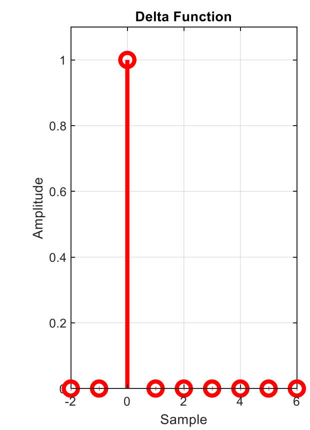
Properties of Convolution

Today's Topics

- Common Delta Functions
- Phase Relationships
- Causality
- Correlation

A Few Common Impulse Functions

- Delta Function –
  Convolution identity $x[n] * \delta[n] = x[n]$

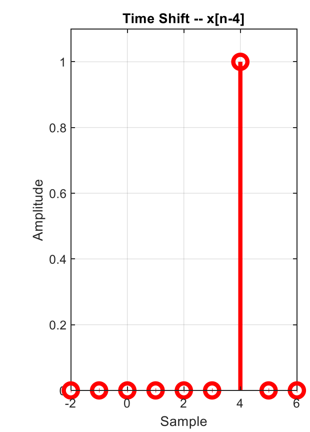


- Amplifier or Attenuator

$$x[n] * k\delta[n] = kx[n]$$

- Time Shift

$$x[n] * \delta[n-s] = x[n-s]$$



In Class Problem

- A system has an impulse response, h[n], given by: 1, 2, 2, 1, 0, -1, 0, 0, for the values of samples 0 to 7.
- Calculate the output of the system in response to the following input signals.
- a. 1, 0, 0, 0, 0, 0, 0, 0, 0
- b. -3, 0, 0, 0, 0, 0, 0, 0, 0
- c. 0, 0, 1, 0, 0, 0, 0, 0, 0
- d. 1, 0, 1, 0, 0, 0, 0, 1, 0

Try doing these by thinking about homogeneity, Additivity and time invariance!

In Class Problem

- $a[n] * h[n] =$ 1 2 2 1 0 -1 0 0 0 0 0 0 0 0 0
- $b[n] * h[n] =$ -3 -6 -6 -3 0 3 0 0 0 0 0 0 0 0 0
- $c[n] * h[n] =$ 0 0 1 2 2 1 0 -1 0 0 0 0 0 0 0
- $d[n] * h[n] =$ 1 2 3 3 2 0 0 0 2 2 1 0 -1 0 0 0

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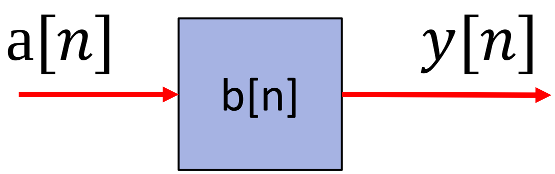
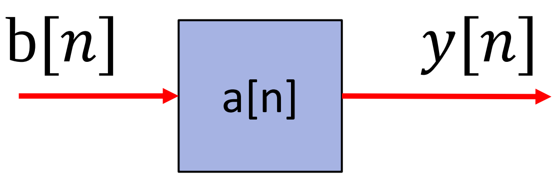
Properties of Convolution
Commutative Property

$$a[n] * b[n] = b[n] * a[n]$$





If $a[n]$ is the input to a system with impulse response $b[n]$

The output is the same as:

If $b[n]$ is the input to a system with impulse response $a[n]$

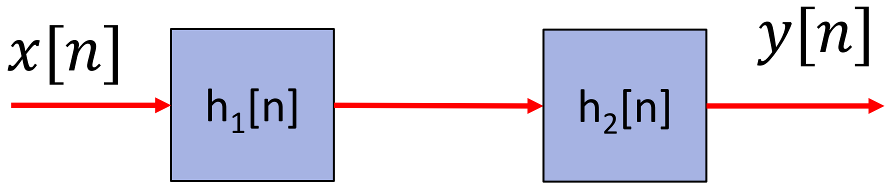
Properties of Convolution
Associative Property

