\text{ mS}\angle 0^\circ]-\mathbf{V}_2[0.539\text{ mS}\angle 21.80^\circ]=4\text{ mA}\angle 0^\circ \quad \textbf{(18.2)}$$

Recall: Currents in RMS

Solving (18.1) and (18.2) yields:

$\mathbf{V_1}$ = 9.96 V_{RMS} < 1.83º
$\mathbf{V_2}$ = 1.84 V_{RMS} < -12.6º

Don't forget to check…
One option – KCL @ Node $\mathbf{V_2}$

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{\mathbf{Z}_3}+\frac{\mathbf{V}_2}{\mathbf{Z}_4}+\mathbf{I}=0$$

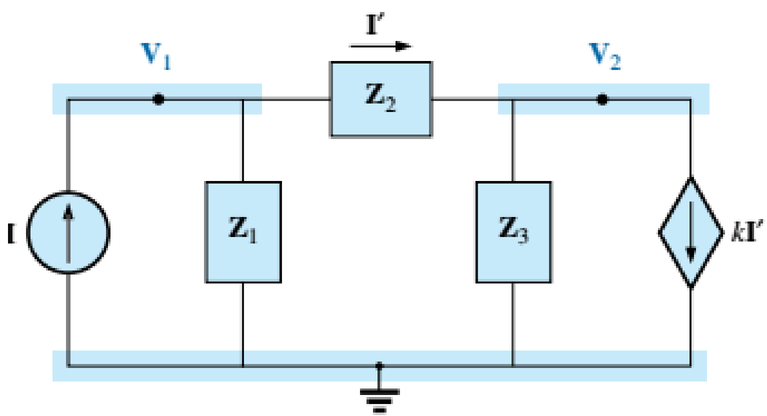
4.10mA_{RMS} < -175º + 368uA_{RMS} < 77.4º + 4mA_{RMS} < 0 º = 0 ?

883×10^{-9} < -41.6º ~ 0?,
Yes (some minor rounding error)

Special Cases – Dependent Current Sources

Dependent Current Sources Present

Step 3 is modified as follows: Treat each dependent current source like an independent source when Kirchhoff's current law is applied to each defined node. However, once the equations are established, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen nodal voltages.



Write the KCL equations and rearrange:

At Node **V1**:

$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2} - \mathbf{I} = 0$$

$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1}{\mathbf{Z}_2}\right] = \mathbf{I}$$

At Node **V2** (note text error on I'):

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + k\mathbf{I}' = 0$$

Substituting for the dependent source yields:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} + k\left[\frac{\mathbf{V}_1 - \mathbf{V}_2}{\mathbf{Z}_2}\right] = 0$$

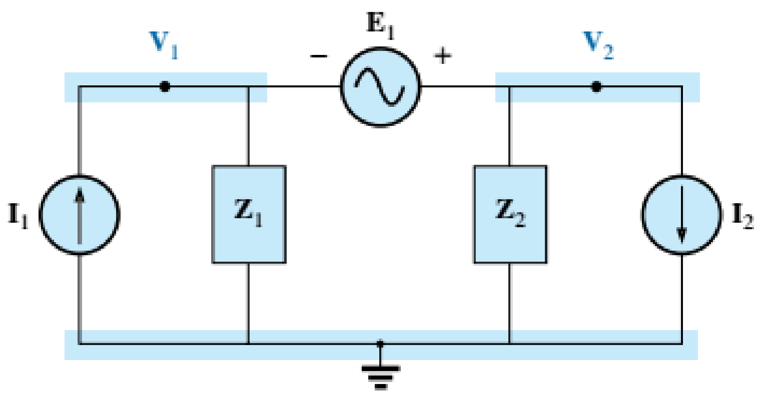
Collecting terms:

$$\mathbf{V}_1\left[\frac{1 - k}{\mathbf{Z}_2}\right] - \mathbf{V}_2\left[\frac{1 - k}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}\right] = 0$$

Special Cases – Independent Voltage Sources

Independent Voltage Sources Present

Step 3 is modified as follows: Treat each source between defined nodes as a short circuit (recall the *supernode* classification in Chapter 8), and write the nodal equations for each remaining independent node. Then relate the chosen nodal voltages to the independent voltage source to ensure that the unknowns of the final equations are limited solely to the nodal voltages.



