Digital Signal Processing

Analog to Digital Conversion and Quantization Noise

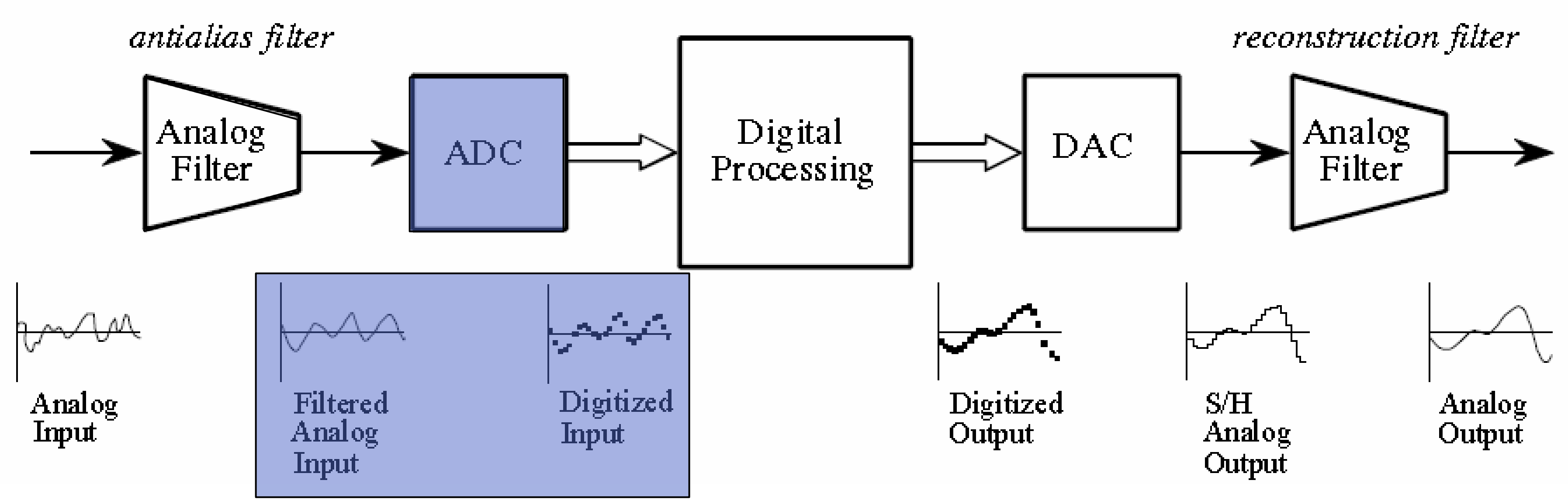
Key Points for Today

- During A/D conversion the input signal is limited to discrete values – Quantization
- Quantization adds noise to the signal
  - Adds to the input signal noise
  - Quantization noise σ is 0.29 LSB
- Quantization noise can be reduced by increasing the number of bits in the ADC
  - $\approx 6\ dB$ per bit

Key Points for Today

- Averaging can improve the SNR by reducing the noise level
- Averaging may not work in all cases
  - Where signals are small compared to the quantization levels
- Adding noise (dithering) to the input signal then averaging can improve this situation.