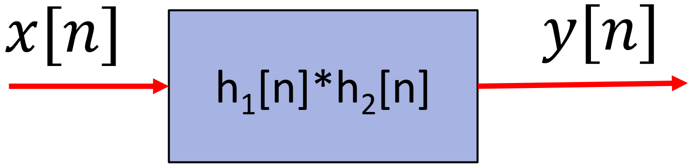
- Associative Property

$$\big[a[n] * b[n]\big] * c[n] = a[n] * \big[b[n] * c[n]\big]$$

Properties of Convolution
Cascaded Systems

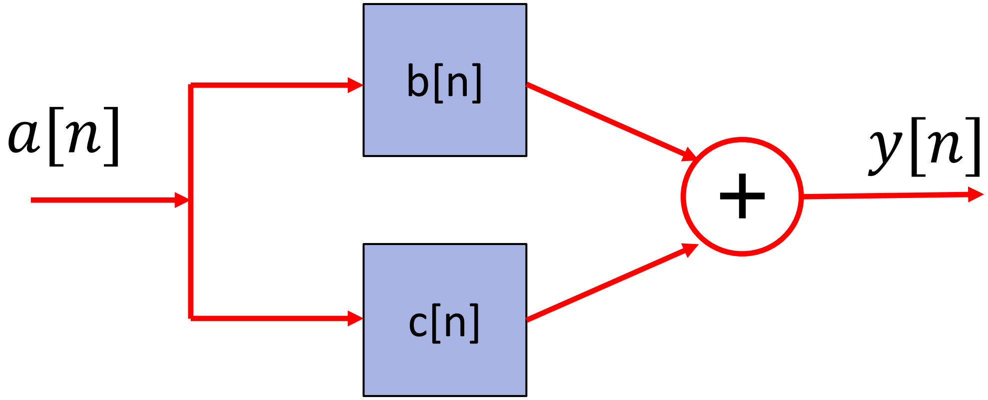
- Two cascaded systems with impulse responses $h_1[n]$ and $h_2[n]$ are equivalent to one system with impulse response $h_1[n] * h_2[n]$

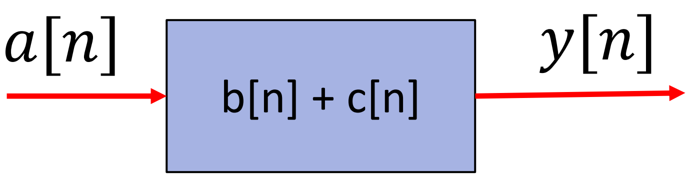




Properties of Convolution
Distributive Property

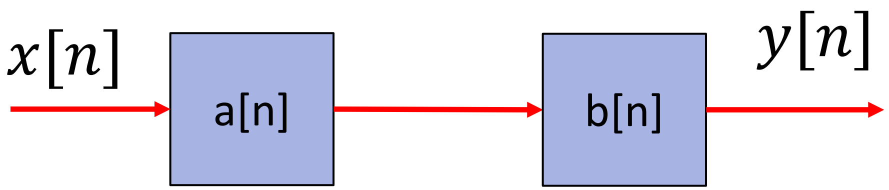
$$a[n] * c[n] + a[n] * b[n] = a[n] * \left[b[n] + c[n]\right]$$

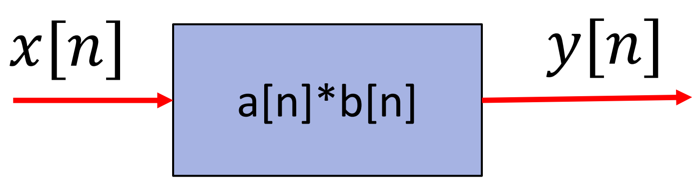




In Class Problem

- There are two impulse responses
  - $a[n] = [0.25,\ .5,\ 0.25]$ and $b[n] = [-1, 1\ ]$
- Find their cascaded response
- Find the output of the cascaded system when
  - $x[n] = [1, 0, 1, 0, 1]$





Convolving the two impulse responses

- Use the output side algorithm.
- Time reverse ("flip") b[n]

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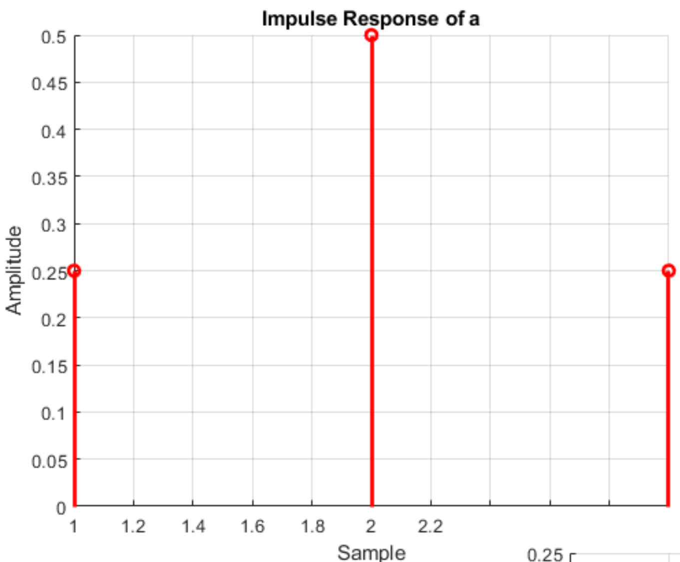
	n	0	1	2	
	a[n]	0.25	0.5	0.25	
b[N-1-n]	1	-1			
	y[n]	-0.25			

