Digital Signal Processing

Averaging and Dithering

Key Points from Last Class

- During A/D conversion the input signal is limited to discrete values – Quantization

- Quantization adds noise to the signal
  - Adds to the input signal noise
  - Quantization noise σ is 0.29 LSB

- Quantization noise can be reduced by increasing the number of bits in the ADC
  - $\approx 6\ dB$ per bit

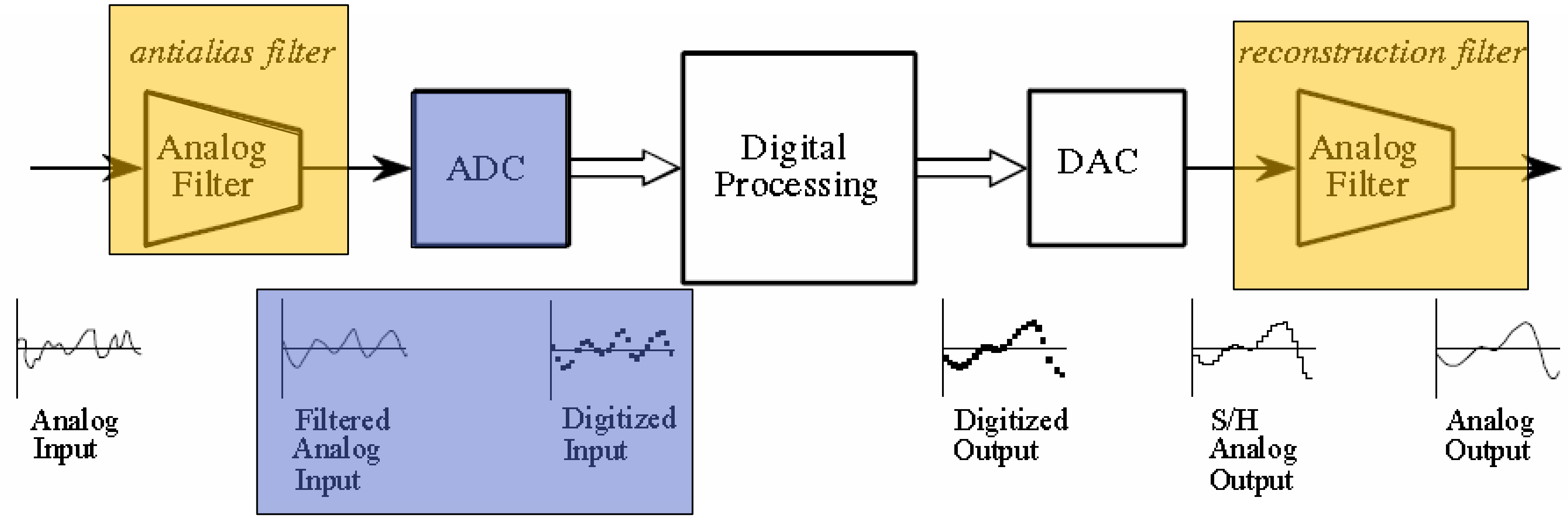
Key Points for Today

- Averaging can improve the SNR by reducing the noise level

- Averaging may not work in all cases
  - Where signals are small compared to the quantization levels

- Adding noise (dithering) to the input signal then averaging can improve this situation.

Overview of ADC/DSP/DAC System



Still focusing on the ADC today

Decreasing Noise Levels in Signals

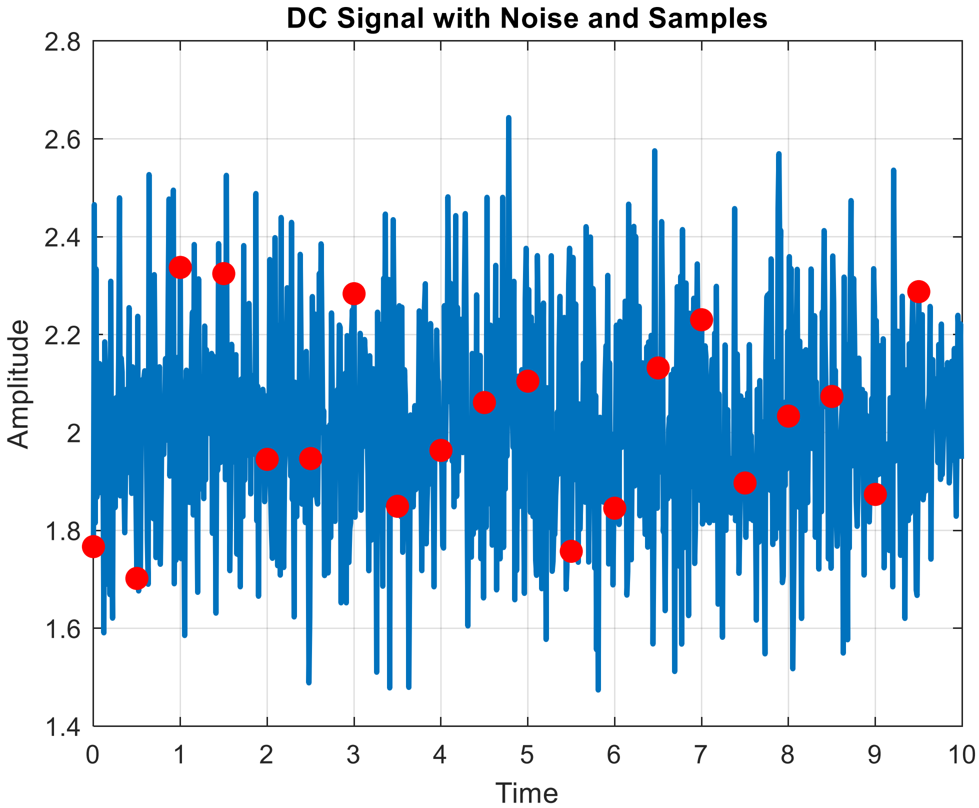
- The typical error equation says

$$\sigma_{est} = \frac{\sigma}{\sqrt{N}}$$

- Taking more samples of the signal and averaging should decrease the noise and improve the SNR

Oversampling and Averaging

- If we have a noisy signal that we are sampling

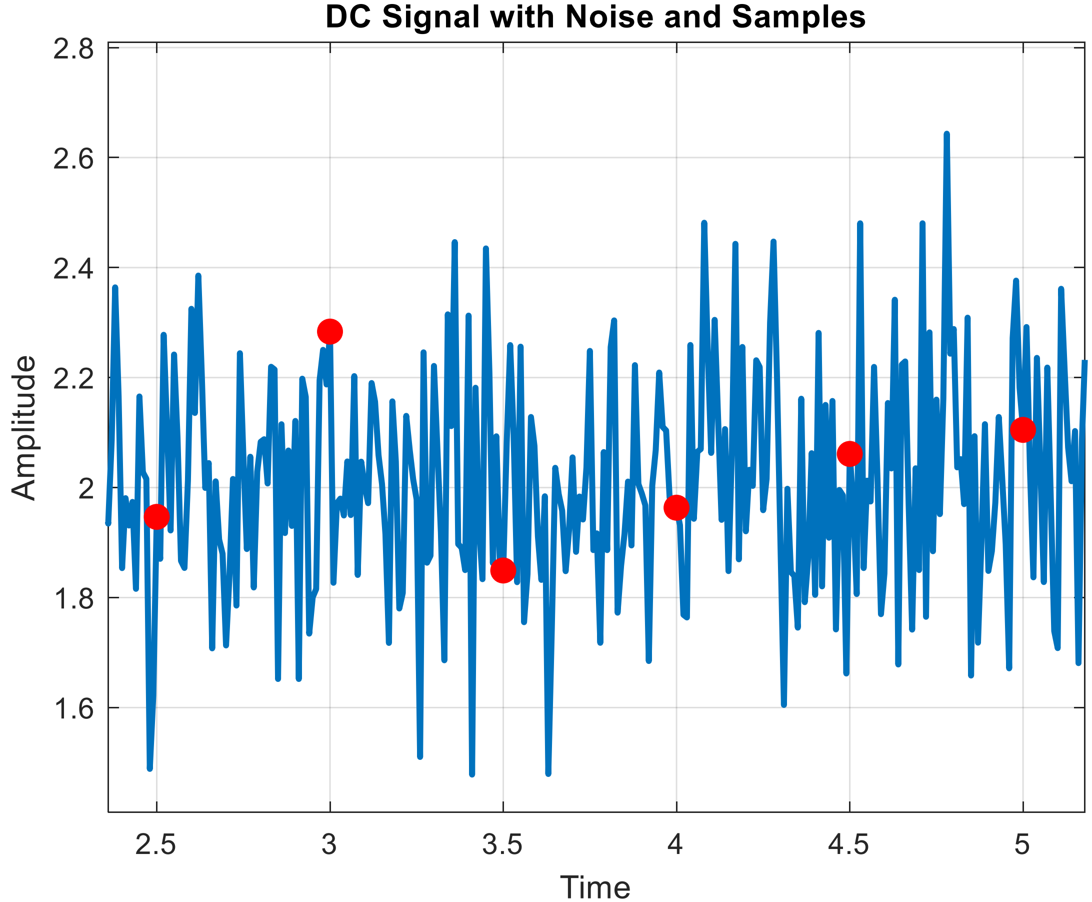


Average Value = 2V

The sample values will have the same standard deviation as the noise

Oversampling and Averaging

- Close up view of signal and samples



Reducing Noise

- I need to keep the same sample rate, but I want to reduce the noise in the signal. How?
- What if I increase my sample rate by 5 then average 5 of those values to create a single sample at the original rate?

$$\sigma_{est} = \frac{\sigma}{\sqrt{N}}$$

Reducing Noise

- What happens to my noise level?

