Average and Effective (rms) Values

- Average Value
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  - Periodic waveforms
  - ICP
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  - ICP
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  - Equating DC and AC average power
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  - ICP (non-sinusoidal)

Average Speed = Area under the curve/Length under the curve

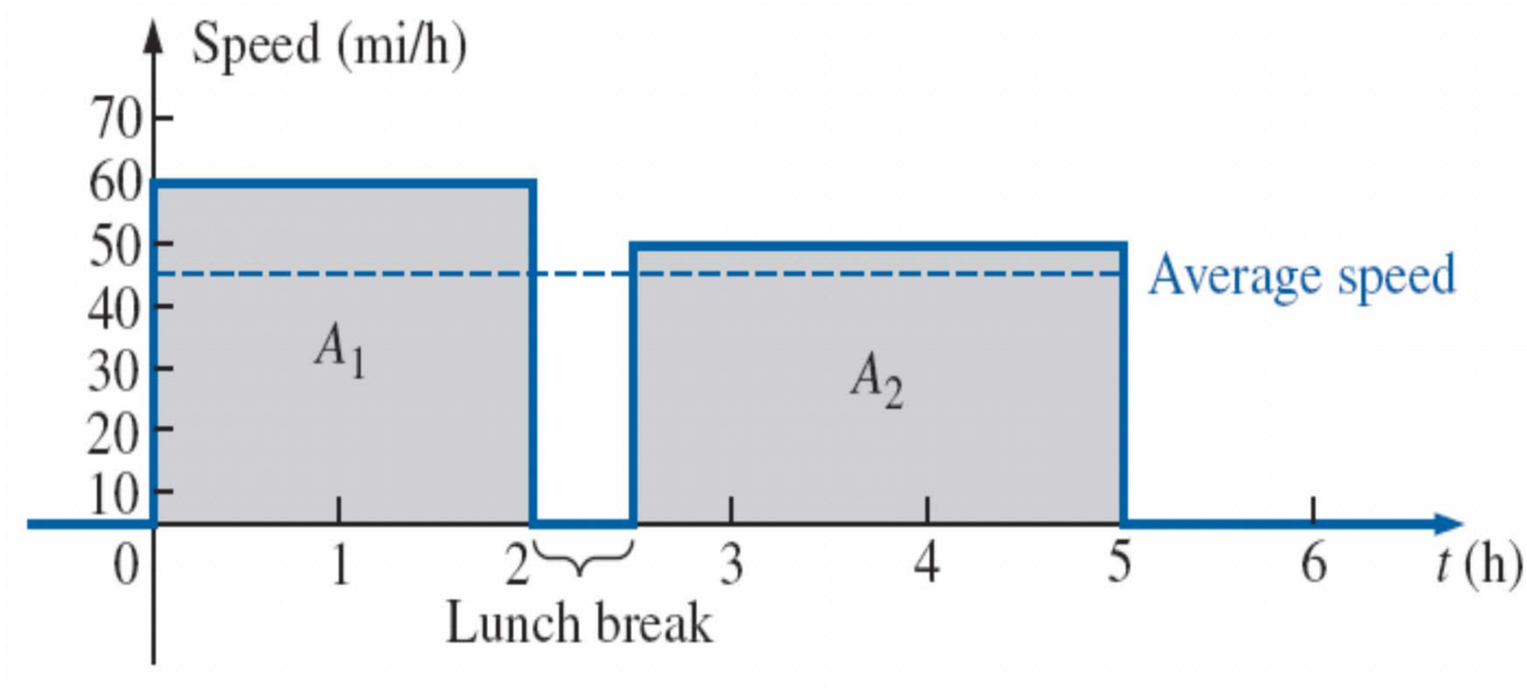


FIG. 13.43 *Plotting speed versus time for an automobile excursion.*

A1 = 60 mi/hr * 2 hrs = 120 miles

A2 = 50 mi/hr * 2.5 hrs = 125 miles

In 5 hours: Ave Speed = 245 mi/5hrs = 49 mph (text error of 45mph)