n	0	1	2	
a[n]	0.25	0.5	0.25	
b[N-1-n]	1	-1		
y[n]	-0.25	-0.25		

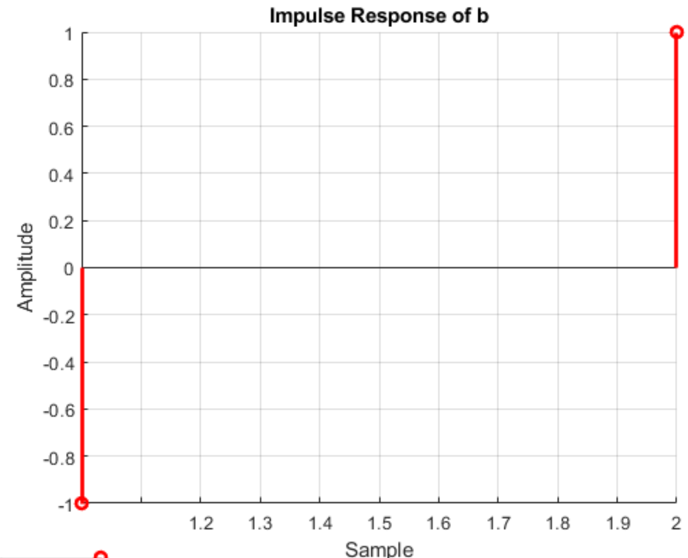
n	0	1	2	
a[n]	0.25	0.5	0.25	
	b[N-1-n]	1	-1	
y[n]	-0.25	-0.25	0.25	

n	0	1	2	
a[n]	0.25	0.5	0.25	
		b[N-1-n]	1	-1
y[n]	-0.25	-0.25	0.25	0.25

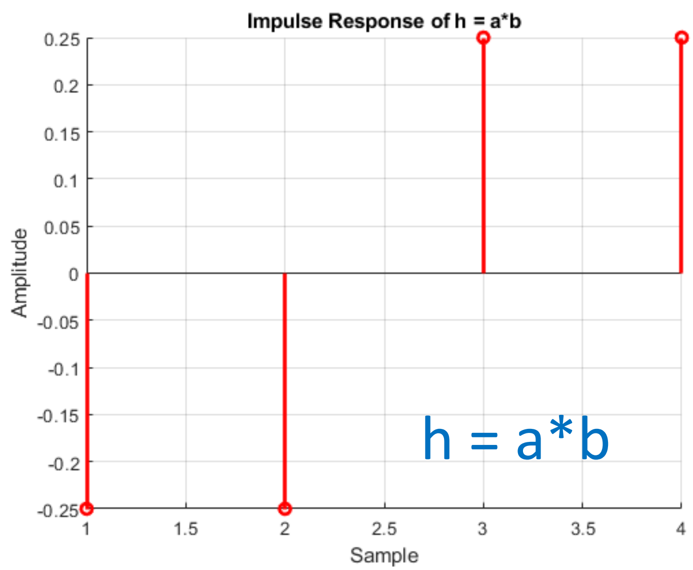
Cascaded Impulse Response



a



b



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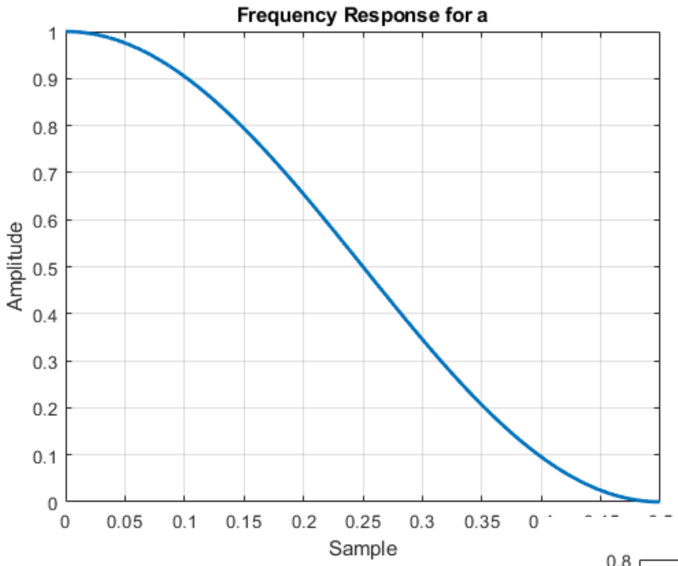
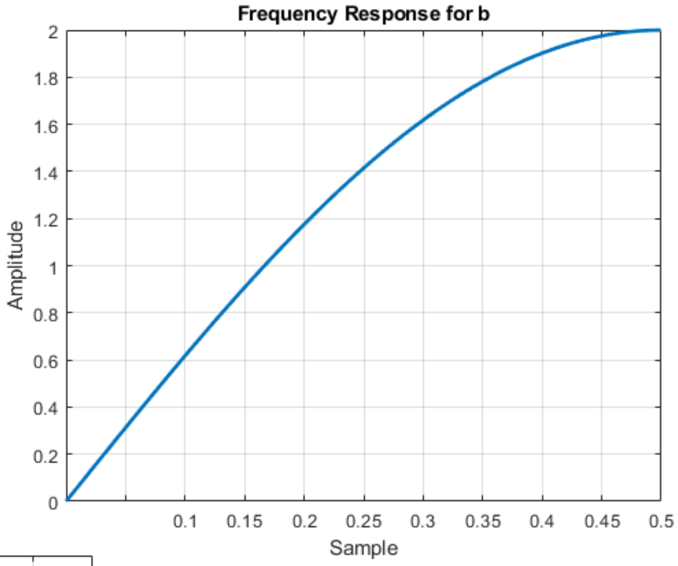
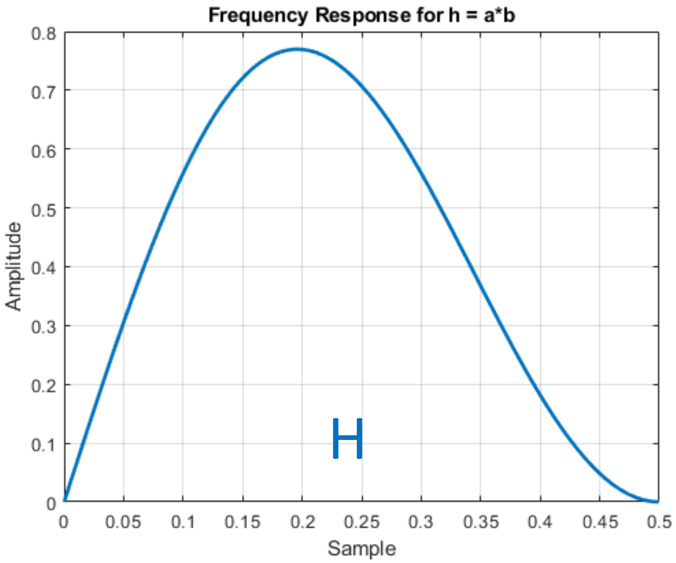
Frequency Responses A, B and H



