Parallel AC Circuits Frequency Response and Equivalent Circuits

- R-L-C Parallel Circuit
  - Frequency response (**Z_T**) - introduction
  - **Z_T** and Fp - calculation
  - Excel spreadsheet for **Z_T** and Fp
  - **ICP – Find C to "cancel" L in a parallel R-L-C (Project #1 Week 2)**
- Parallel-Series Equivalent Circuits
  - Discussion/example
  - Limitations
  - **ICP – Find an equivalent series circuit to a parallel R-L (Project #1 Week 2)**

R-L-C Parallel Circuit – Frequency Response (Z_T)

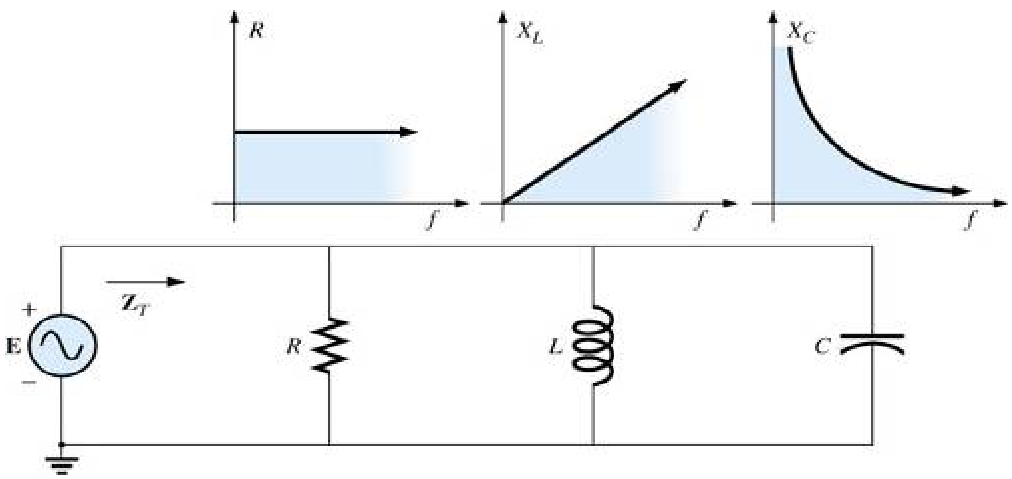


FIG. 16.30 *Frequency response for parallel R-L-C elements.*

At Low Frequencies:

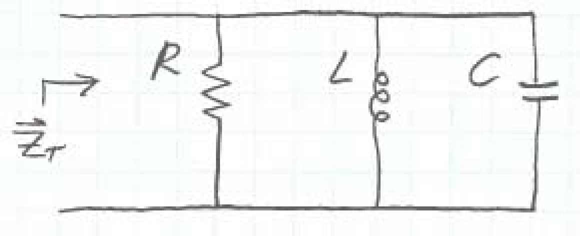
- L dominates $\mathbf{Z_T}$
  - Small $|\mathbf{Z_T}|$
  - $\angle \mathbf{Z_T} \sim +90$ Degrees

At High Frequencies:

- C dominates $\mathbf{Z_T}$
  - Small $|\mathbf{Z_T}|$
  - $\angle \mathbf{Z_T} \sim -90$ Degrees

What about when $X_C = X_L$?

R-L-C Parallel Circuit – Frequency Response (Z_T)



$$X_C = \frac{1}{2\pi f C} \quad \Omega$$

$$X_L = 2\pi f L \quad \Omega$$

$$\vec{Z}_T = \frac{1}{\vec{Y}_T}$$

$$\vec{Y}_T = \frac{1}{R} + \frac{1}{\vec{Z}_L} + \frac{1}{\vec{Z}_C}$$

$$= \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$\vec{Y}_T = \frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

$$\vec{Y}_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$|\vec{Y}_T| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}$$

$$|\vec{Z}_T| = \frac{1}{|\vec{Y}_T|}$$

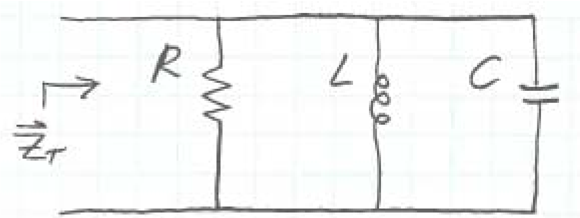
$$\measuredangle \vec{Y}_T = \tan^{-1}\left(\frac{\frac{1}{X_C} - \frac{1}{X_L}}{\frac{1}{R}}\right)$$

$$= \tan^{-1}\left(\frac{R}{X_C} - \frac{R}{X_L}\right)$$

$$\therefore \measuredangle \vec{Z}_T = -\tan^{-1}\left(\frac{\frac{1}{X_C} - \frac{1}{X_L}}{\frac{1}{R}}\right)$$