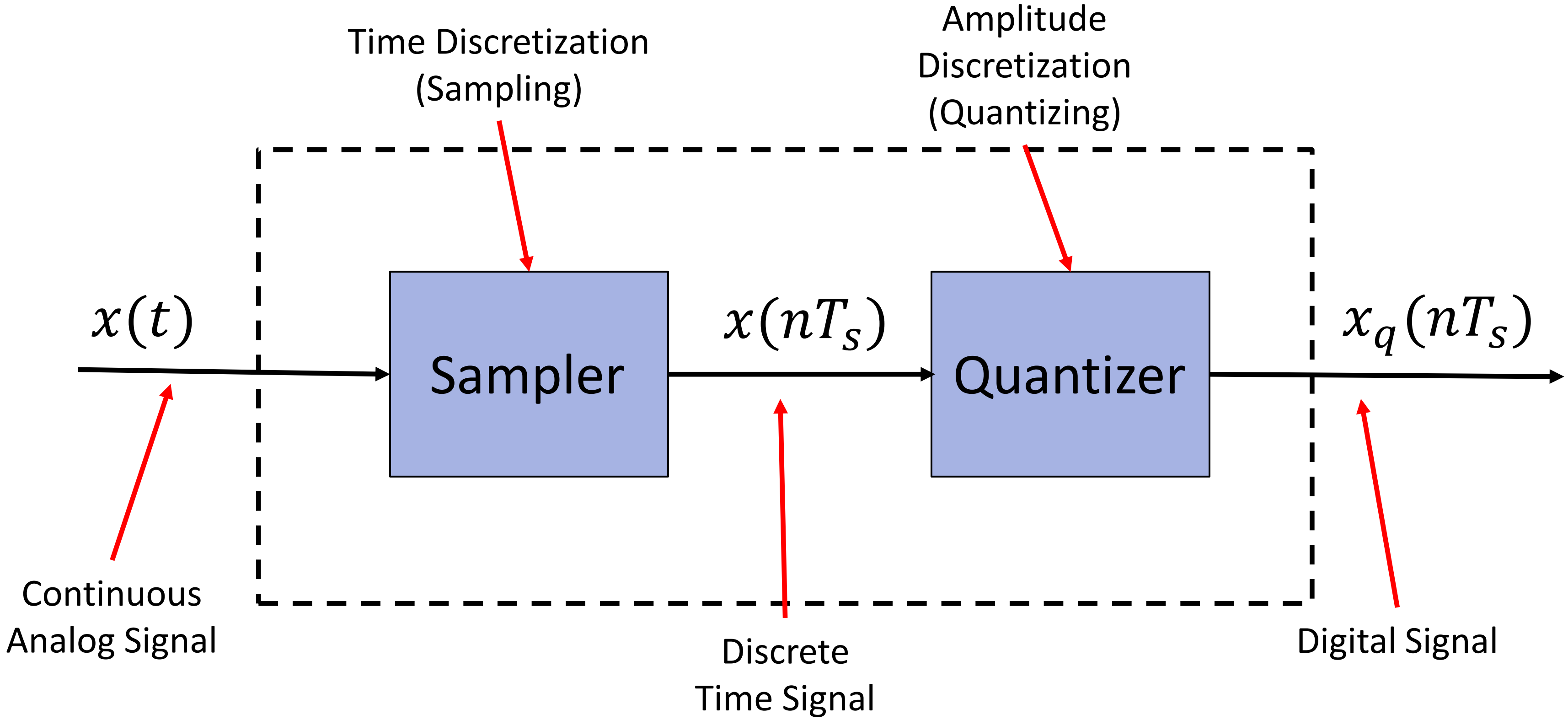
Overview of ADC/DSP/DAC System



Still focusing on the ADC today

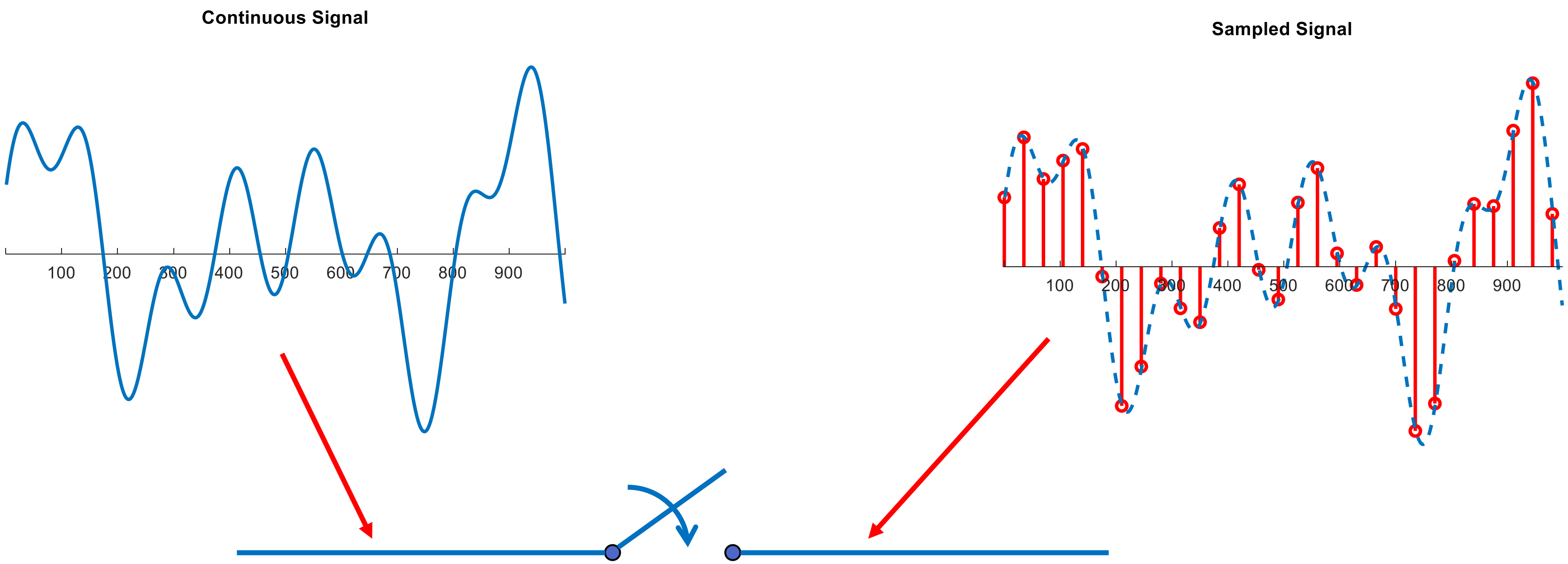
Analog to Digital Conversion

- Conversion from analog to digital requires two forms of discretization, Time and Amplitude



Sampling

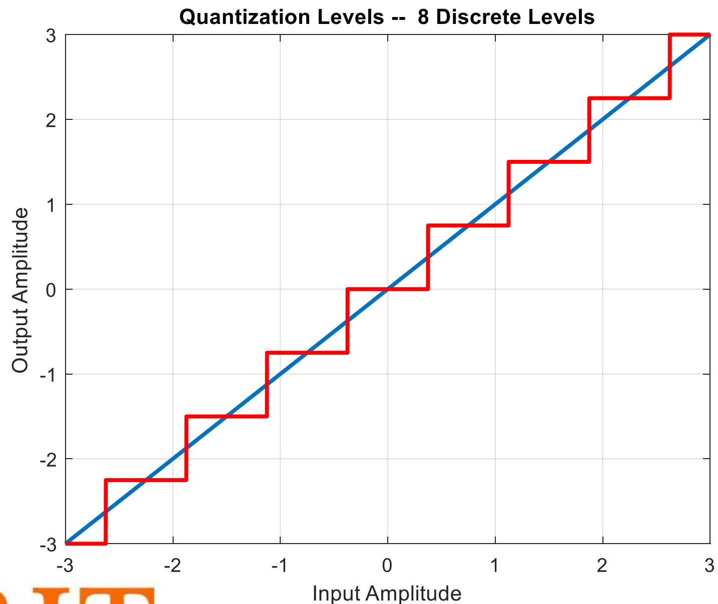
- The continuous analog signal is “viewed” or “sampled” at a periodic time T_s
- T_s is the sampling interval. $F_s = 1/T_s$ is the sample rate