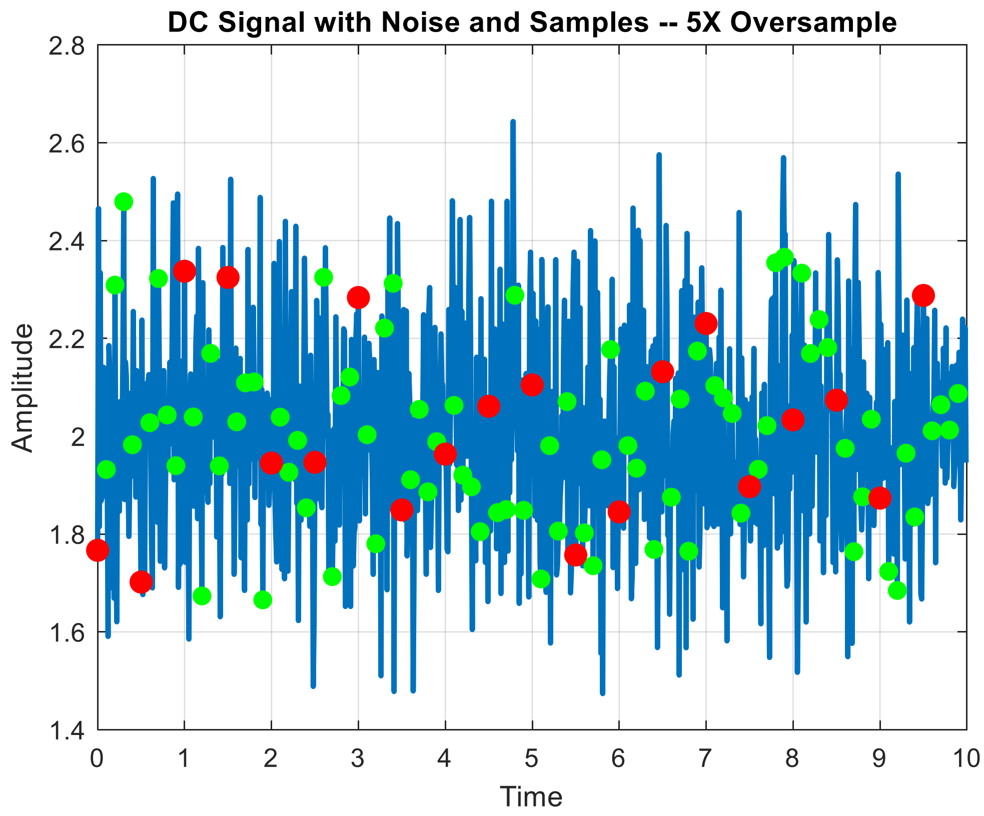
- Using the typical error equation

$$\sigma_{est} = \frac{\sigma}{\sqrt{N}}$$

- The standard deviation of the noise should decrease by a factor of $1/\sqrt{N}$.

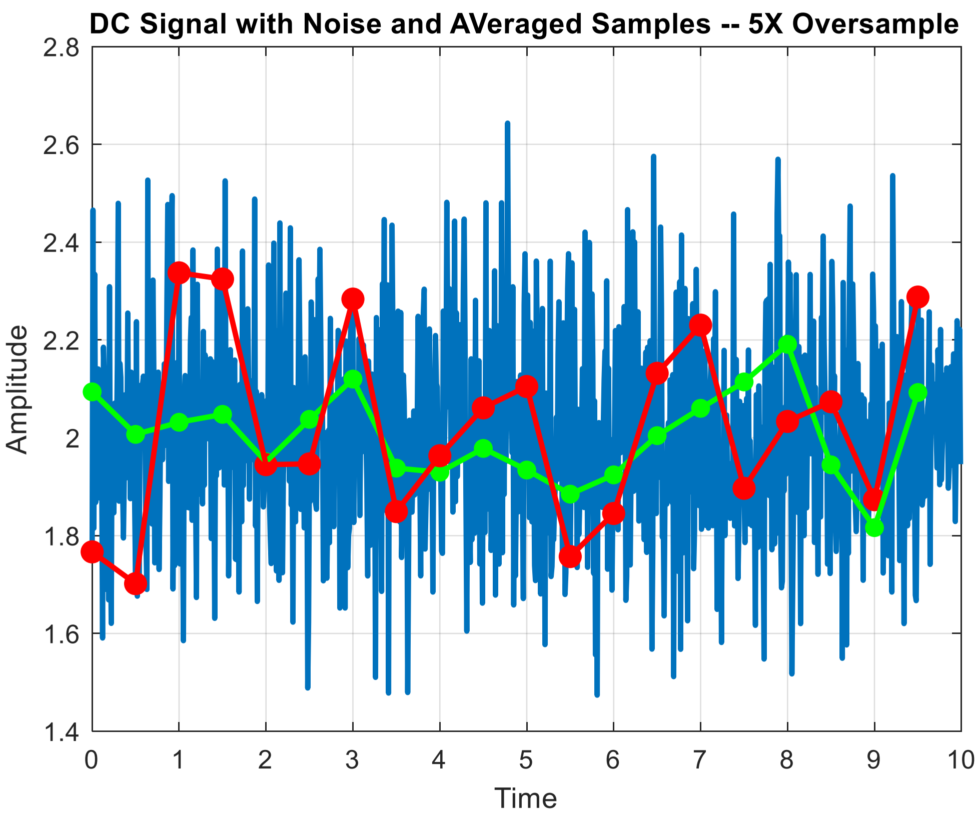
Oversample by 5X

- Change the sample rate (increase by 5X)



Oversample by 5X

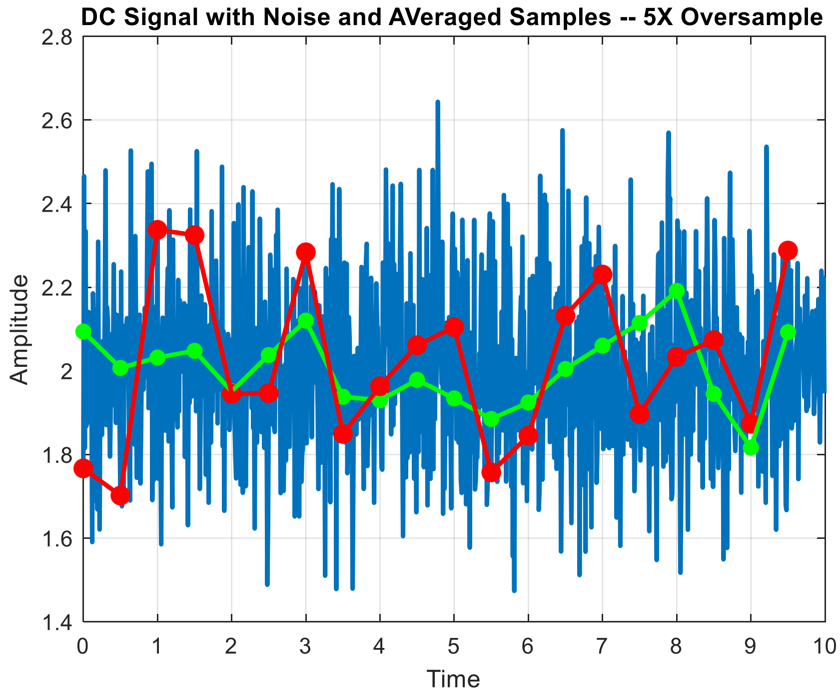
- Change the sample rate (increase by 5X)



Now average 5 samples at the higher rate to create one sample at the original rate