A



B



H

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Convolving with the input sequence

- Impulse decompose the input sequence

n	0	1	2	3	4
a[n]	1	0	1	0	1

n	0	1	2	3	4	5	6	7
h[n]	-0.25	-0.25	0.25	0.25				

n	0	1	2	3	4
a1[n]	1	0	0	0	0
a2[n]	0	0	0	0	0
a3[n]	0	0	1	0	0
a4[n]	0	0	0	0	1
a5[n]	0	0	0	0	1

n	0	1	2	3	4	5	6	7
y1[n]	-0.25	-0.25	0.25	0.25	0	0	0	0
y2[n]	0	0	0	0	0	0	0	0
y3[n]	0	0	-0.25	-0.25	0.25	0.25	0	0
y4[n]	0	0	0	0	0	0	0	0
y5[n]	0	0	0	0	-0.25	-0.25	0.25	0.25
SUM y[n]	-0.25	-0.25	0	0	0	0	0.25	0.25

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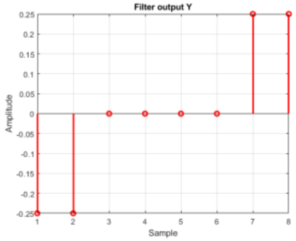
Cascaded Filter Output

- Plot the output samples of the cascaded filter with input x[n]



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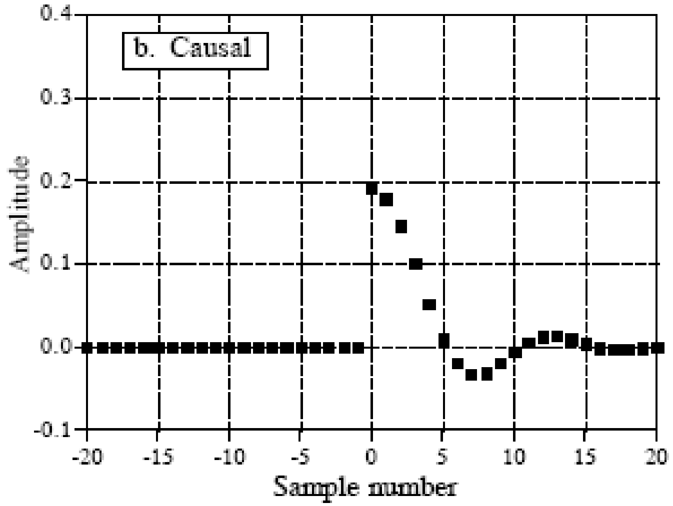
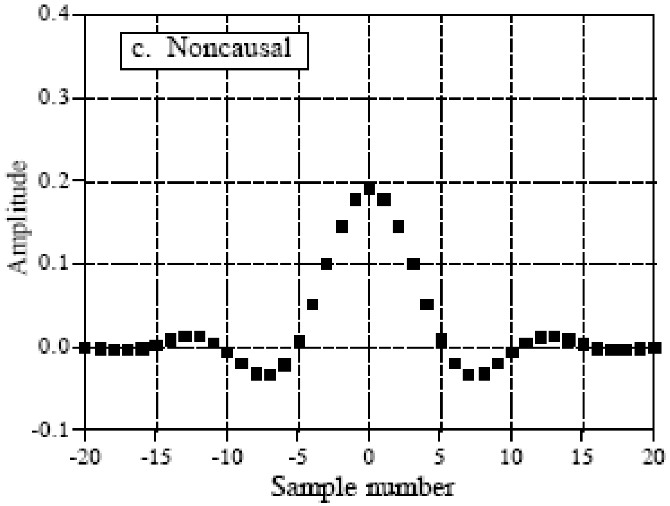
Causality