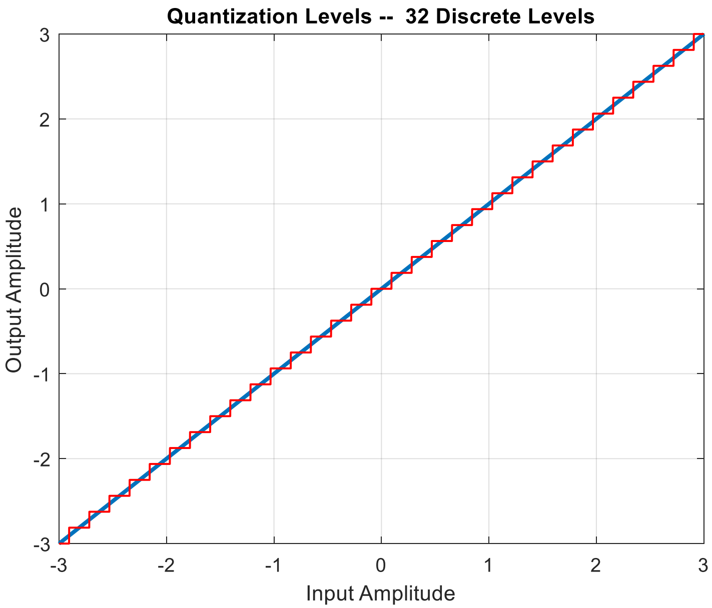


$$F_s = 1/T_s$$

Quantizing

- When represented digitally the amplitude of the signal is limited to discrete levels
- Amplitudes in between the discrete levels are rounded to the nearest discrete level





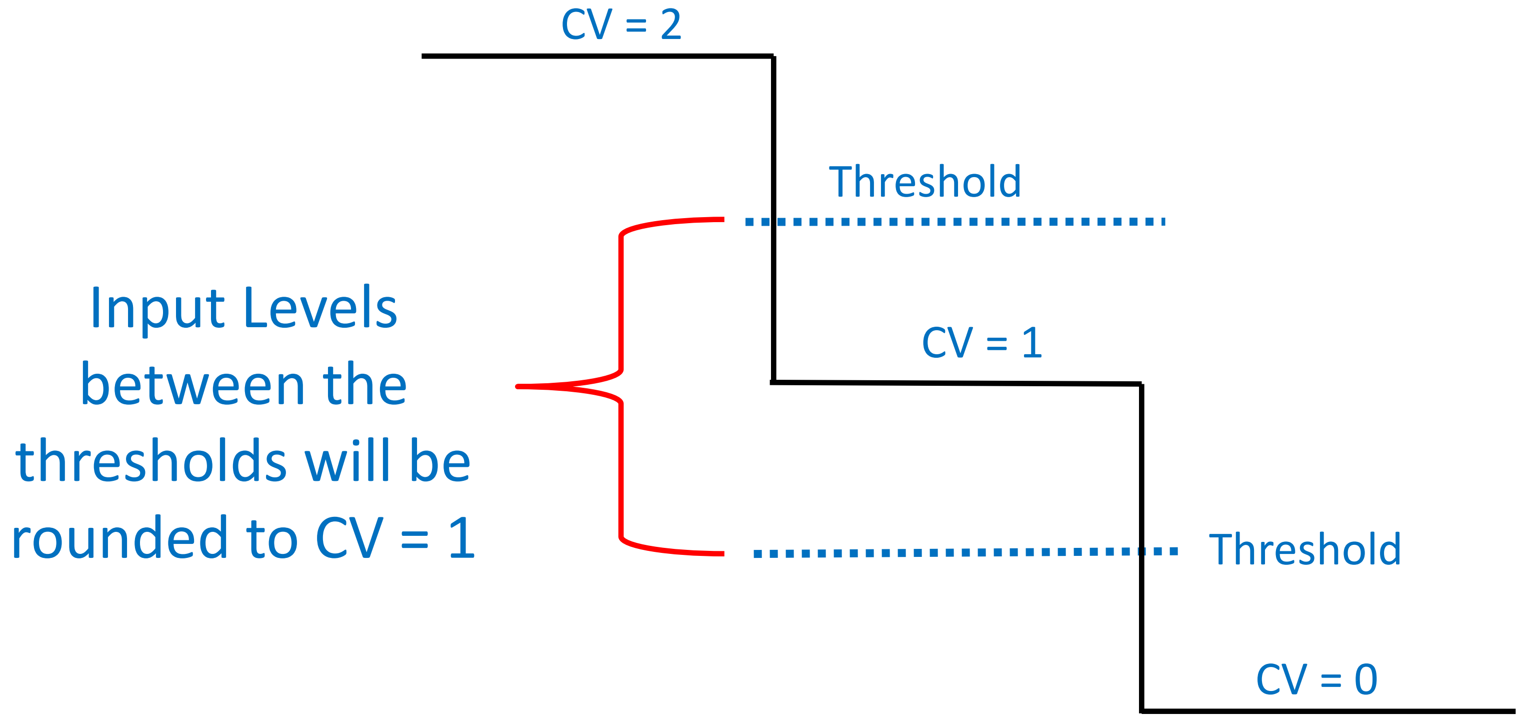
ADC Conversion Impairments

- If sampling is done properly, then the sampled signal contains all of the original information
  - The continuous signal can be reproduced exactly

- Quantizing can cause some loss in information.
  - Usually in the form of additive noise