Oversample by 5X and Average

- The SD of the averaged samples has decreased



Standard deviation of the original samples

$$\sigma_{1X} = .16$$

Standard deviation of the averaged samples

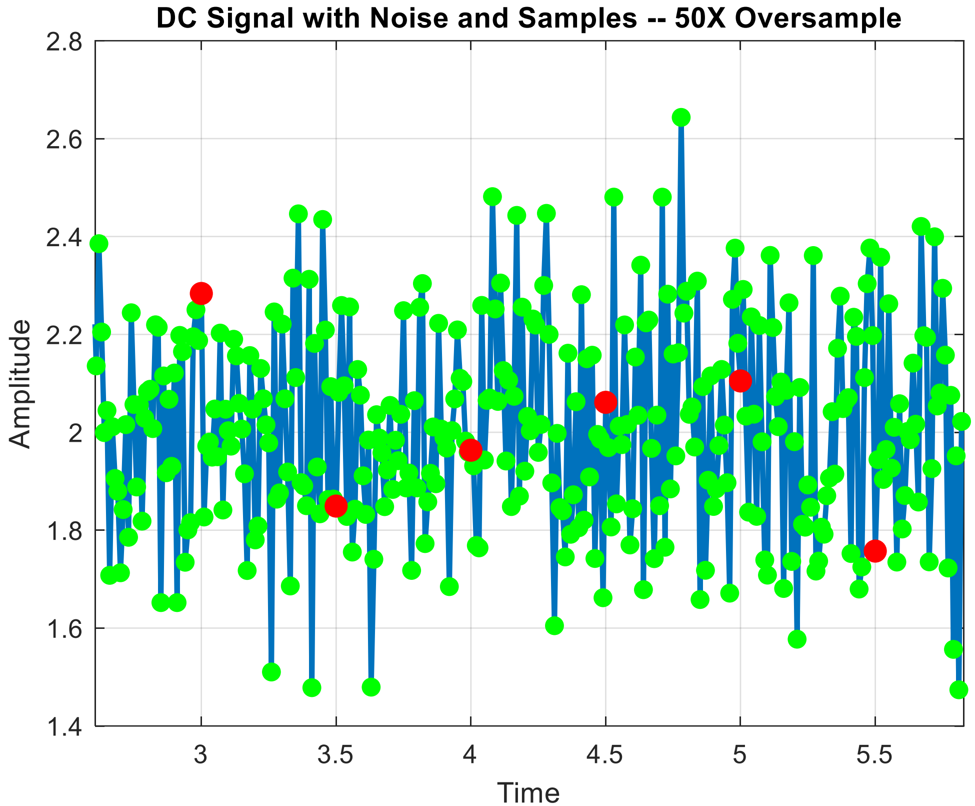
$$\sigma_{5X} = .072$$

$$\frac{\sigma_{5X}}{\sigma_{1X}} = \frac{.072}{0.16} = .45$$

$$\frac{1}{\sqrt{5}} = .4472$$

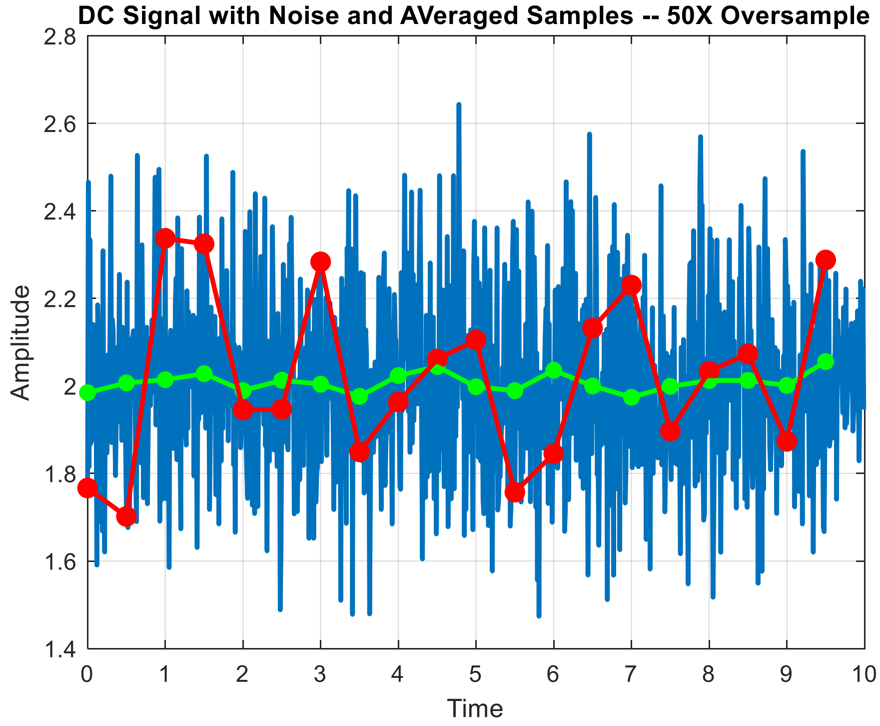
50X Oversample

- Increase the oversample rate to 50X
- Average 50 samples



50X Oversample

- Average 50 samples to get 1 sample at the original rate



Standard deviation of the original samples

$$\sigma_{1X} = .18$$

Standard deviation of the averaged samples

$$\sigma_{50X} = .026$$

$$\frac{\sigma_{50X}}{\sigma_{1X}} = \frac{.026}{0.18} = .144$$

$$\frac{1}{\sqrt{50}} = .14$$

Arduino Code For Averaging

```
//******************************************
void loop()
{
  syncSample();

  //sample = analogRead(LM61);
  sample = analogReadAve();
```

Code hangs out waiting for a sample interrupt

When the interrupt occurs then accumulate 160 samples of the ADC

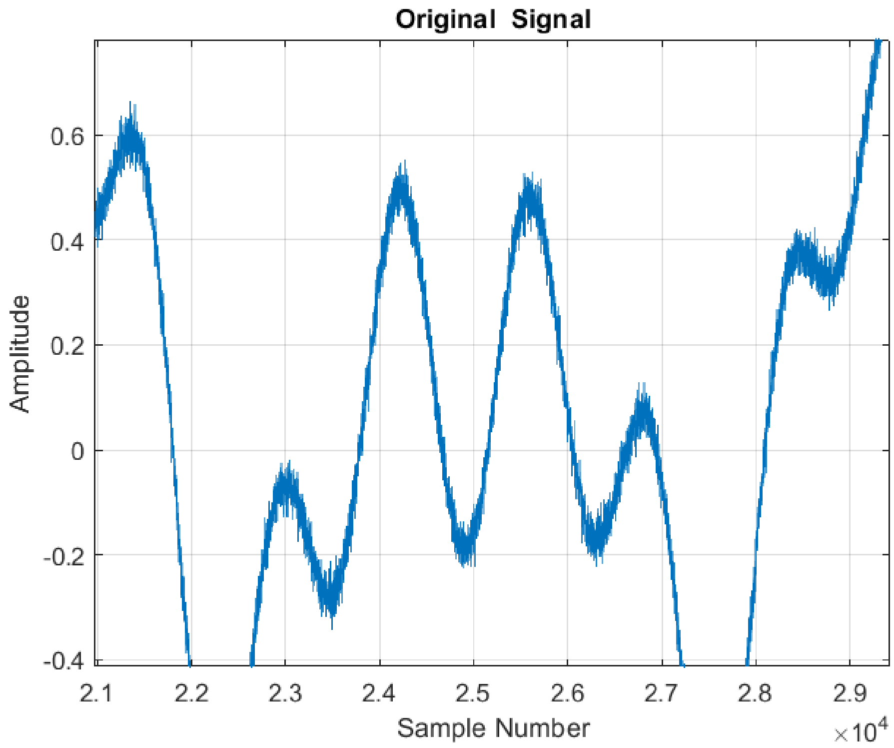
```

//**********************************************
float analogReadAve(void)
{
  float sum = 0.0;
  for (int i = 0; i < NUM_SUBSAMPLES; i++) sum += analogRead(LM61);
  return sum/NUM_SUBSAMPLES; // averaged subsamples
}

//**************************************************************
```

Then average the values

Dithering to Improve Quantization Errors

Sample Averaging Example

- Start with a signal with some noise on it and quantize it with a resolution of 0.1 Volts