For Periodic Waveforms - Calculate the Average over 1 Period

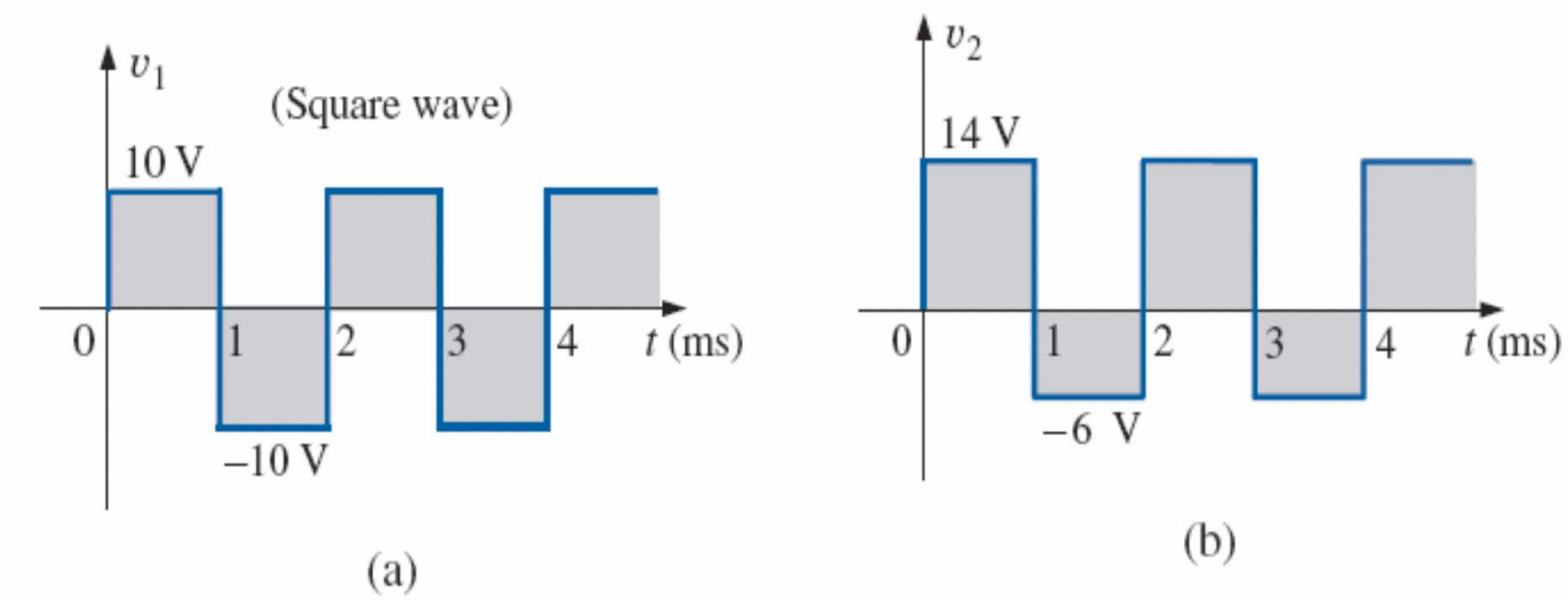


FIG. 13.44 *Example 13.14.*

$$\text{Ave(v1)} = \frac{(10V)(1ms)+(-10V)(1ms)}{2ms}$$

$$\text{Ave(v2)} = \frac{(14V)(1ms)+(-6V)(1ms)}{2ms}$$

Ave(v1) = 0V

Ave(v2) = 4V

ICP – Calculate the Average Value of i(t)