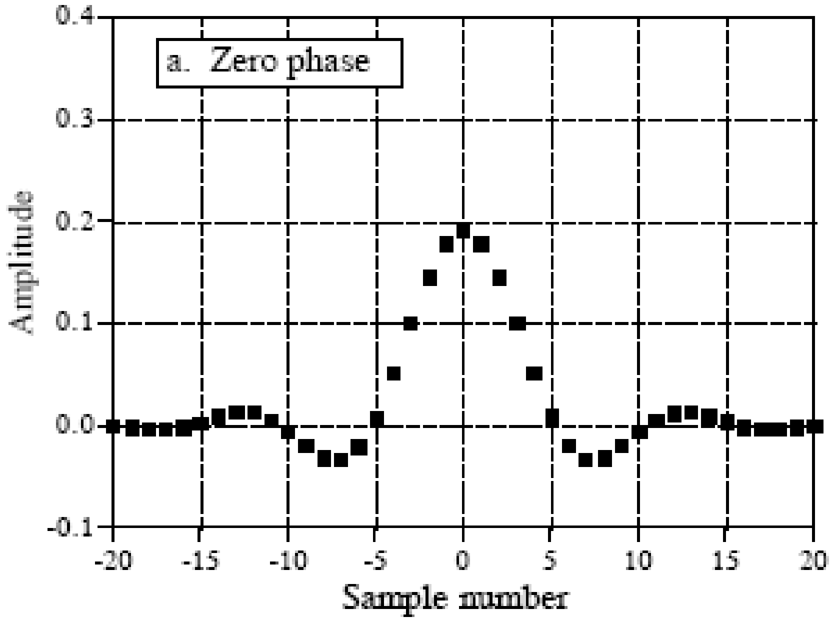
- Any effect must happen after the cause
- Causal : Impulse response has no negative sample numbers
- Non-Causal : Impulse response uses negative and positive sample numbers





Phase Responses

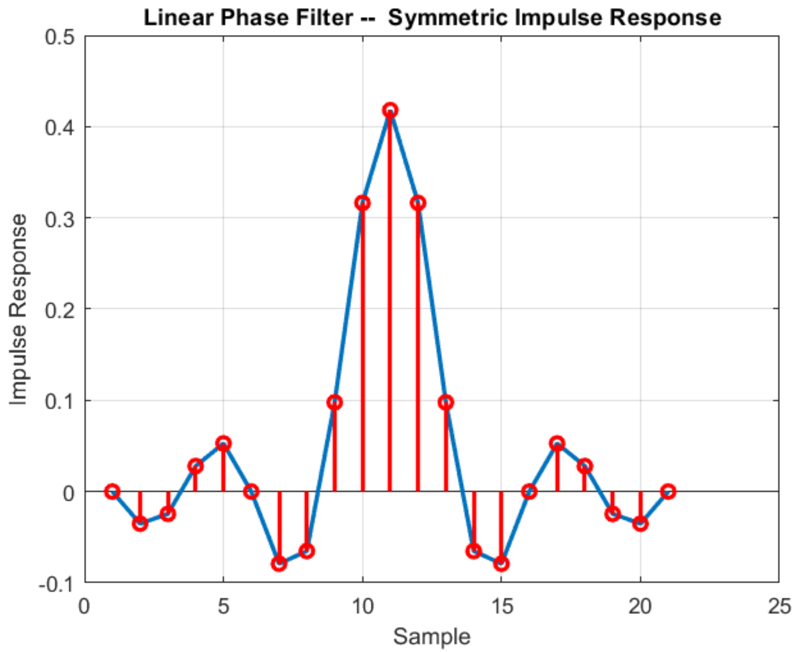
- If the impulse response is symmetric about sample zero then there is zero phase shift of the output. The phase of the frequency response will be zero over all frequencies



Symmetric around zero (but non-causal)

Phase Responses

- If the impulse response is symmetric about sample point (not zero) then there is a phase shift and the phase shift is linear with frequency