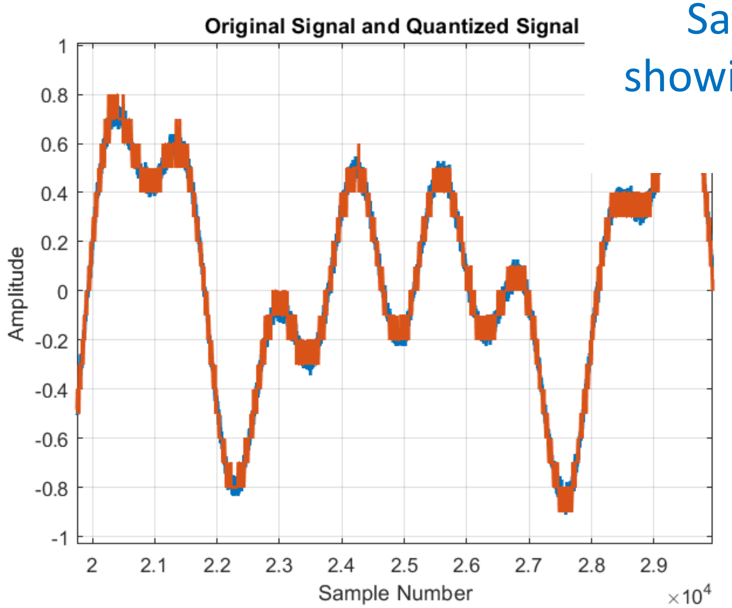
Signal Level $\sigma_s = 0.45V$

Input noise $\sigma_n = .022V$

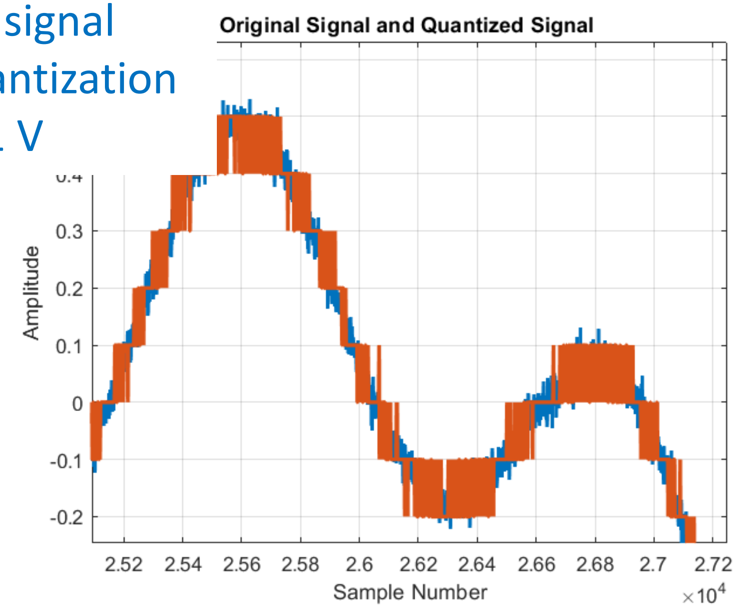
SNR = σ_s^2/σ_n^2

SNR$_{dB}$ $= 26\ dB$

Signal After Quantization



Sampled signal showing quantization at 0.1 V



Input Noise $\sigma = .022$

Quantization Noise $\sigma = .29(.1) = .029V$

Theoretical Total Noise = .037

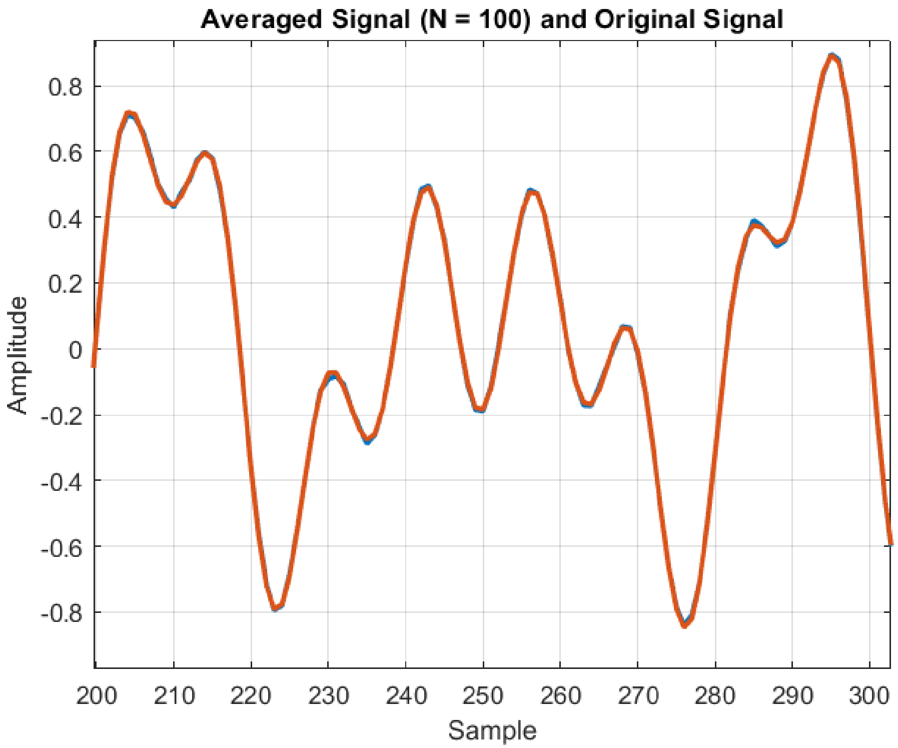
Measured Total Noise = .037

SNR_{dB} = 21.7 dB

5.3 dB degradation in SNR due to quantization

Use Averaging
N=100 samples

Increase the sample rate by 100 times! – Average 100 samples at the new rate – Results in the original sample rate



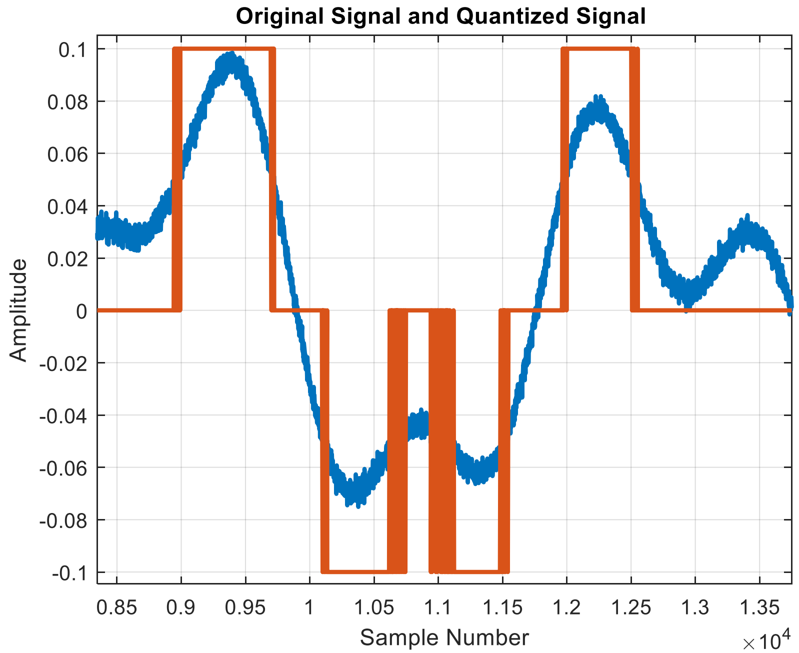
$$\sigma_{ave} = \frac{\sigma}{\sqrt{N}} = \frac{.037}{10} = .0037$$

Expected SD after averaging = .0037
Measured Noise SD after averaging = .006
SNR(dB) = 37 dB

Averaging improved the overall SNR!
Both input and quantization noise reduced by averaging

When This May not Work?

- What if the signal were smaller and closer to the quantization level?



Measured $\sigma = .3043$ which is what we would expect for quantization noise.

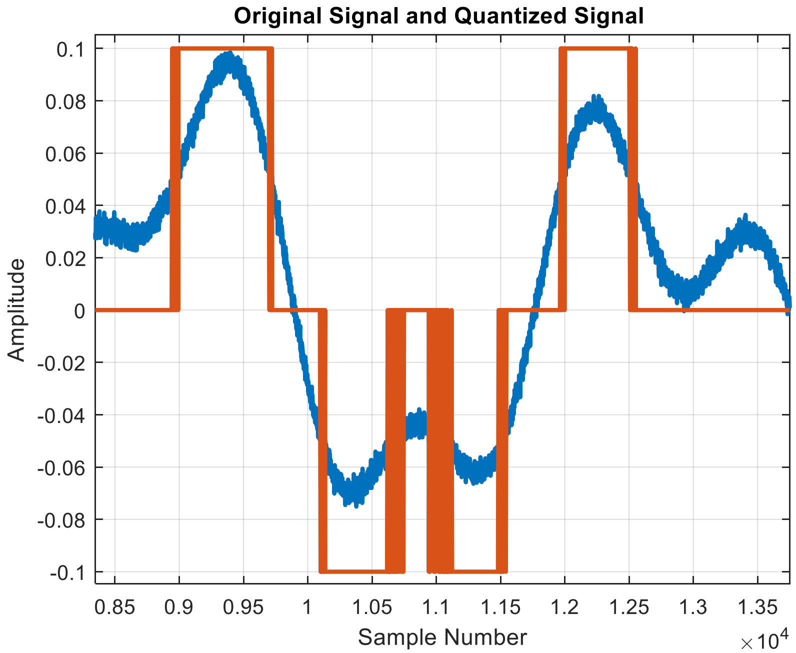
SNR dB = 3.3 dB
Original SNR dB = 27 dB ☹