$$= -\tan^{-1}\left(\frac{R}{X_C} - \frac{R}{X_C}\right)$$

R-L-C Parallel Circuit – Frequency Response (Z_T)



$$X_C = \frac{1}{2\pi f C} \quad \Omega$$

$$X_L = 2\pi f L \quad \Omega$$

WHEN $X_L = X_C$,

$$\vec{Y}_T = \frac{1}{R} + j(0) = \frac{1}{R}$$

HENCE $\vec{Z}_T = \dfrac{1}{\vec{Y}_T} = \underline{\underline{R}}$

SET $X_C = X_L$ @ $f = f_p$: $\dfrac{1}{2\pi f_p C} = 2\pi f_p L$

$$\frac{1}{2\pi C} = 2\pi (f_p)^2 L$$

OR $f_p^2 = \dfrac{1}{(2\pi)(2\pi) L C}$

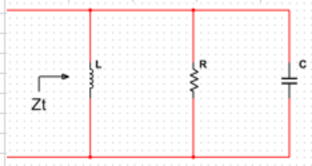
$$\therefore \quad f_p = \frac{1}{2\pi\sqrt{LC}} \quad Hz$$

R-L-C Parallel Circuit – Frequency Response (Z_T)

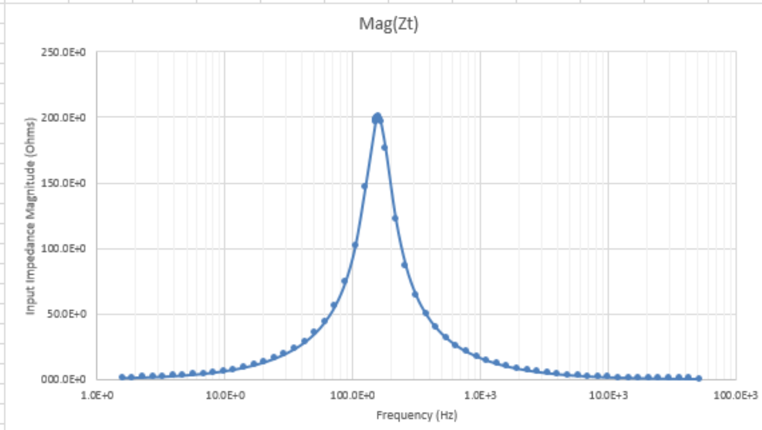
Demo - Zin RLC Parallel.xlsx

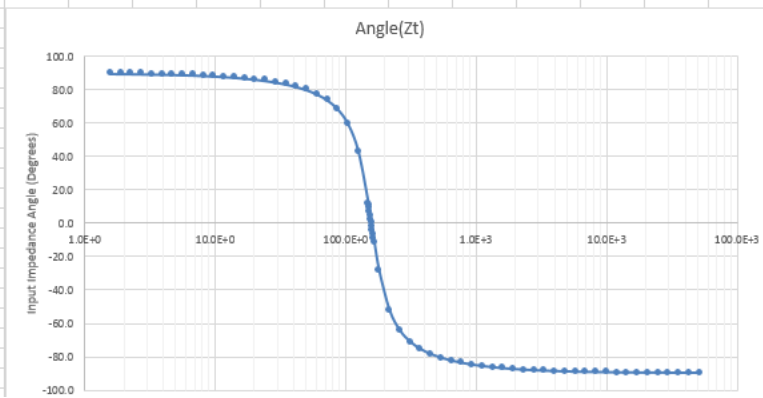
B C D E F G H I J K L M N O P Q R S T U V W

	INPUT R,L and C Values		
R	200.0E+0	Ohms	
L	100.0E-3	Henries	
C	10.0E-6	Farads	
fp	159.2E+0	Hz	(XL = XC) at fp