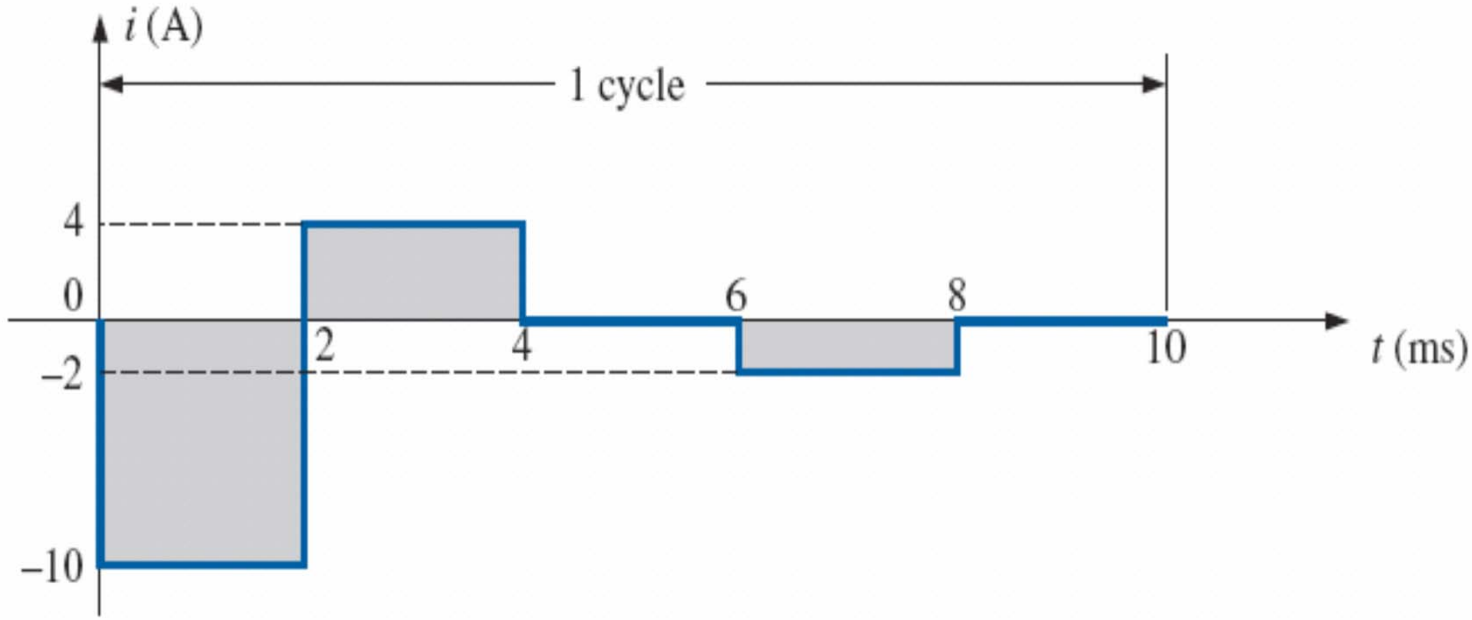


FIG. 13.47 *Example 13.15(b).*

How about the Average Value of a Half-Sinusoid?

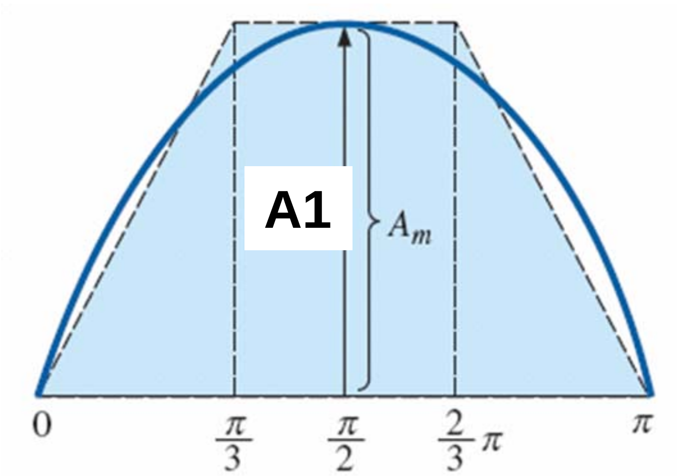


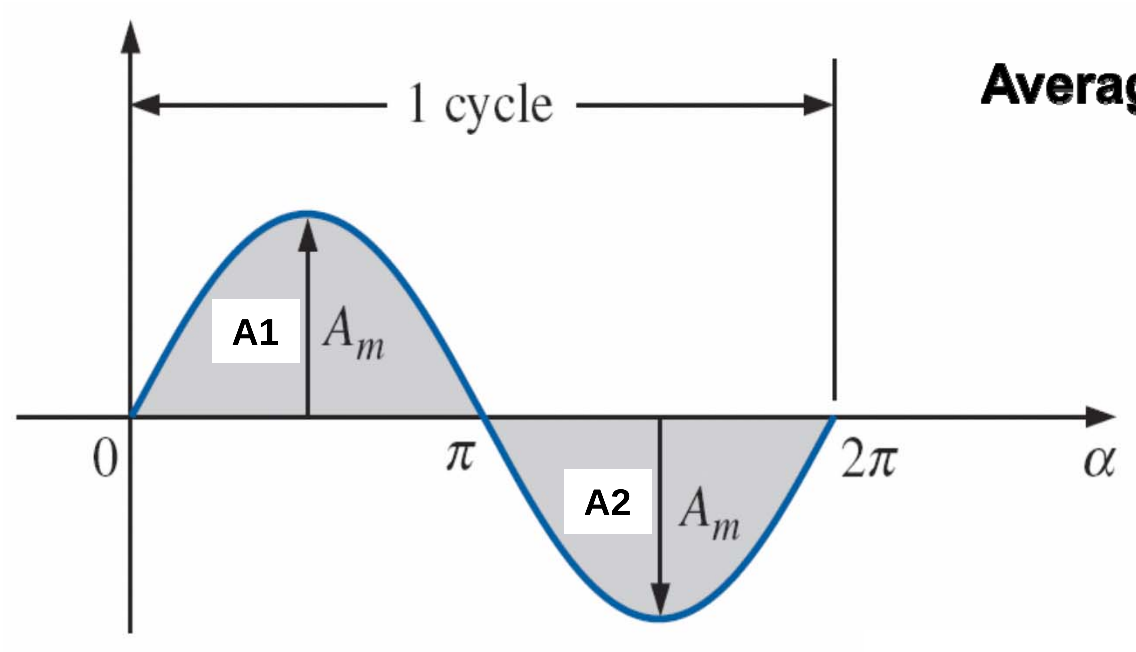
FIG. 13.51 *A better approximation for the shape of the positive pulse of a sinusoidal waveform.*