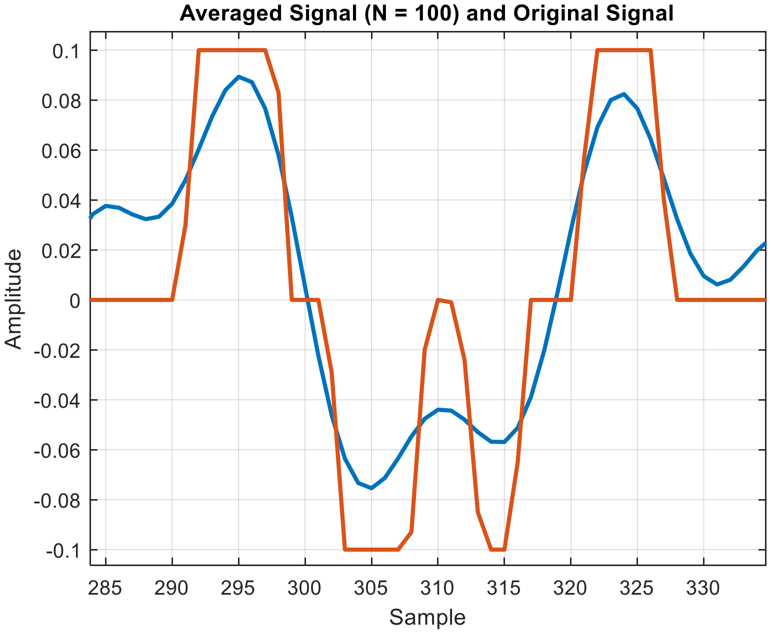
The signal does not cross the quantization threshold often

It is effectively “hidden” within the quantization levels

Maybe I can fix this with Averaging?
Averaging 100 Samples

- Taking the average of 100 of the same value still results in the same value

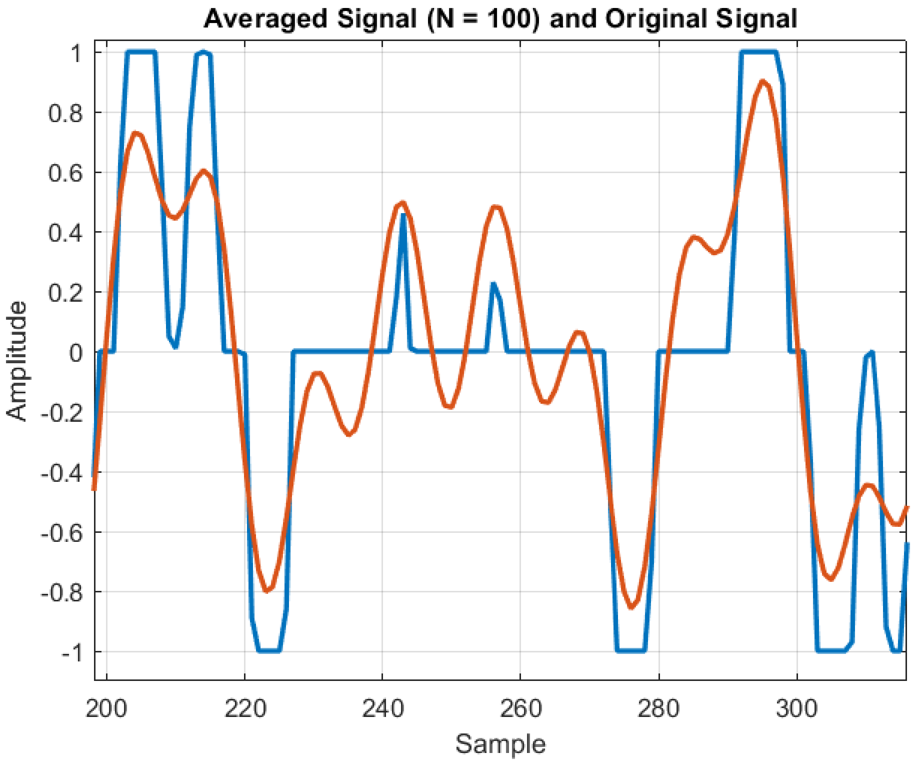




The averaged signal shows little to no improvement in reproducing the original signal

Maybe I can fix this with Averaging? Averaging 100 Samples

- Taking the average of 100 of the same value still results in the same value



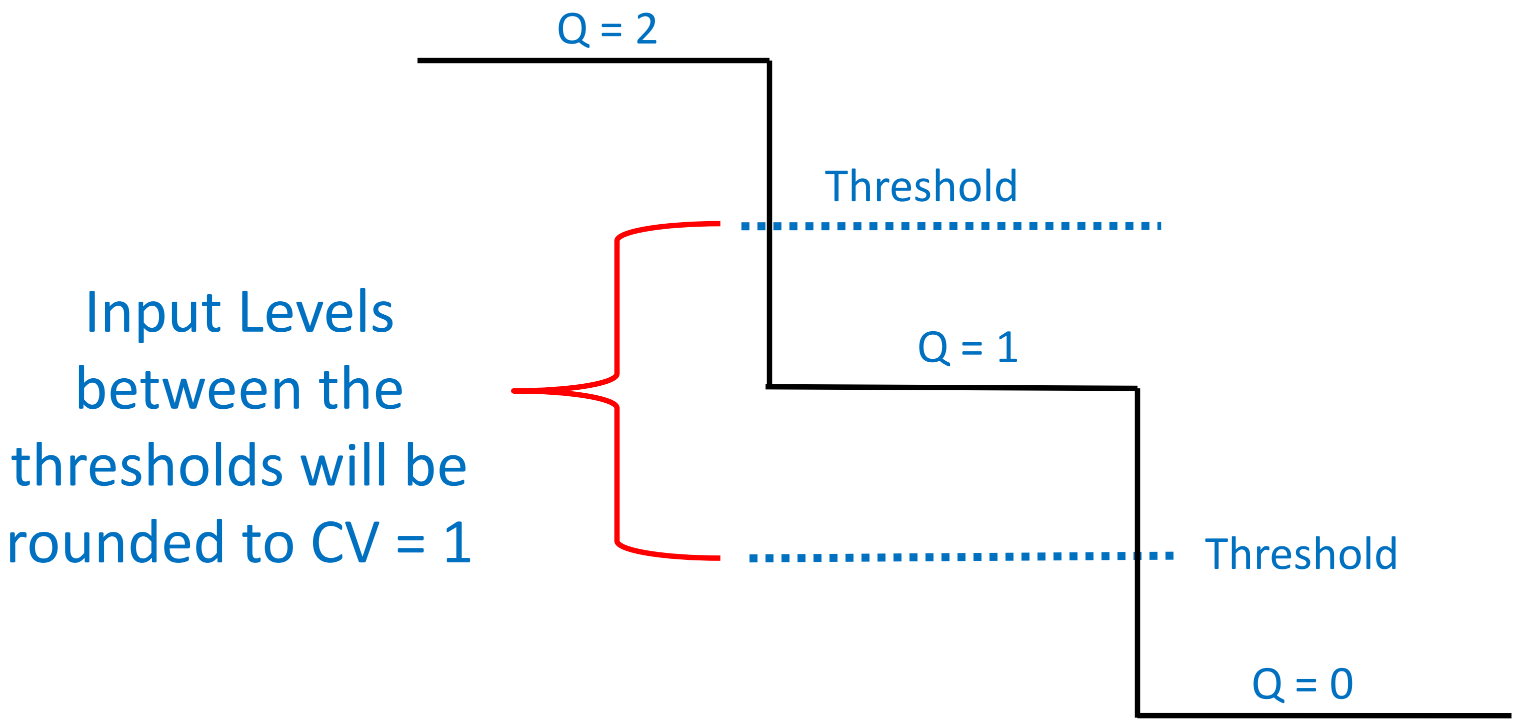
The noise SD after averaging is 0.26
We expected it to be .03

SNR dB = 3.3 dB
Original SNR dB = 27 dB ☹

No improvement in SNR!