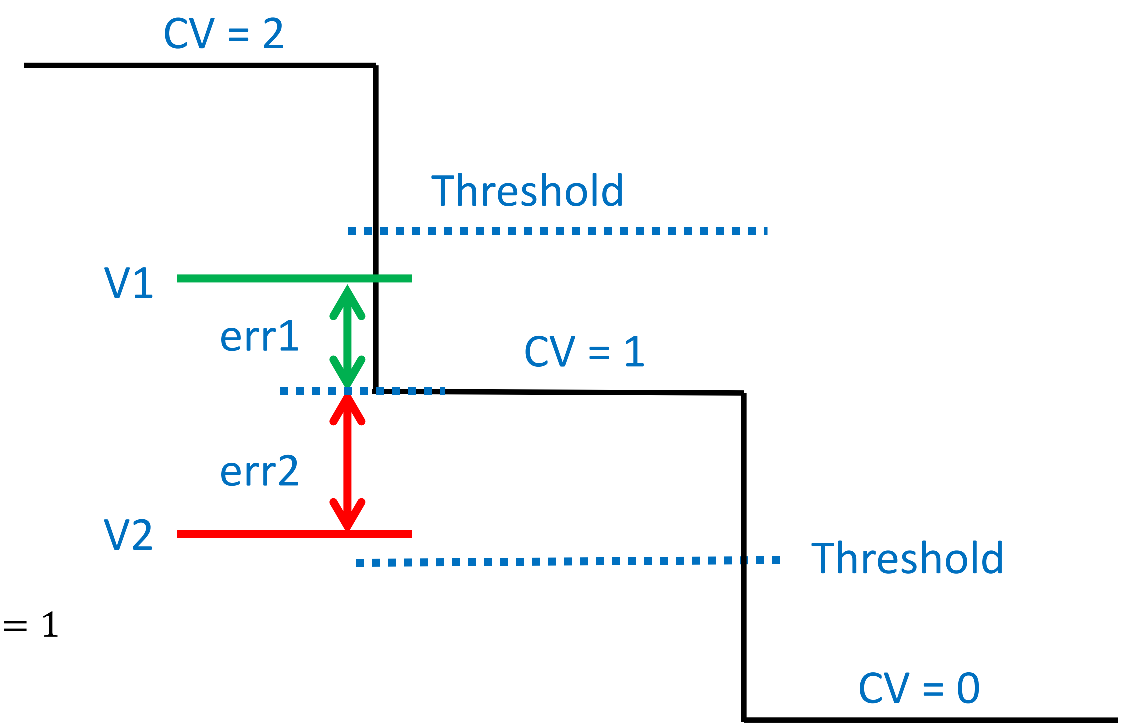
Quantization Error

- Input levels are rounded to a nearest code value (CV)

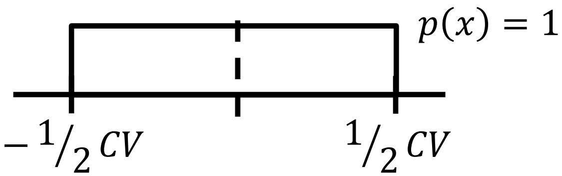


Quantization Error

- For an input level, the error will be between ½ CV and $-{}^{1}/_{2}$ CV

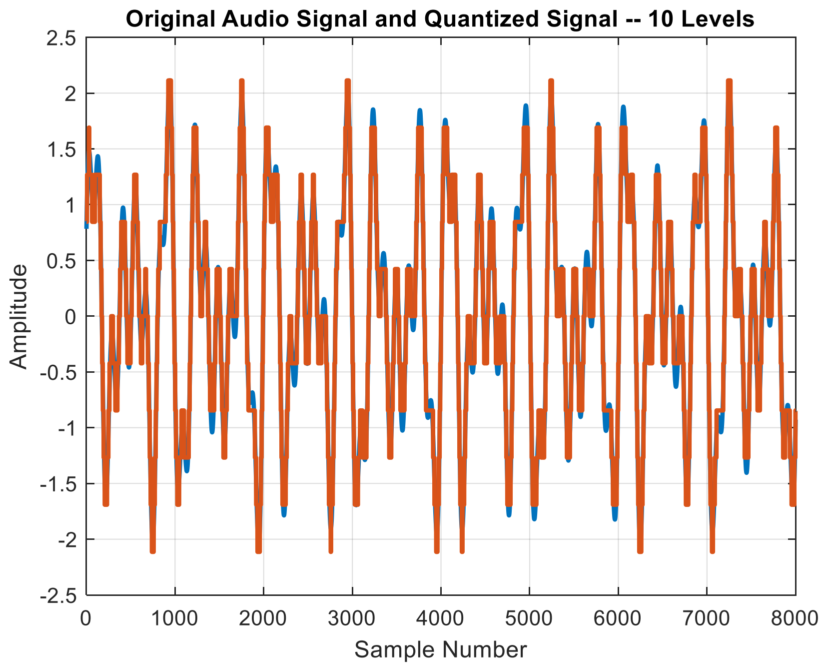


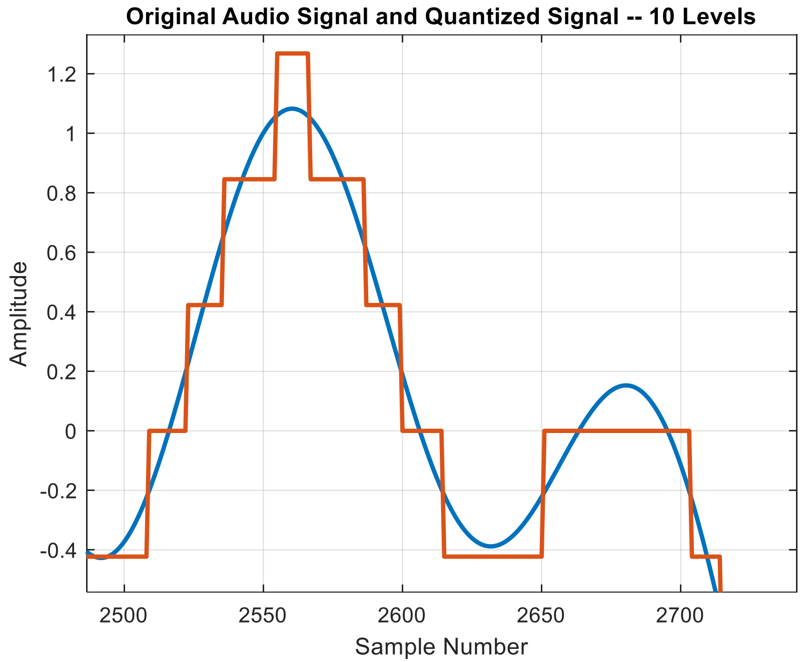
The error will be uniformly distributed between $\pm {}^{1}/_{2}\, CV$