Relating E1 to the node voltages:

$$\mathbf{V}_2 - \mathbf{V}_1 = \mathbf{E}_1$$

We now have two equations and two unknowns (**V1** and **V2**), solve…

Replacing **E1** with a s/c (think superposition):

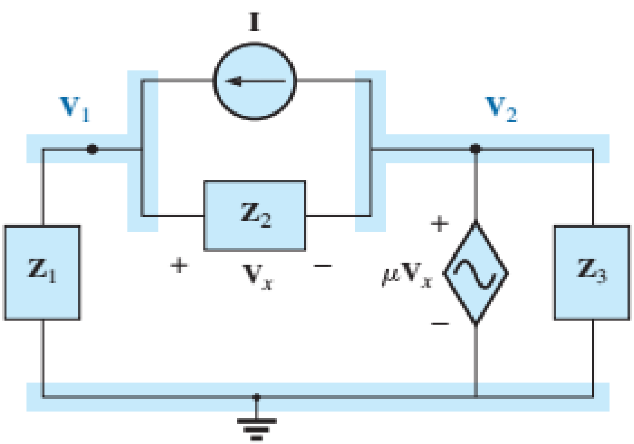
KCL at the *supernode*:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{\mathbf{V}_2}{\mathbf{Z}_2} + \mathbf{I}_2$$

Special Cases – Dependent Voltage Sources

Dependent Voltage Sources Present

Step 3 is modified as follows: The procedure is essentially the same as that applied for independent voltage sources, except that now the dependent sources have to be defined in terms of the chosen nodal voltages to ensure that the final equations have only nodal voltages as their unknown quantities.



Replacing u**Vx** with a s/c (think superposition):

KCL at the *supernode (node **V1**)*:

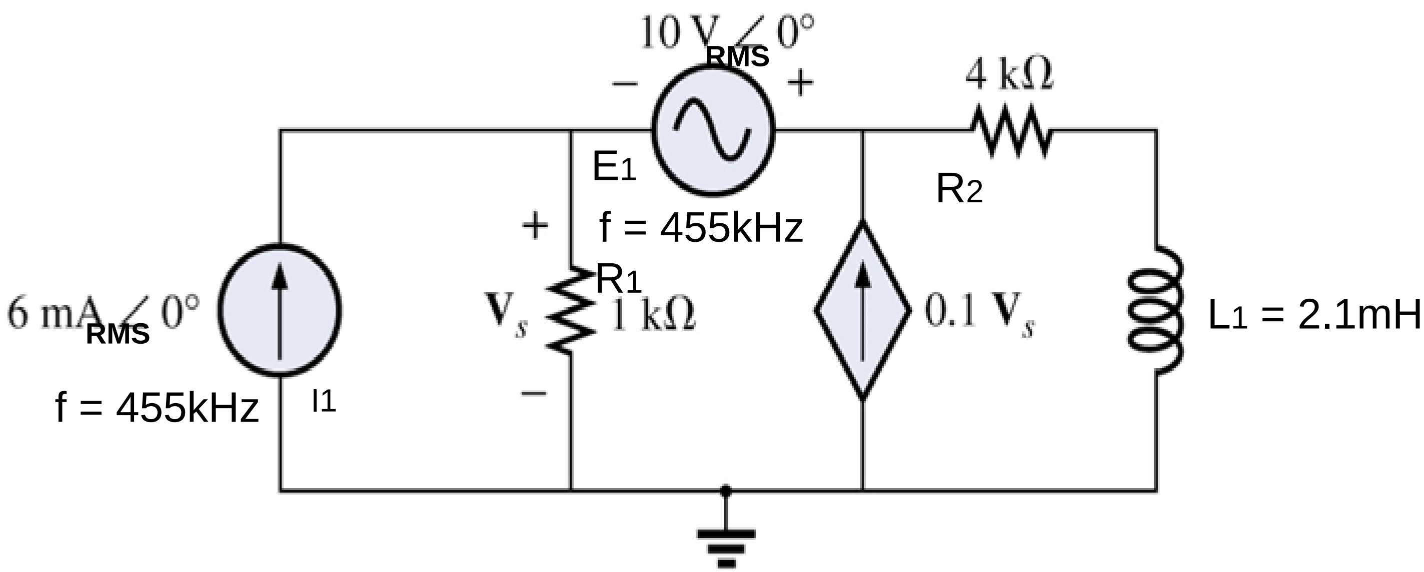
$$\frac{\mathbf{V}_1}{\mathbf{Z}_1} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{\mathbf{Z}_2} - \mathbf{I} = 0$$

Relating **uVx** to the node voltages:

$$\mathbf{V}_2 = \mu\mathbf{V}_x = \mu[\mathbf{V}_1 - \mathbf{V}_2]$$

We now have two equations and two unknowns (**V1** and **V2**), solve…

In Class Problem (also a modified homework problem)



Find:
- The current through the inductor

Approach:
- Use Nodal Analysis
- Combine R2 and L1