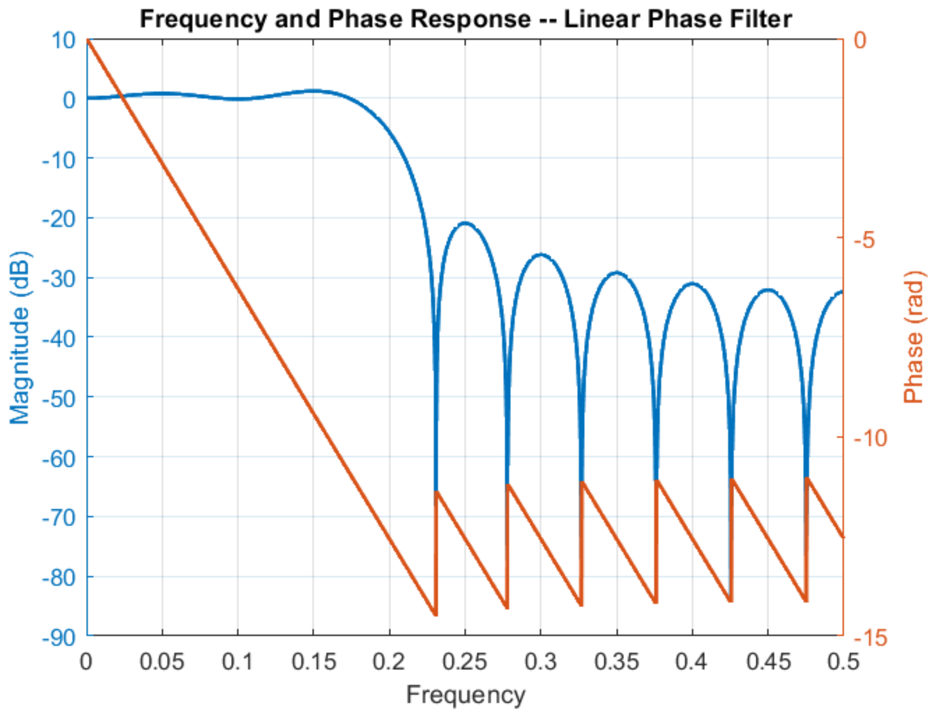


Symmetric, but not around zero (but causal)

This is important in data systems to avoid intersymbol interference

Phase Responses
Symmetric Impulse Response

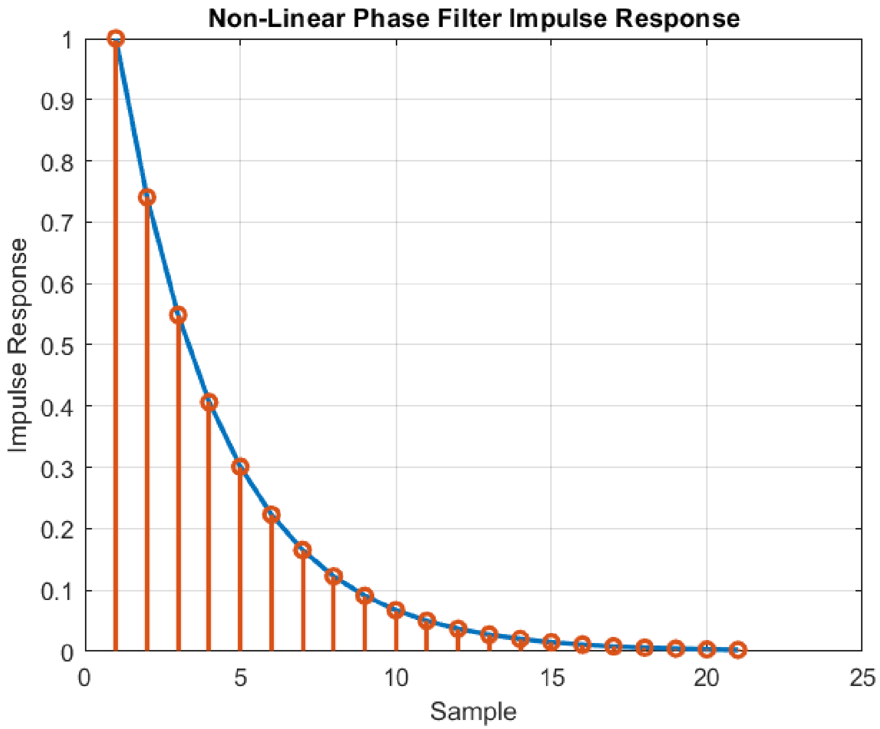
- If the impulse response is symmetric about sample point (not zero) then there is a phase shift and the phase shift is linear with frequency



The phase changes linearly with frequency Sawtooth caused by phase “wrapping over 2π

Phase Responses

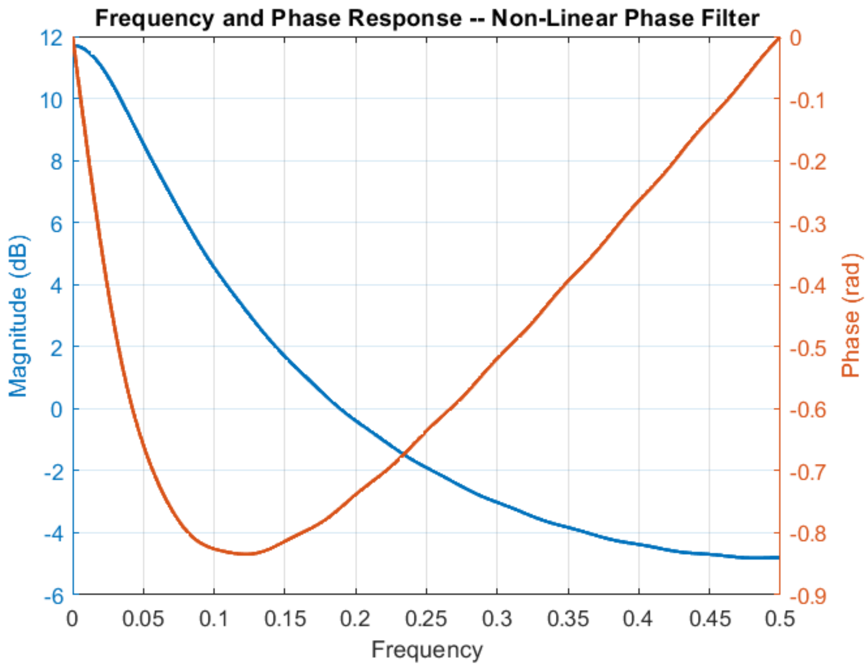
- If the impulse response does not have symmetry there is a phase shift and it is non-linear with frequency