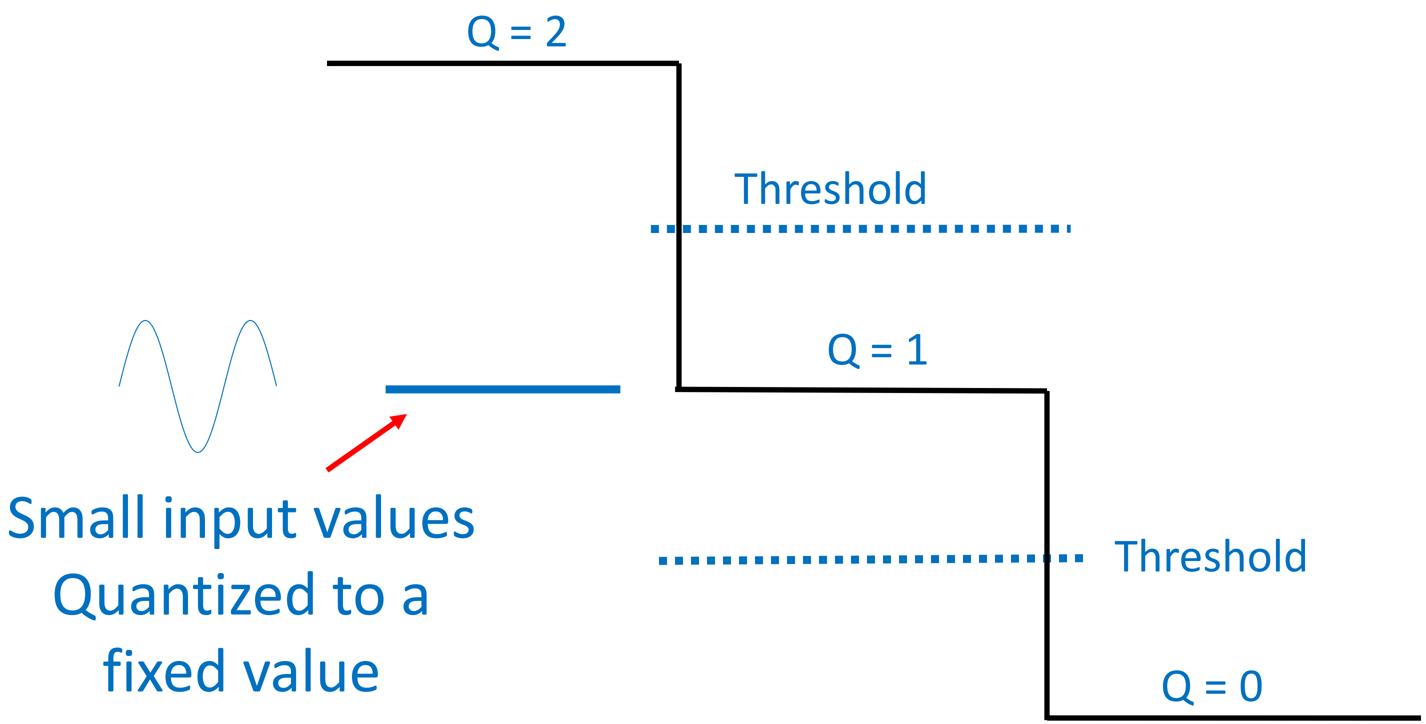
Quantization Error

- Input levels are rounded to a nearest code value (CV)



Small Signals are Quantized to a fixed Value

- If I have small signal that is not crossing the threshold then I lose that information



What Can We Do?

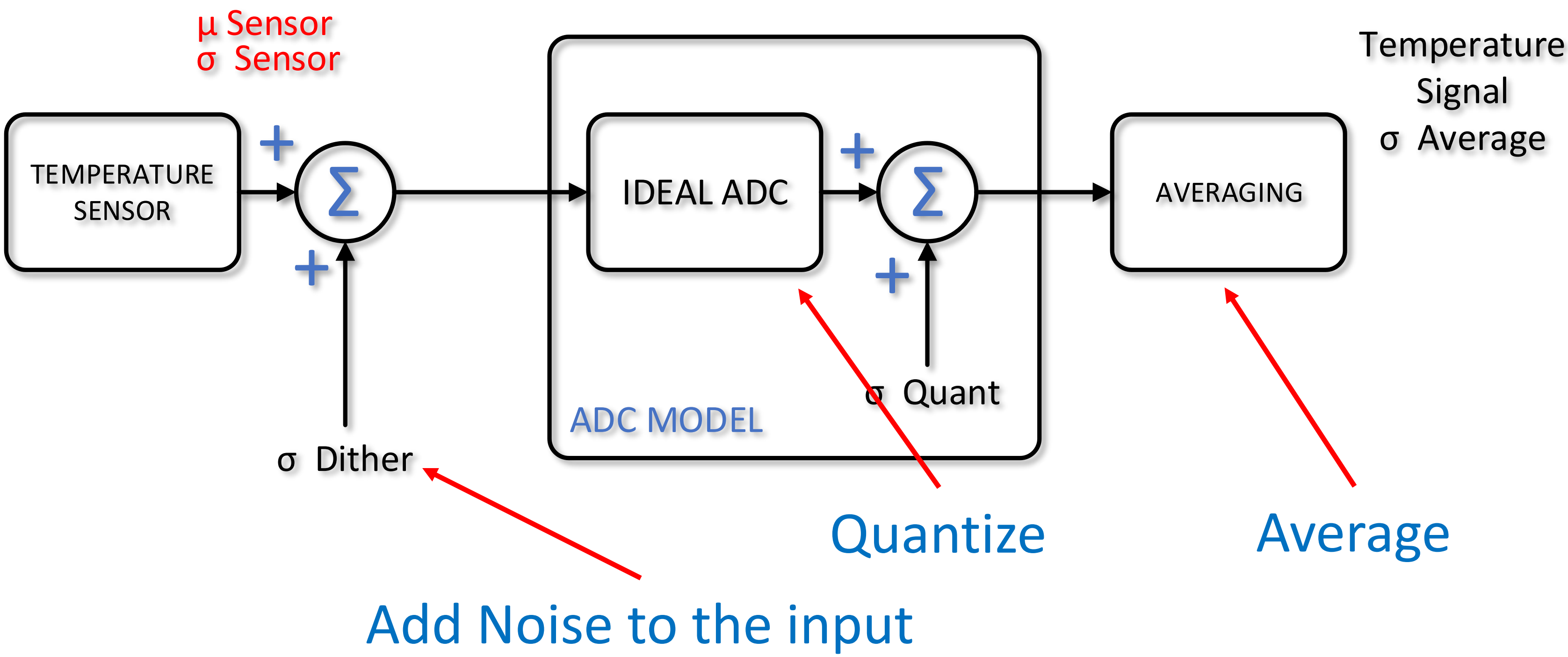
- If we could randomly force the signal into other quantization levels, then maybe we could get averaging to work?

- Add noise to the signal to get the signal to cross into other quantization levels -- Dithering

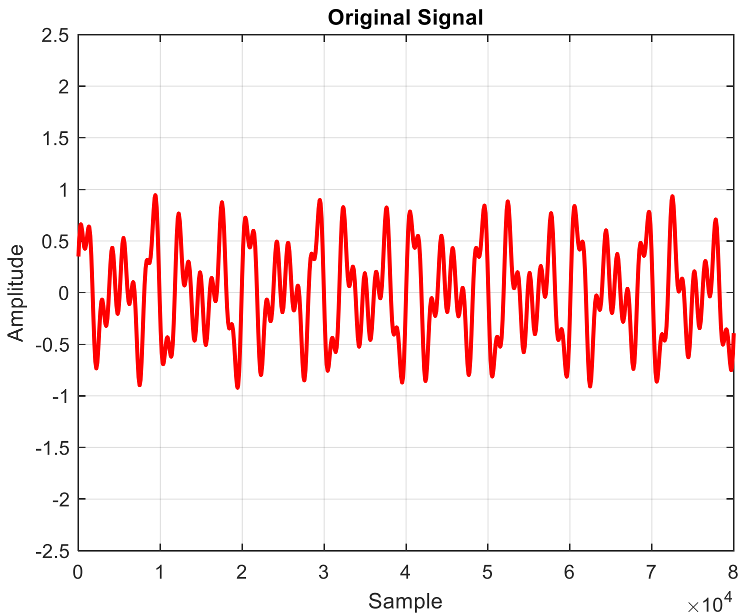
- Dithering noise can come from analog noise in the system or be added to the signal digitally using a DAC.