Recall: Ave Value = Area / Length

$$A_1 = \int_0^{\pi} A_m \sin(\alpha)\, d\alpha$$
$$= A_m \int_0^{\pi} \sin(\alpha)\, d\alpha$$
$$= A_m \left[-\cos(\alpha)\right|_0^{\pi}$$
$$= -A_m \left[\cos(\alpha)\right|_0^{\pi}$$
$$= -A_m[\cos(\pi) - \cos(0)]$$
$$-A_m[-1-1]$$
$$= \boxed{2A_m}$$

Average Value = 2*Am/pi ~ 0.637*Am

How about the Average Value of a Sinusoid?



$$\textbf{Average Value} = \frac{A1+A2}{2*pi}$$

FIG. 13.53 *Example 13.16.*

ICP – Calculate the Average Value of v(t)

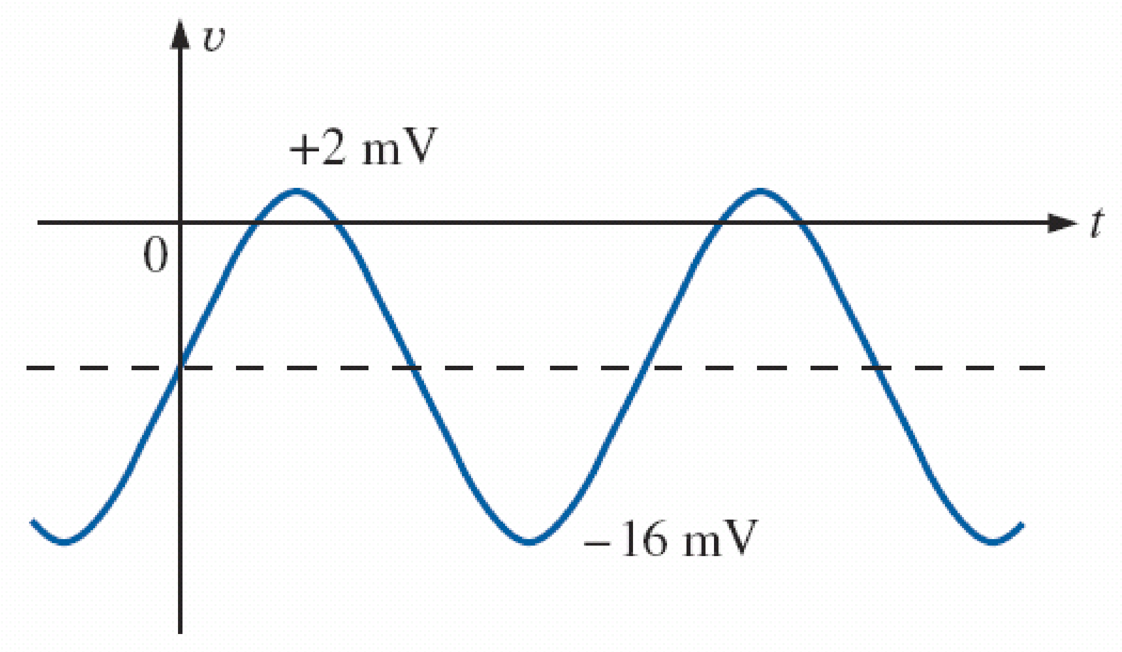


FIG. 13.54 *Example 13.17.*