Quantization Error Example

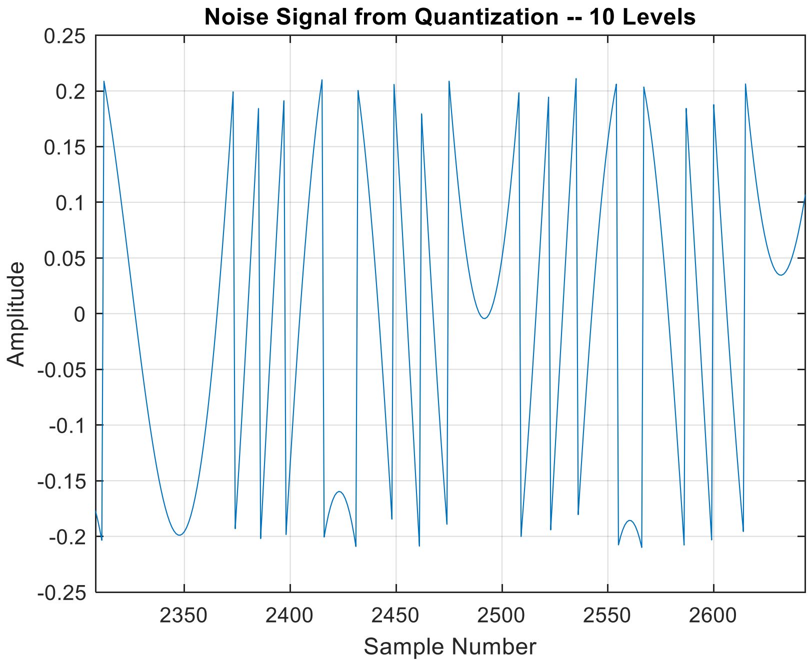
- A signal is quantized to 10 levels

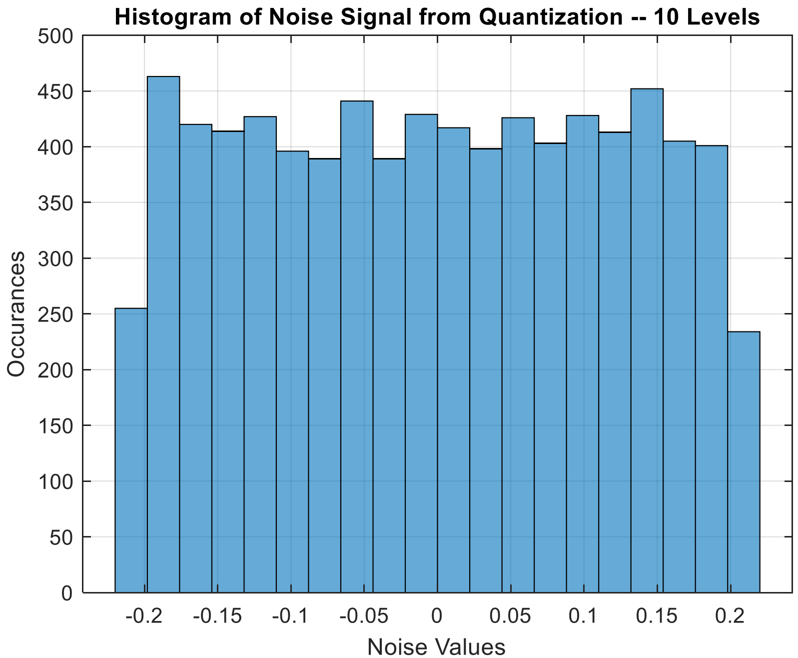




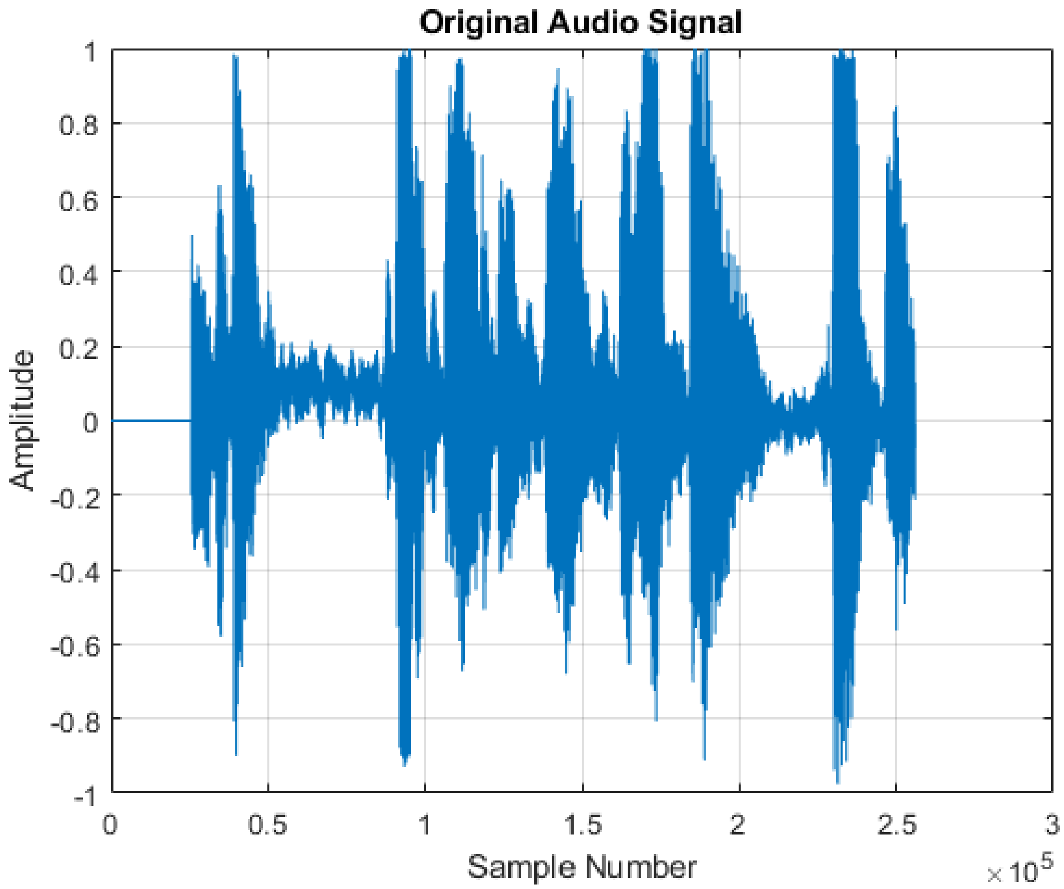
Quantization Error in Time and Histogram