Not symmetric impulse response

Phase Responses

- If the impulse response does not have symmetry there is a phase shift and it is non-linear with frequency



Phase does <u>not</u> change linearly with frequency

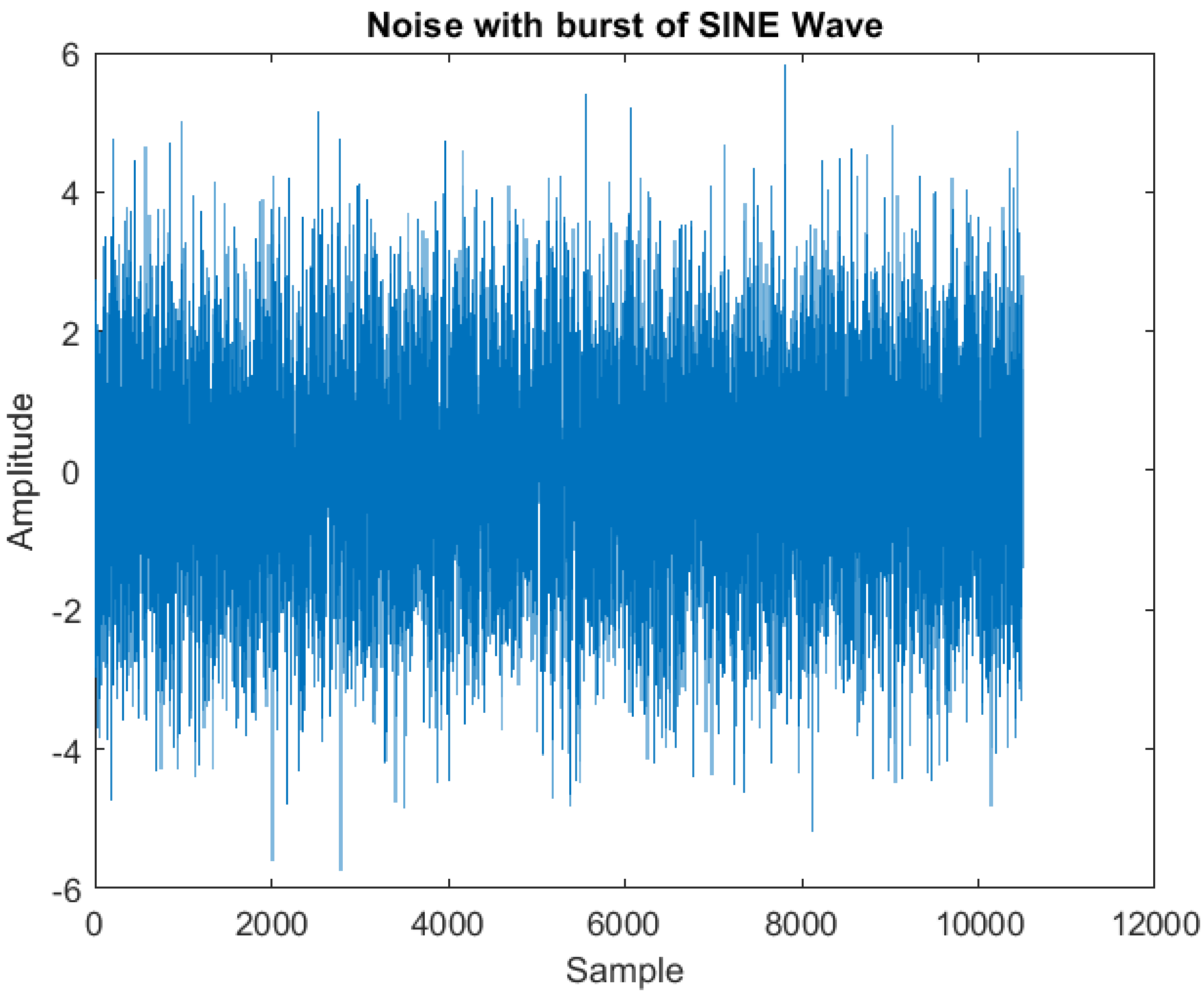
Correlation

- Correlation is closely related to convolution
- Same operation as convolution but the impulse response <u>is not flipped in time</u>
- Can be used to find a known signal in a noisy environment