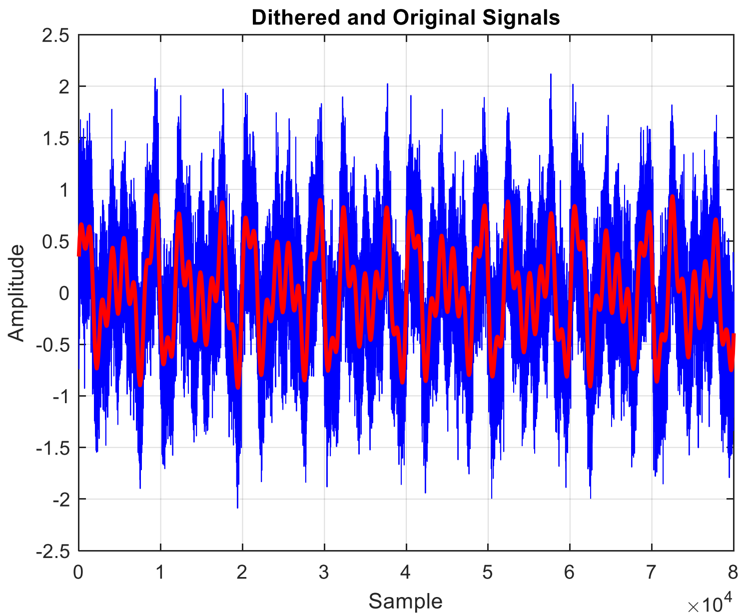
Dithering, ADC and Averaging

- Try adding dither noise to improve SNR



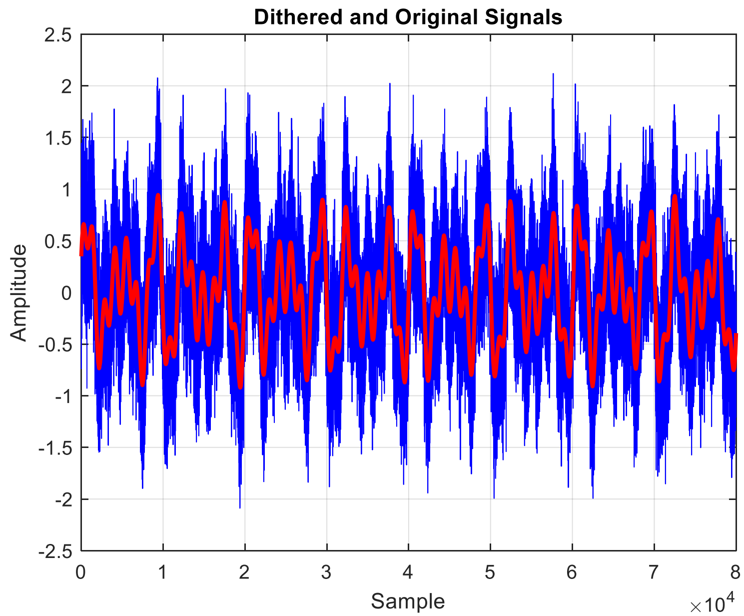
Dither Noise Added to Input Signal

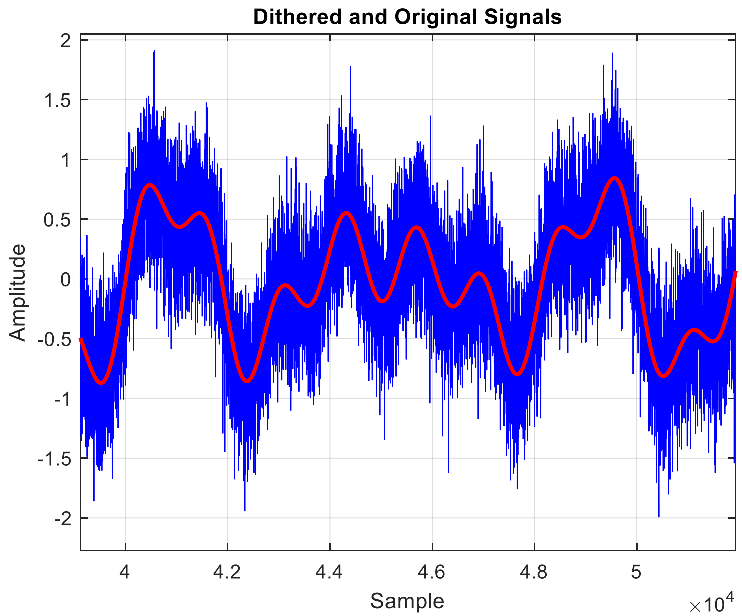




Digitally generated gaussian random noise with a σ of .35 CV is added to the input signal

Dither Noise Added to Input Signal

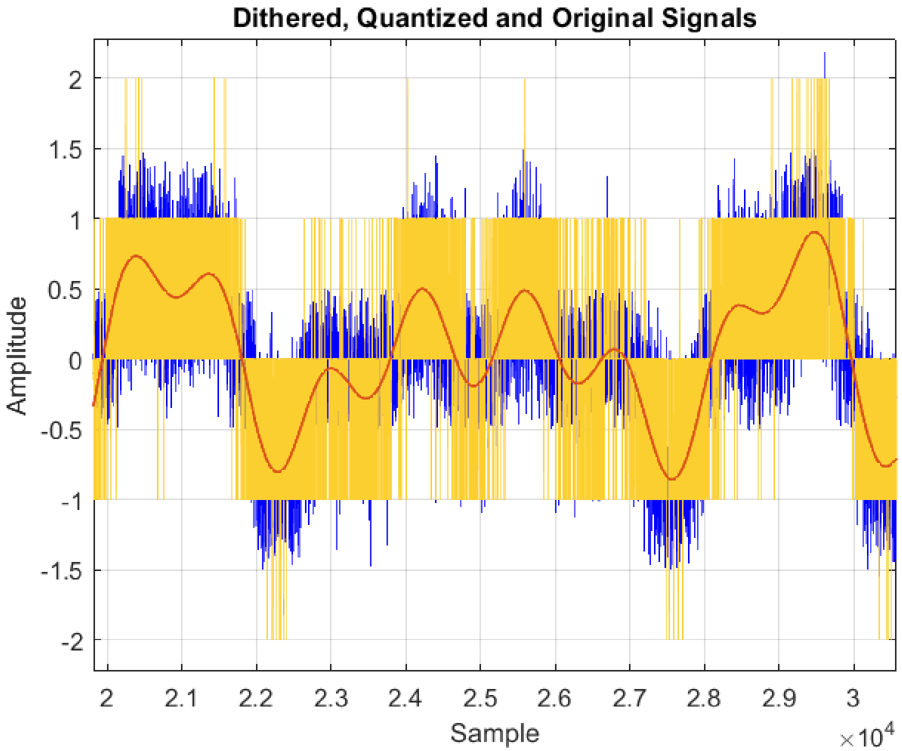




Digitally generated gaussian random noise with a σ of .35 CV is added to the input signal

Dithered Signal After Quantization

- The dither noise has forced the input to cross the quantization threshold multiple times

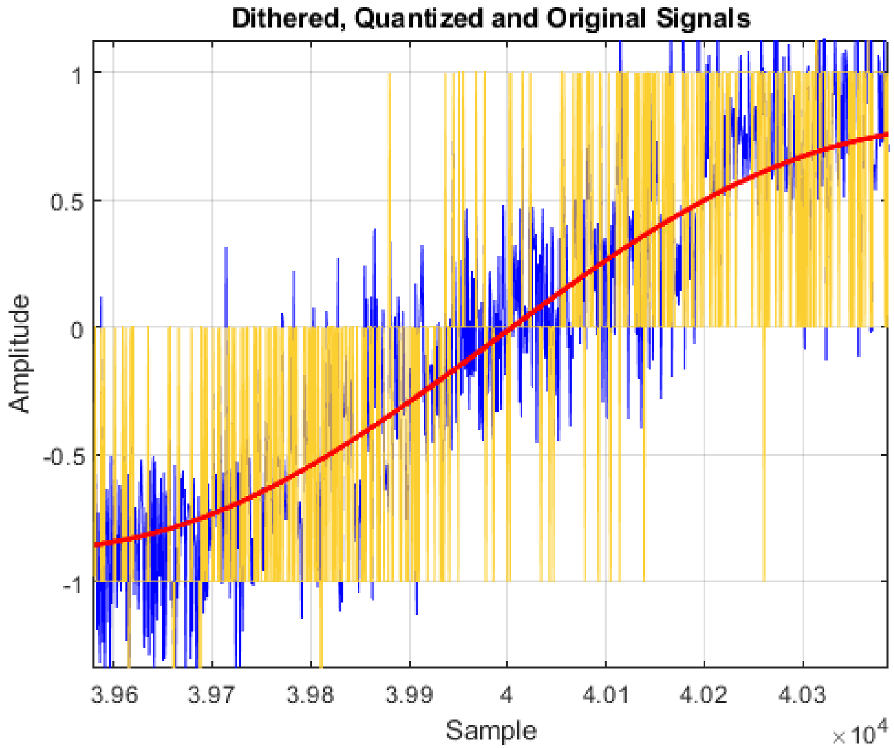


Quantization output in YELLOW.