Oscilloscope for DC Waveforms

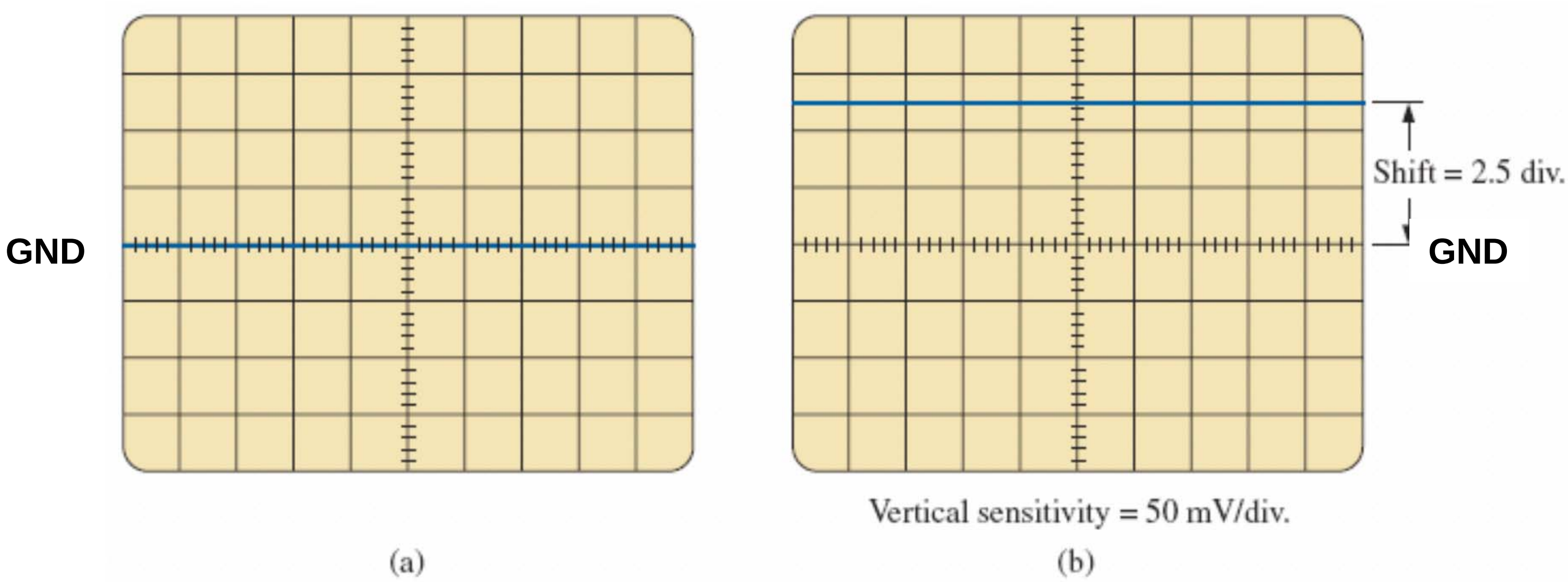


FIG. 13.57 *Using the oscilloscope to measure dc voltages; (a) setting the GND condition; (b) the vertical shift resulting from a dc voltage when shifted to the DC option.*

DC Coupling Mode

VDC = Vave =
2.5 div * 50mV/div = 125mV

Oscilloscope for Mixed Waveforms

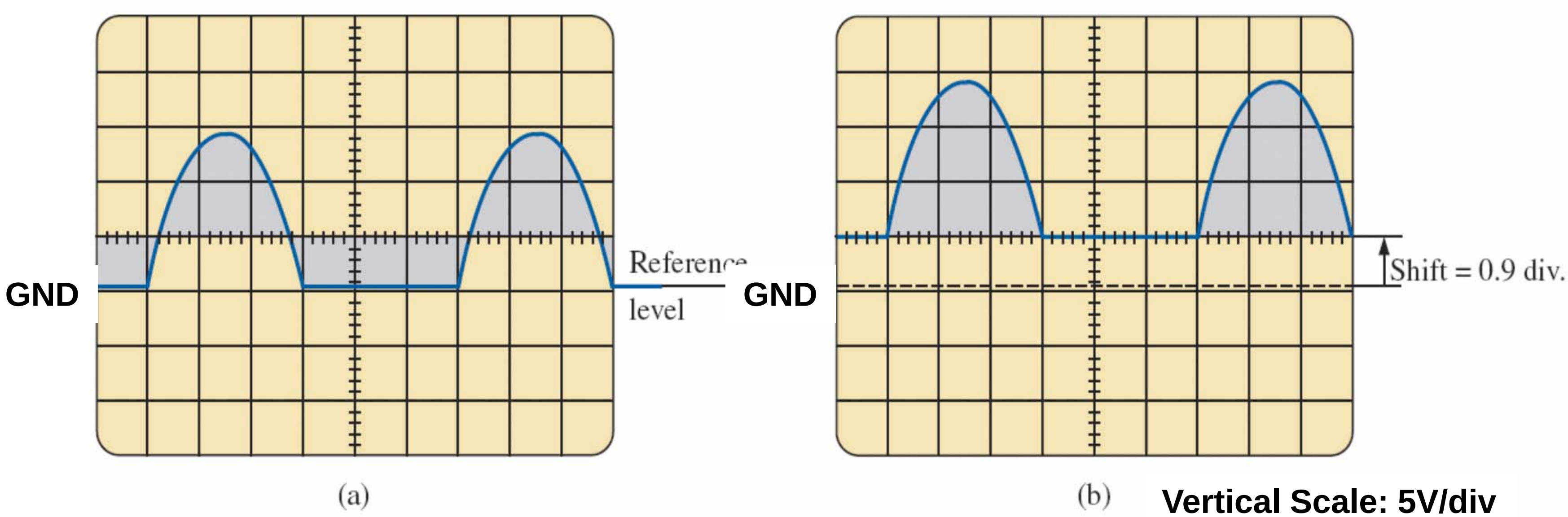


FIG. 13.58 *Determining the average value of a nonsinusoidal waveform using the oscilloscope: (a) vertical channel on the ac mode; (b) vertical channel on the dc mode.*