- The error between the input signal and the quantized value is often considered to be a uniformly distributed random variable
- The amplitude ranges from $\pm 1/2CV$





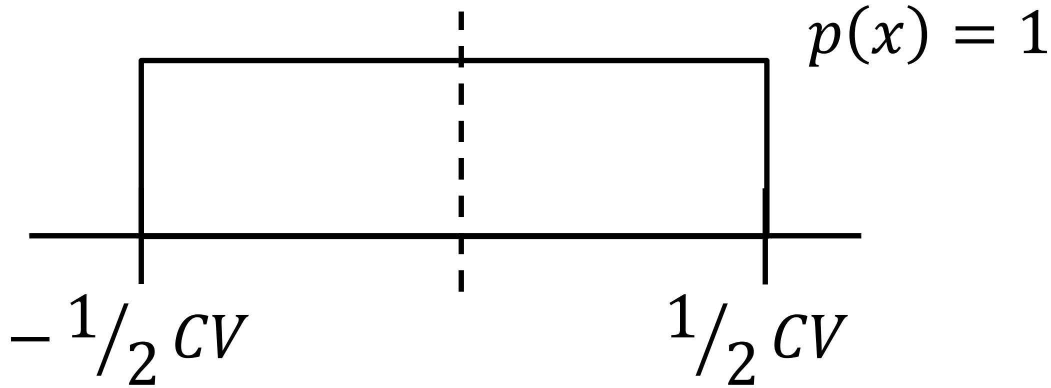
Quantizing Audio MATLAB Demo



How much noise is that?

- We can compute the noise in terms of code values using the probability density function

The error is uniformly distributed between $-{}^{1}/_{2}\,CV$ and ½ CV



The variance is computed using the PDF

$$\sigma^2 = \int_{-1/2}^{1/2} x^2 p(x)\,dx$$

How much noise is that?

- We can compute the noise in terms of code values using the probability density function

$$\sigma^2 = \int_{-1/2}^{1/2} x^2 p(x)\, dx \quad \Rightarrow \quad \sigma^2 = \frac{x^3}{3}\bigg|_{-1/2}^{1/2}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{1}{8} - \left(-\frac{1}{8}\right)\right) = \frac{1}{12} \quad \Rightarrow \quad \sigma = \sqrt{\frac{1}{12}}$$

$$\boxed{\sigma = 0.29\ CV}$$

How Many Quantization Levels are There?

- If an ADC has a resolution of N bits then there are 2^N quantization levels

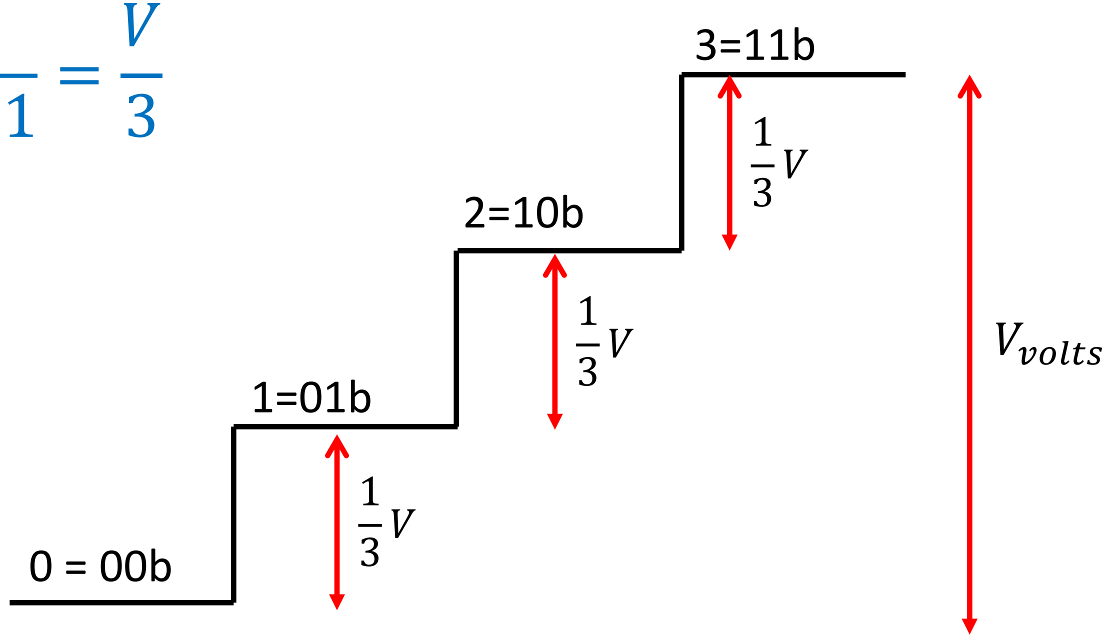
- Example if an ADC has 8-bits then:

$$2^{nBits} = 2^8 = 256$$

What is the value of 1 LSB or 1 Code Level?