	f (Hz)	Xl (Ohms)	Xc (Ohms)	Mag(Yt)	Mag(Zt)	Angle(Yt)	Angle(Zt)
	1.6E+0	1.0E+0	10.0E+3	999.9E-3	**1.0E+0**	-89.7	**89.7**
	1.9E+0	1.2E+0	8.3E+3	833.2E-3	**1.2E+0**	-89.7	**89.7**
	2.3E+0	1.4E+0	6.9E+3	694.3E-3	**1.4E+0**	-89.6	**89.6**
	2.8E+0	1.7E+0	5.8E+3	578.6E-3	**1.7E+0**	-89.5	**89.5**
	3.3E+0	2.1E+0	4.8E+3	482.1E-3	**2.1E+0**	-89.4	**89.4**
	4.0E+0	2.5E+0	4.0E+3	401.7E-3	**2.5E+0**	-89.3	**89.3**
	4.8E+0	3.0E+0	3.3E+3	334.6E-3	**3.0E+0**	-89.1	**89.1**
	5.7E+0	3.6E+0	2.8E+3	278.8E-3	**3.6E+0**	-89.0	**89.0**
	6.8E+0	4.3E+0	2.3E+3	232.2E-3	**4.3E+0**	-88.8	**88.8**
	8.2E+0	5.2E+0	1.9E+3	193.4E-3	**5.2E+0**	-88.5	**88.5**
	9.9E+0	6.2E+0	1.6E+3	161.0E-3	**6.2E+0**	-88.2	**88.2**
	11.8E+0	7.4E+0	1.3E+3	133.9E-3	**7.5E+0**	-87.9	**87.9**
	14.2E+0	8.9E+0	1.1E+3	111.4E-3	**9.0E+0**	-87.4	**87.4**
	17.0E+0	10.7E+0	934.6E+0	92.5E-3	**10.8E+0**	-86.9	**86.9**
	20.4E+0	12.8E+0	778.9E+0	76.8E-3	**13.0E+0**	-86.3	**86.3**
	24.5E+0	15.4E+0	649.1E+0	63.6E-3	**15.7E+0**	-85.5	**85.5**
	29.4E+0	18.5E+0	540.9E+0	52.5E-3	**19.1E+0**	-84.5	**84.5**
	35.3E+0	22.2E+0	450.7E+0	43.1E-3	**23.2E+0**	-83.3	**83.3**
	42.4E+0	26.6E+0	375.6E+0	35.3E-3	**28.4E+0**	-81.8	**81.8**
	50.8E+0	31.9E+0	313.0E+0	28.5E-3	**35.0E+0**	-79.9	**79.9**
	61.0E+0	38.3E+0	260.8E+0	22.8E-3	**43.8E+0**	-77.3	**77.3**
	73.2E+0	46.0E+0	217.4E+0	17.9E-3	**56.0E+0**	-73.7	**73.7**
	87.9E+0	55.2E+0	181.1E+0	13.5E-3	**73.8E+0**	-68.3	**68.3**
	105.4E+0	66.2E+0	150.9E+0	9.8E-3	**101.7E+0**	-59.4	**59.4**
	126.5E+0	79.5E+0	125.8E+0	6.8E-3	**146.8E+0**	-42.8	**42.8**
	151.8E+0	95.4E+0	104.8E+0	5.1E-3	**196.5E+0**	-10.7	**10.7**
	151.4E+0	95.1E+0	105.2E+0	5.1E-3	**196.1E+0**	-11.4	**11.4**
	152.9E+0	96.1E+0	104.1E+0	5.1E-3	**197.5E+0**	-9.1	**9.1**
	154.4E+0	97.0E+0	103.1E+0	5.0E-3	**198.6E+0**	-6.9	**6.9**
	156.0E+0	98.0E+0	102.0E+0	5.0E-3	**199.4E+0**	-4.6	**4.6**
	157.6E+0	99.0E+0	101.0E+0	5.0E-3	**199.8E+0**	-2.3	**2.3**
fp	159.2E+0	100.0E+0	100.0E+0	5.0E-3	**200.0E+0**	0.0	**0.0**
	160.7E+0	101.0E+0	99.0E+0	5.0E-3	**199.8E+0**	2.3	**-2.3**
	162.4E+0	102.0E+0	98.0E+0	5.0E-3	**199.4E+0**	4.6	**-4.6**
	164.0E+0	103.0E+0	97.1E+0	5.0E-3	**198.6E+0**	6.8	**-6.8**
	165.6E+0	104.1E+0	96.1E+0	5.1E-3	**197.5E+0**	9.0	**-9.0**
	167.3E+0	105.1E+0	95.1E+0	5.1E-3	**196.2E+0**	11.3	**-11.3**
	182.2E+0	114.5E+0	87.4E+0	5.7E-3	**175.8E+0**	28.5	**-28.5**
	218.6E+0	137.4E+0	72.8E+0	8.2E-3	**122.4E+0**	52.2	**-52.2**
	262.4E+0	164.8E+0	60.7E+0	11.6E-3	**86.5E+0**	64.4	**-64.4**
	314.8E+0	197.8E+0	50.6E+0	15.6E-3	**64.3E+0**	71.2	**-71.2**