AC Coupling Mode

DC (Direct Coupling) Mode

VDC = Vave =
0.9 div * 5V/div = 4.5V

Effective or (RMS) Values

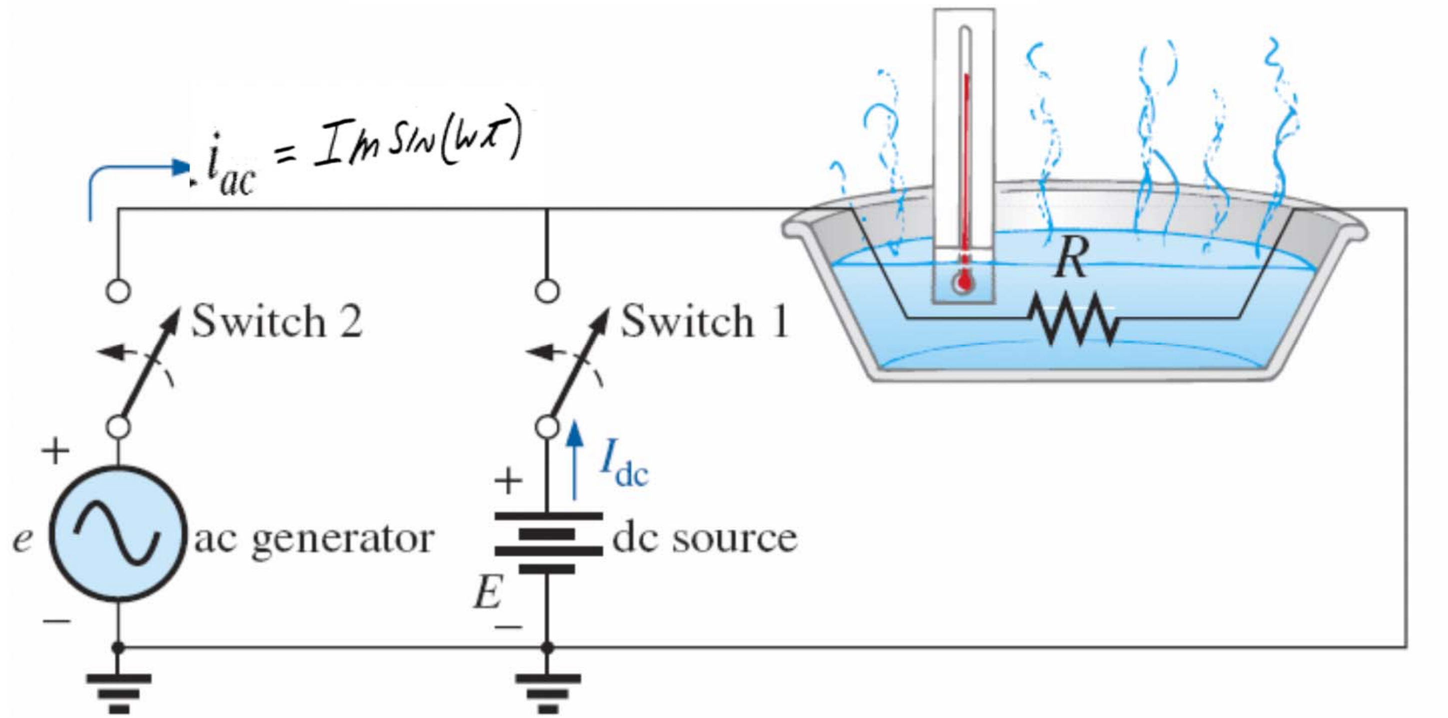


FIG. 13.59 *An experimental setup to establish a relationship between dc and ac quantities (Average Power).*

$$P_{ac} = (i_{ac})^2 R = [I_m \sin(\omega t)]^2 R = I_m^2 \sin^2(\omega t) R$$

but $\sin^2(x) = \frac{1}{2}[1 - \cos(2x)]$

$$\therefore P_{ac} = \frac{I_m^2 R}{2}[1 - \cos(2\omega t)] = \boxed{\frac{I_m^2 R}{2} - \frac{I_m^2 R}{2}\cos(2\omega t)}$$

POWER DELIVERED TO R AT ANY TIME "t"