- An ADC with 2 bits has 2^2 quantization levels
- Each "bit" or code value is

$$CV = \frac{V}{2^2 - 1} = \frac{V}{3}$$



How Many Quantization Levels are There?

- If an ADC has a resolution of 8 bits then there are 2^8 quantization levels. It has a full scale voltage of 5V
- The value in volts of one code value is then:

$$CV = \frac{V}{2^{nBits} - 1} = \frac{5v}{2^8 - 1} = \frac{5v}{255} = 19.6mV$$

What is the Quantization Noise?

- If an ADC has a resolution of 8 bits and a full scale voltage of 5V
- What is the quantization noise of the ADC?

$$CV = \frac{V}{2^{nBits} - 1} = \frac{5v}{2^8 - 1} = \frac{5v}{255} = 19.6mV$$

$$\sigma = 0.29\ CV$$

$$\sigma = 0.29 \times 19.6mV = 5.68mV$$

In Class Problem
Quantization Noise Level

- Assume an ADC that covers the range from 0 to 5V.   Assume that the ADC has 10 bits

- How many quantization levels are there?
- How large, in volts is 1 code value?
- How much quantization noise will there be?

In Class Problem
Quantization Noise Level

- Assume an ADC that covers the range from 0 to 5V. Assume that the ADC has 10 bits

- How many quantization levels are there?

The number of quantization levels is

$$2^{NumBits} = 2^{10} = 1024$$

- How large, in volts is 1 quantization level or code value?

1 code value = 1 quantization level ⟹ $\boxed{1\ CV = \dfrac{5v}{2^{10} - 1} = 4.89\ mV}$

NOPRINT

In Class Problem
Quantization Noise Level

- Assume an ADC that covers the range from 0 to 5V. Assume that the ADC has 10 bits

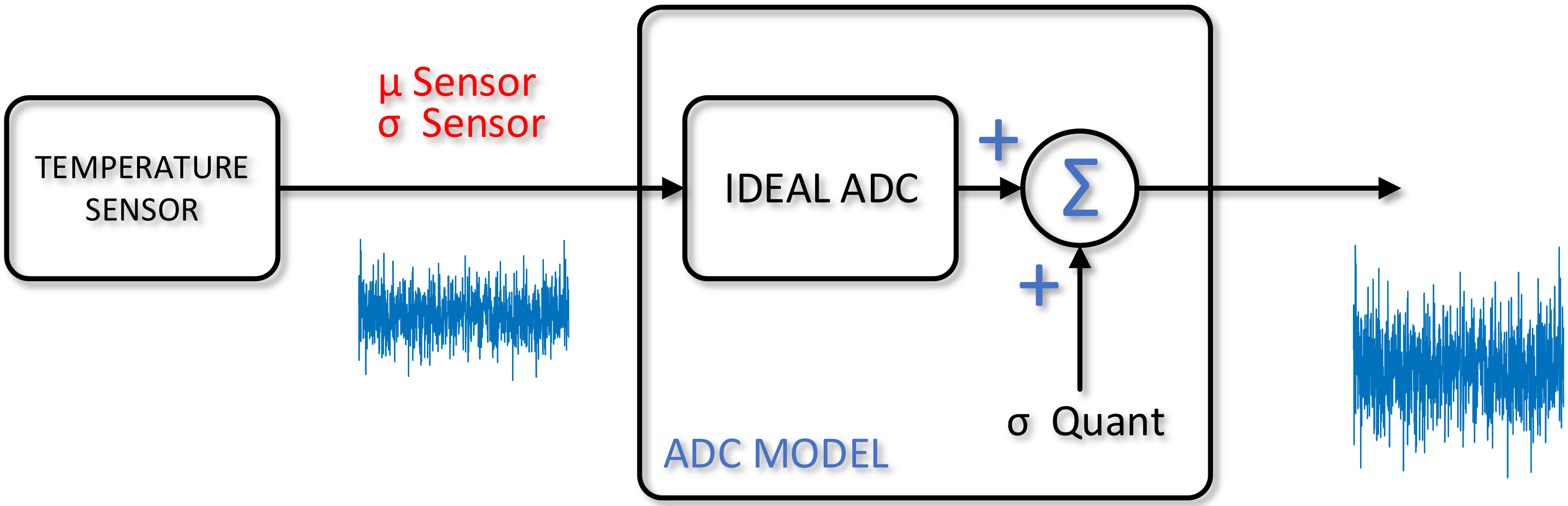
- How much quantization noise is there?

The quantization noise level is then

$$\boxed{\sigma = .29\, CV = 0.29 \times 4.89\, mV = 1.4 mV_{rms}}$$

What if the Input Signal is Noisy?

- If the input the ADC has noise, what is the impact of quantization noise?



Noise adds in quadrature

$$\sigma_{total} = \sqrt{\sigma_{input}^2 + \sigma_{quant}^2}$$

What if the Input Signal is Noisy?

- If the input the ADC has noise, what is the impact of quantization noise?

- Recall the noise levels add in quadrature

$$\sigma_{total} = \sqrt{\sigma_1^2 + \sigma_2^2}$$

- If the input has noise, then the total noise in the signal will be

$$\sigma_{total} = \sqrt{\sigma_{input}^2 + \sigma_{quant}^2}$$

What is the SNR Degradation due to Quantization Noise?