Correlation Demo

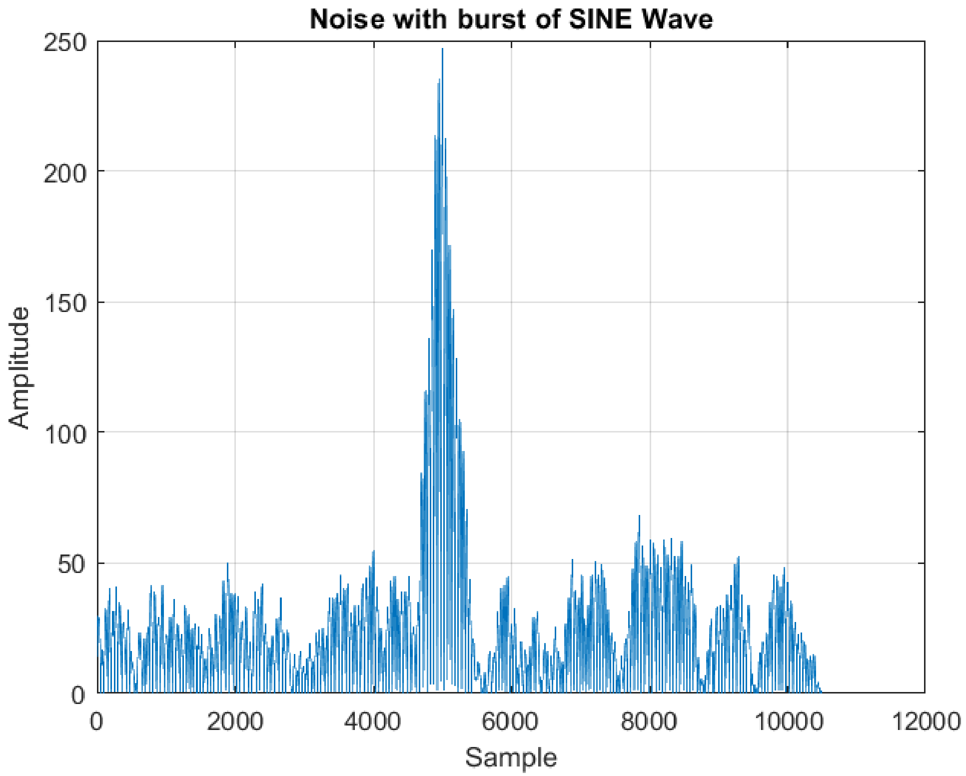
- I have a short burst of a sine wave buried in noise
- I know that my signal is a sinewave, so I correlate my noisy signal with the known signal
- The peak of the correlation will be at the location of the sine wave burst

Noisy Signal with Sine Wave Burst



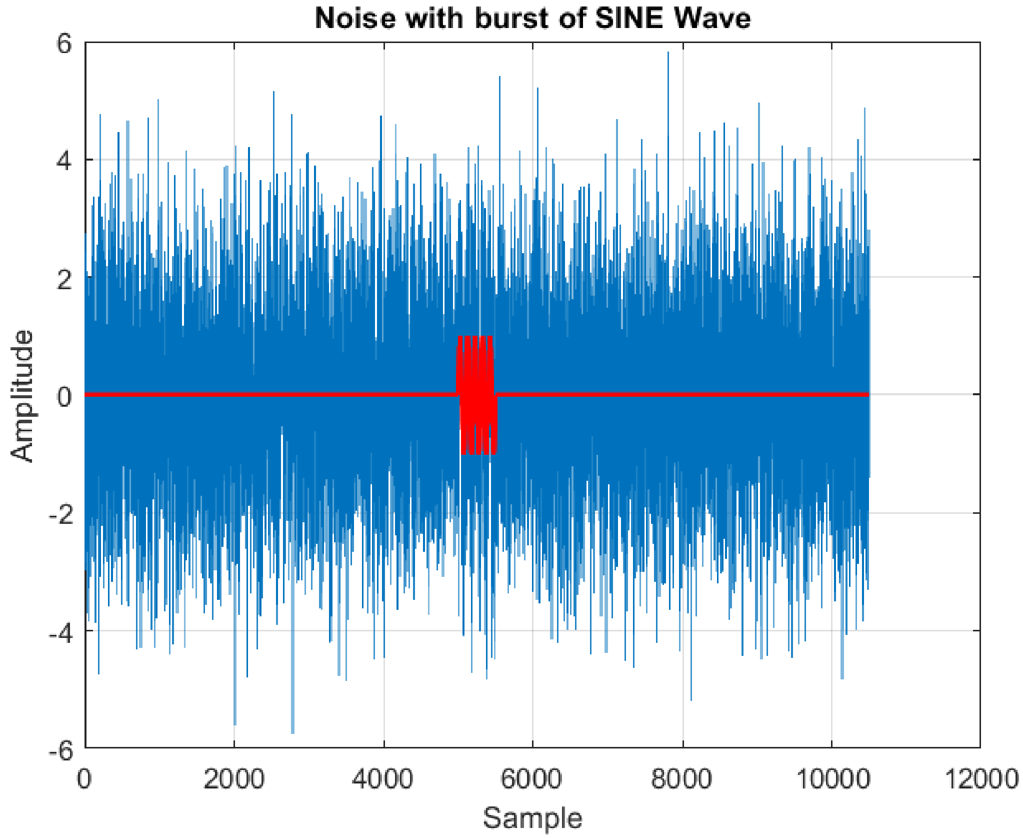
Correlation Output

- Peak of the correlation output is the location of the burst



Burst Location in Time

- Actual burst location within noise signal



Topic Summary

- Properties of Convolution
  - Commutative, Associative, Distributive and Cascaded
- Phase Relationships
  - Phase linearity is associated with impulse response symmetry
- Correlation
  - Similar to convolution. Can be used to find signals in noise.