Effective or (RMS) Values

WE HAVE: $P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos(2\omega t)$

$\therefore$ THE AVERAGE VALUE OF P_{ac} IS:

$$P_{ac(AVE)} = \frac{I_m^2 R}{2}$$, SINCE THE AVE VALUE OF THE SECOND TERM IS ZERO

EQUATE THIS TO P_{DC} (TO FIND THE "EQUIVALENT" AC CURRENT REQUIRED TO CAUSE THE SAME EFFECT)

$$\frac{I_m^2 R}{2} = I_{DC}^2 R$$

$$\therefore I_m^2 = \frac{I_{DC}^2 R \cdot 2}{R}$$

OR $I_m = \sqrt{2 I_{DC}^2} = \sqrt{2}\, I_{DC}$

WE HAVE: $\boxed{I_m = \sqrt{2}\, I_{DC} \quad \text{OR} \quad I_{DC} = \frac{1}{\sqrt{2}} I_m}$

$I_{EFFECTIVE}$ OR EQUIVALENT DC

CALLED THE EFFECTIVE VALUE OF THE SINUSOIDAL AC WAVEFORM CURRENT

Effective or (RMS) Values

IN GENERAL, FOR ANY PERIODIC WAVEFORM, $i(t)$, THE EFFECTIVE VALUE (OR RMS VALUE) IS:

$$I_{EFF} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}} = \sqrt{\frac{\text{AREA}\left(i(t)\right)^2}{T}}$$

FOR A SINUSOID - RECALL

$$\left.\begin{aligned} I_{\underset{RMS}{EFF \text{ or}}} &= \frac{1}{\sqrt{2}} I_m \\ E_{\underset{RMS}{EFF \text{ or}}} &= \frac{1}{\sqrt{2}} E_m \end{aligned}\right\}$$

Find the RMS Value of:

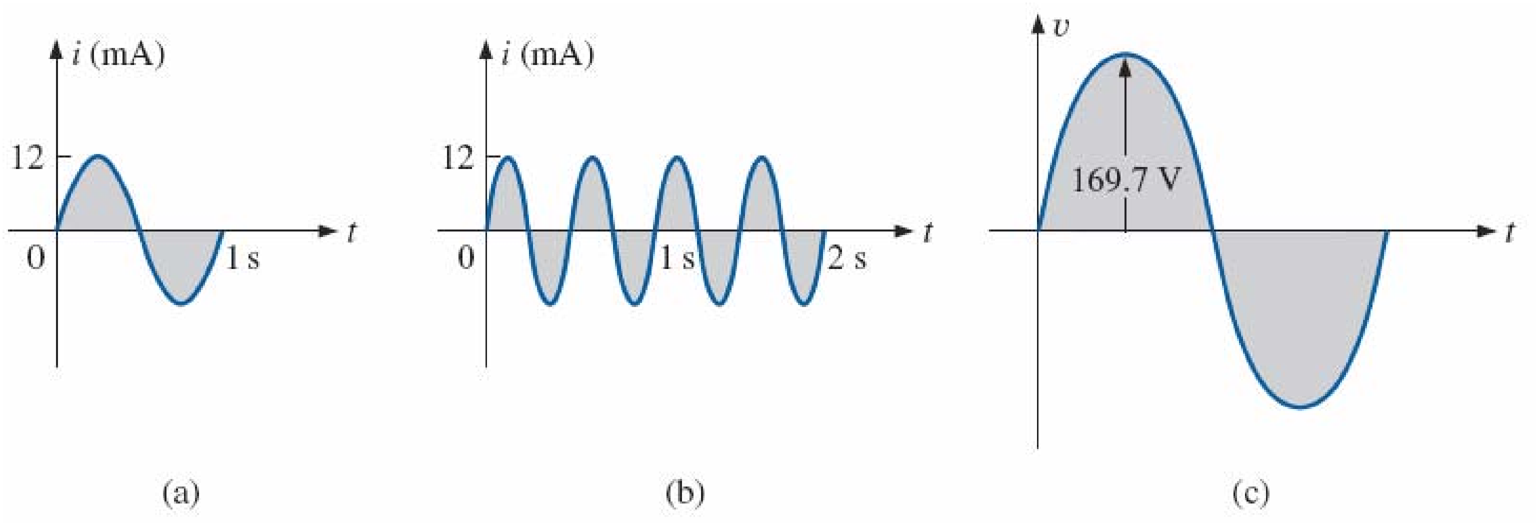


FIG. 13.60 *Example 13.20.*

12mA * 1/SQRT(2) = 8.49mA

12mA * 1/SQRT(2) = 8.49mA

169.7V * 1/SQRT(2) = 120V

ICP – Find Em and Im (The peak values of e and i)