Instead of the signal being stuck at a single quantization levels it crosses it many times

Dithered Signal After Quantization

- Expanded view shows many more quantization level threshold crossings



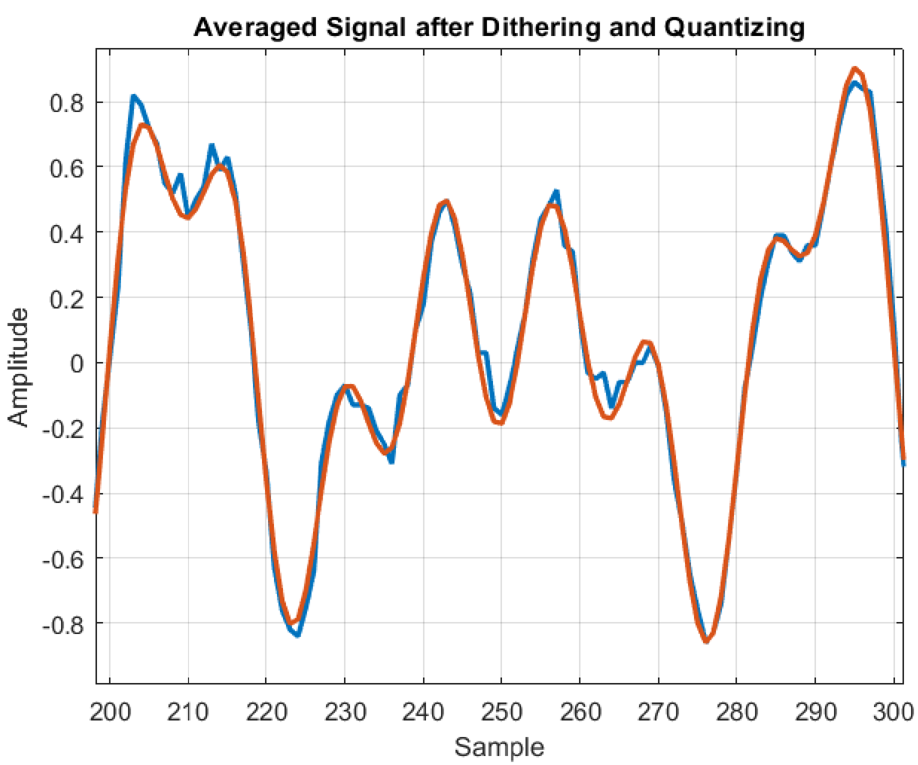
Quantization output in YELLOW.

Instead of the signal being stuck at a single quantization levels it crosses it many times

Try averaging now!

Output after Dithering and Averaging

- Average 100 samples again



The dithering and averaging has allowed us to recover the original signal even though it was small compared to a quantization level

Measured SD after dither/ave = .047
Expected SD after dither/ave = .0457

SNR dB = 19.45
Original SNR dB =27 dB

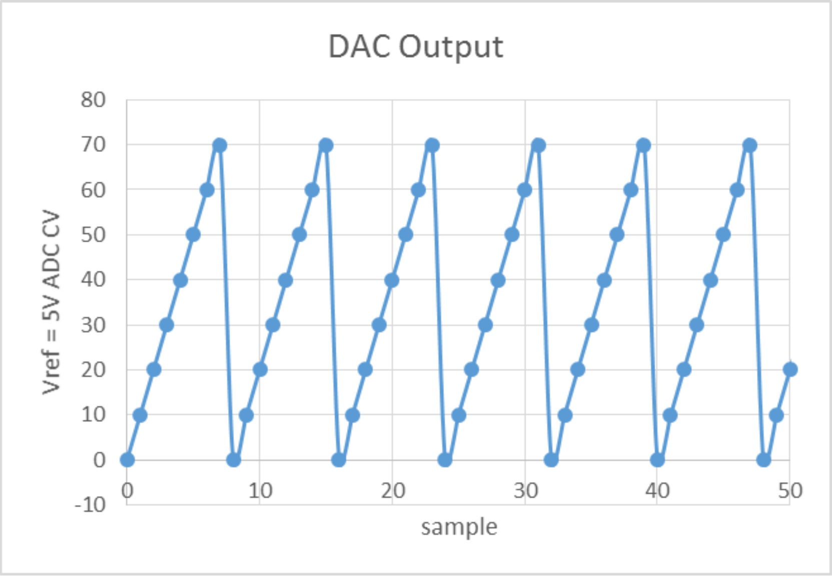
Still some degradation but much improved over no dithering

Dithering Summary

- When signal inputs are small compared to quantization levels the signal is dominated by quantization noise
- Averaging alone doesn't help to improve the SNR
- By adding a small amount of random noise (dithering), the quantization error can be made uniformly distributed.
- Averaging is then effective in improving SNR

Arduino

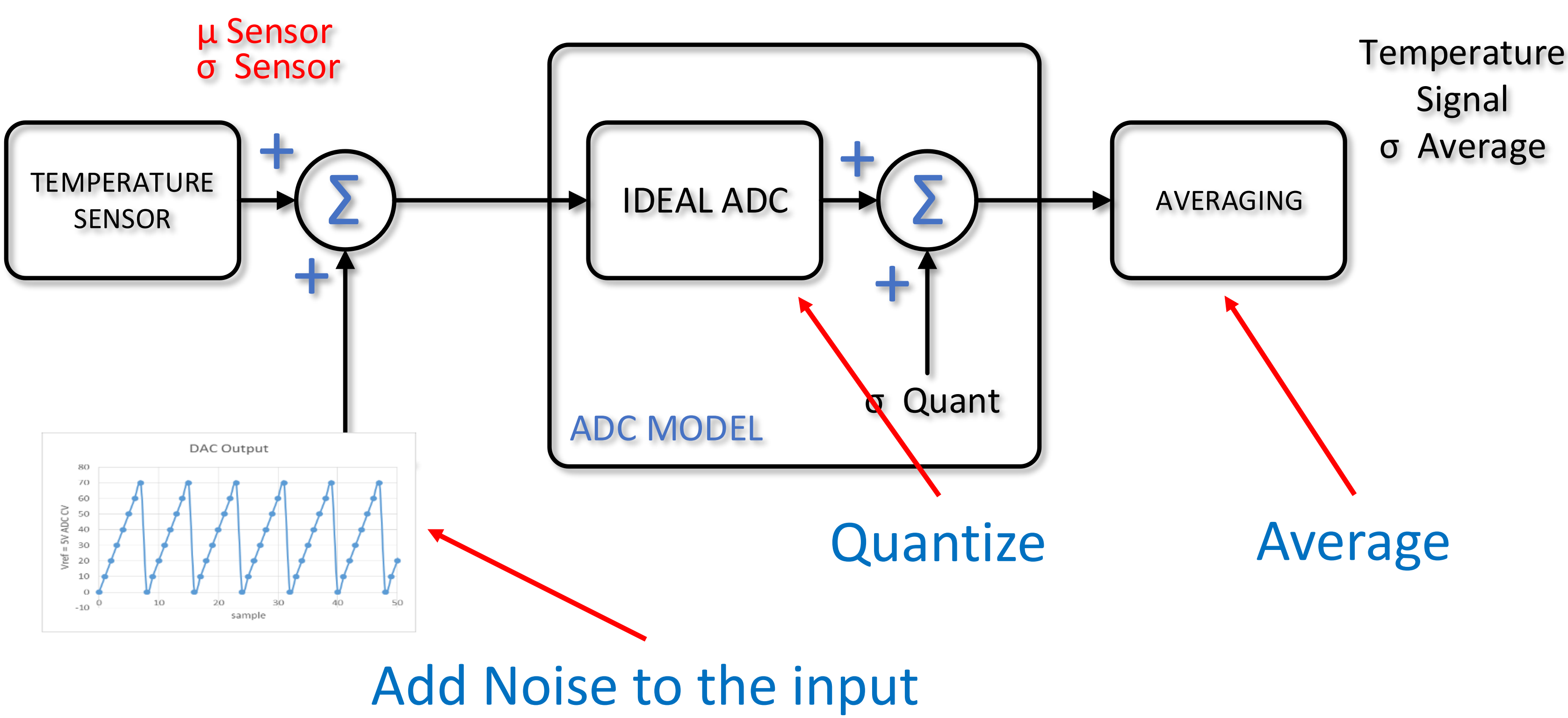
- In the Arduino we generate noise using a ramp signal out of the DAC



- We add that signal to the input and then oversample average

Dithering Noise Added

- Try adding dither noise to improve SNR



Summary of Oversample Averaging/Dithering

- Averaging can improve the SNR by reducing the noise level

- Averaging may not work in all cases
  - Where signals are small compared to the quantization levels

- Adding noise (dithering) to the input signal then averaging can improve this situation.