- The chart shows the amount of SNR degradation due to quantization noise for various levels of input noise

Input Noise Level (mV)	Quantization Noise Level (mV)	Input to Quantization Noise Ratio	Total Noise (mV)	SNR Degradation (dB)
0.142	1.417	0.1	1.424	20.0
0.709	1.417	0.5	1.585	6.99
1.417	1.417	1	2.005	3.01
2.835	1.417	2	3.169	0.97
4.252	1.417	3	4.482	0.46
5.670	1.417	4	5.844	0.26
7.087	1.417	5	7.227	0.17
14.174	1.417	10	14.245	0.04
28.348	1.417	20	28.383	0.01

The degradation is large when the input noise is small relative to Q-Noise

The degradation is 3dB when the input noise is equal to Q-Noise

The degradation becomes small even for noise that is 4 times Q-Noise ($\approx .25\ dB$)

What if the Input Signal is Noisy?

- If the noise on the input signal is large relative to the quantization noise, then the SNR degradation is small.

- Example: If the input noise is 5 times greater than the quantization noise, the SNR degradation is $< 0.2\ dB$

- What can be done if the quantization noise is too large?

Improving Noise Performance

- The quantization noise is a function of the number of bits in the ADC

- Increasing the number of bits will decrease quantization noise

- Example 5V ADC with 8, 10 and 12-bit resolution

$$\sigma_{q8} = 5.69\, mV \quad \sigma_{q10} = 1.42\, mV \quad \sigma_{q12} = 0.35\, mV$$

Improving Noise Performance

- The quantization noise is a function of the number of bits in the ADC
- Example 5V ADC with 8, 10 and 12-bit resolution

$$\sigma_{q8} = 5.69\,mV \quad \sigma_{q10} = 1.42\,mV \quad \sigma_{q12} = 0.35\,mV$$

SNR Improvements

$$\left\{\begin{aligned} 20\log_{10}\frac{\sigma_{q8}}{\sigma_{q10}} &= 20\log_{10}\frac{5.69}{1.42} = 12\,dB \\ 20\log_{10}\frac{\sigma_{q8}}{\sigma_{q12}} &= 20\log_{10}\frac{5.69}{0.35} = 24\,dB \end{aligned}\right.$$

SNR Comparison for Different ADC Resolution

- The SNR increases ≈ 6 dB for each bit of ADC resolution

Number of Bits	Code Level (mV)	Quantization Noise (mV)	SNR Improvement over 8 bits (dB)	Improvement per Bit
8	19.61	5.69	0.00	N/A
10	4.888	1.417	12.07	6.03
12	1.221	0.354	24.11	6.03
16	0.076	0.022	48.20	6.02
24	0.0003	0.0001	96.36	6.02

In Class Problem
How many bits are required?

- An ADC has an input range of $-3V$ to $3V$.
- An input signal has a noise level of 3 mV.
- How many bits are needed to keep the SNR degradation to less than 3 dB?

In Class Problem
How many bits are required?

- How many bits are needed to keep the SNR degradation, D to less than 3 dB?

The SNR degradation will come from the additional noise from quantization. Find the numerical ratio of the degradation for 3 dB. That is the ratio of the total noise to the input noise

$$\frac{\sigma_{total}}{\sigma_{in}} < 10^{\frac{D}{20}} < 10^{\frac{3}{20}} = 1.413$$

We know the input noise level (3mV). Compute the total noise

$$\sigma_{total} = 1.413\sigma_{in} = 1.413(3\,mV) \qquad \sigma_{total} = 4.24\,mV$$

The total noise cannot be more than 4.24 mV

NOPRINT

In Class Problem
How many bits are required?

- How many bits are needed to keep the SNR degradation, D to less than 3 dB?

Now compute the amount of quantization noise that would give you 4.24 mV of total noise knowing the input noise level of 3mV. Noise adds in quadrature.

$$\sigma_{total} = \sqrt{\sigma_{in}^2 + \sigma_q^2} \qquad \sigma_q = \sqrt{\sigma_{total}^2 - \sigma_{in}^2}$$

$$\sigma_q = \sqrt{(4.24\,mV)^2 - (3\,mV)^2} = 3\,mV$$

The quantization noise must be less than or equal to 3mV

NOPRINT

In Class Problem
How many bits are required?

- How many bits are needed to keep the SNR degradation, D to less than 3 dB?

Knowing the quantization noise (3mV) and how it relates to the number of bits and the input voltage range (6V), compute the number of bits required

$$\sigma_q = 0.29 \times 1CV \qquad \sigma_q = 0.29 \times \frac{6V}{2^n - 1}$$

$$2^n \geq \frac{0.29}{3\ mV}(6V) + 1 \ = 581 \qquad n \geq \log_2 581 = 9.18$$

$$n = 10\ bits$$

At least 10 bits is need to keep the degradation under 3 dB

Table of Degradation vs Number of Bits

- The computed value of 9.18 bits is not realizable so use n = 10 bits

Input Range (V)	**6**
Input Noise (mV)	**3**

Number of Bits	Code Level (mV)	Quantization Noise (mV)	Input Level (mV)	Total Noise (dB)	SNR Degradation (dB)
8	23.53	6.82	3.00	7.45	7.91
9	11.74	3.405	3.00	4.54	3.60
9.18	10.36	3.005	3.00	4.25	3.02
10	5.87	1.701	3.00	3.45	1.21
11	2.93	0.850	3.00	3.12	0.34
12	1.47	0.4249	3.00	3.03	0.09

Summary of Today

- During A/D conversion the input signal is limited to discrete values – Quantization

- Quantization adds noise to the signal
  - Adds to the input signal noise
  - Quantization noise σ is 0.29 LSB

- Quantization noise can be reduced by increasing the number of bits in the ADC
  - $\approx 6\ dB$ per bit