R-L-C Parallel Circuit – From Project 1 WK2 Prelab

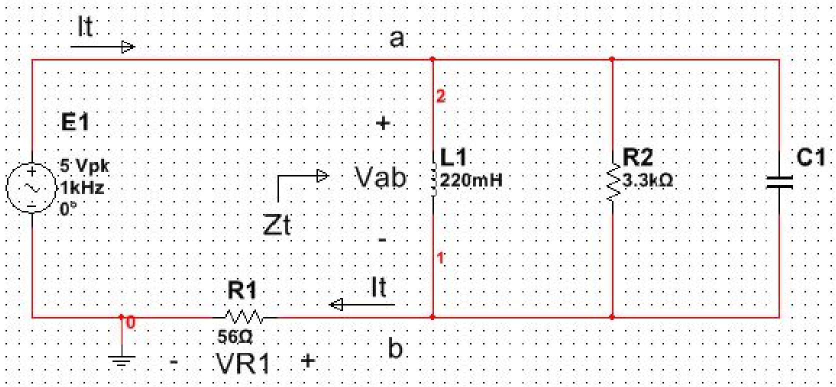


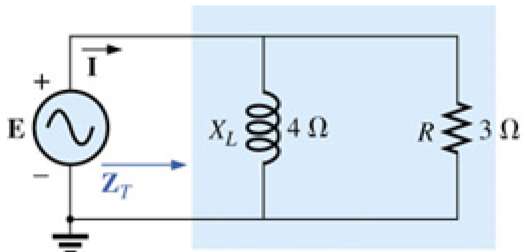
Figure 4 - Parallel R-L-C Circuit

7. Determine the value of C_1 in the parallel R-L-C circuit of Figure 4 to cancel the reactance of L_1 so that at 1kHz, the phase angle between the voltage, V_{ab} and the current I_T is zero degrees.

Parallel-Series "Equivalent" Circuits

Consider a parallel R-L Circuit, R=3 Ohms and **F = 10Hz**, L = 63.66 mH:

In the phasor domain:



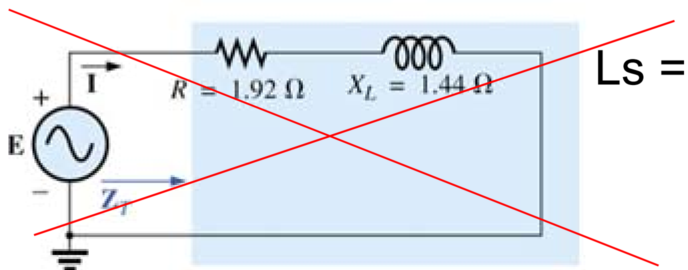
$X_L = 2\pi f L = 4\Omega$

$\mathbf{Y_T} = \frac{1}{j4\Omega} + \frac{1}{3\Omega}$

$\mathbf{Y_T}$ = (333.3E-3 - j250E-3) S

$\mathbf{Z_T = 1/Y_T}$ = (1.92 + j1.44) Ω

Suggesting the following "equivalent" circuit:



Ls = 22.9 mH

What about the same parallel R-L circuit at **F = 20Hz?**

$X_L = 2\pi f L = 8\Omega$

$\mathbf{Y_T} = \frac{1}{j8\Omega} + \frac{1}{3\Omega}$

$\mathbf{Y_T}$ = (333.3E-3 - j125E-3) S

$\mathbf{Z_T = 1/Y_T}$ = (2.63 + j0.986) Ω

Suggests:
Rs= 2.63
Ls = 7.85 mH

Be careful, equivalence only holds at ONE FREQUENCY

Parallel-Series "Equivalent" Circuits – From Project 1 WK2 Prelab

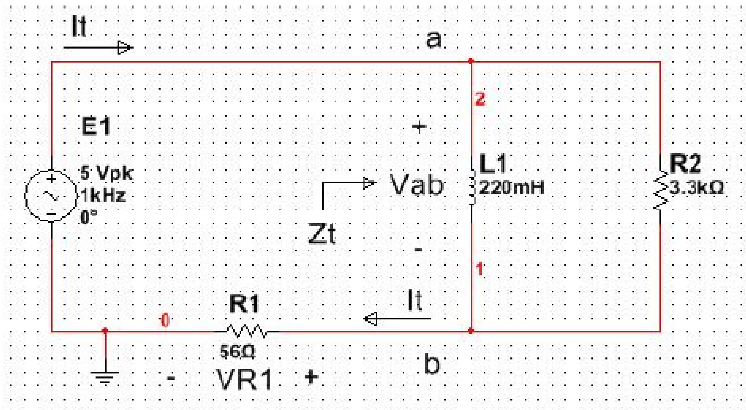


Figure 3 - Parallel R-L Circuit

5. Analyze the parallel R-L circuit of Figure 3:
   a. Determine Z_T (in polar form)
   b. Create an impedance diagram for the network (Does this result suggest that there is an equivalent series circuit?).