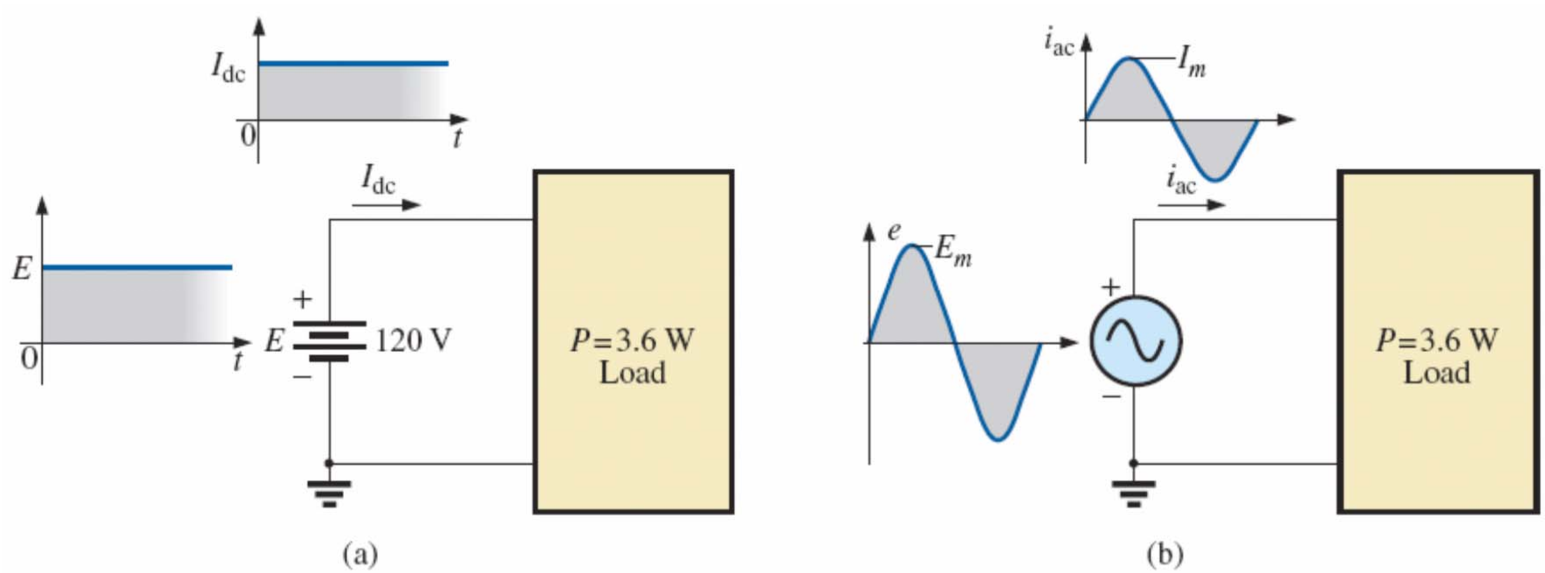


FIG. 13.61 *Example 13.21.*

ICP – Find Vrms

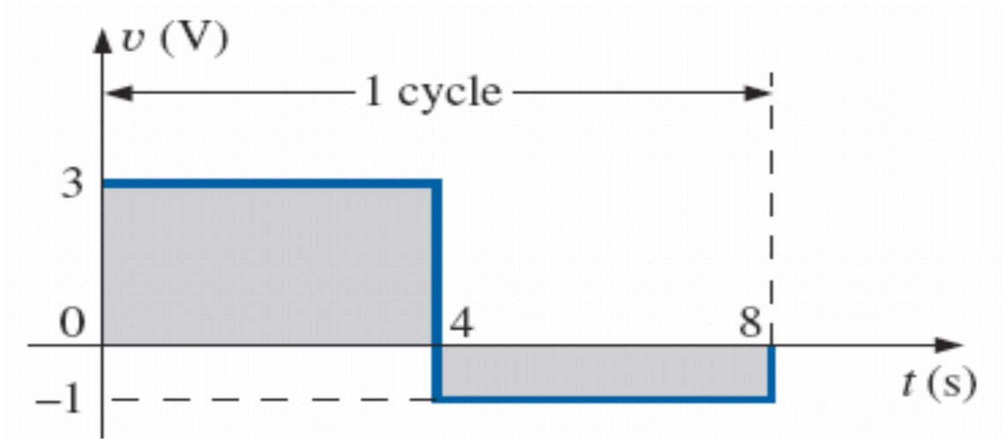


FIG. 13.62 *Example 13.22.*

Recall: $V_{RMS} = \sqrt{\dfrac{AREA[v(t)]^2}{T}}$