Exam 02 Review
In Class Problems

P1 - Convolution

- Convolve the following two signals, x[n] and y[n]. Use any algorithm with which you are comfortable (e.g impulse decomposition, input side, output side algorithm).

$x[n] = 1, 3, -2, 1, 0, 2$

$h[n] = 1, -1, 2, 2, 1$

- If the input is multiplied by 3 what with the output of the convolution be?
- An input sequence has a length of 1200 samples. An impulse response has 81 samples. The input sequence is convolved with the impulse response. How many samples are in the output? How many samples at the beginning of the output are considered “end effects”.

Convolution with Impulse Decomposition

NO PRINT

Input	Time Shifted Impulses									
1	1	-1	2	2	1					
3		1	-1	2	2	1				
-2			1	-1	2	2	1			
1				1	-1	2	2	1		
0					1	-1	2	2	1	
2						1	-1	2	2	1

Input	Time Shifted and Scaled Impulses									
1	1	-1	2	2	1	0	0	0	0	0
3	0	3	-3	6	6	3	0	0	0	0
-2	0	0	-2	2	-4	-4	-2	0	0	0
1	0	0	0	1	-1	2	2	1	0	0
0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	2	-2	4	4	2
	1	2	-3	11	2	3	-2	5	4	2

Convolution with Impulse Decomposition

NO PRINT

- If the input is multiplied by 3, what will the output of the convolution be?

Input	Time Shifted and Scaled Impulses									
1	1	-1	2	2	1	0	0	0	0	0
3	0	3	-3	6	6	3	0	0	0	0
-2	0	0	-2	2	-4	-4	-2	0	0	0
1	0	0	0	1	-1	2	2	1	0	0
0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	2	-2	4	4	2
	1	2	-3	11	2	3	-2	5	4	2

The output will be multiplied by 3 because convolution is linear

3	6	-9	33	6	9	-6	15	12	6

P1 - Convolution

NO PRINT

- An input sequence has a length of 1200 samples. An impulse response has 81 samples. The input sequence is convolved with the impulse response. How many samples are in the output?
  - The output will have 1200 + 81 -1 = 1280 samples
- How many samples at the beginning of the output are considered “end effects”.
  - The end effects will be the shorter of the two sequences minus 1. ie. 81-1 = 80. They will be on both ends of the output sequence

P2 - Properties of the DFT

- If an input sequence x[n] consists of 120 samples and the sample rate is 120 kHz, what is the frequency resolution of the DFT?
- How can I improve the resolution of the DFT to better than 40 Hz.
- If the input values to the DFT are multiplied by 2 what happens to the output of the DFT?

P2 - Properties of the DFT NO PRINT

- If an input sequence x[n] consists of 120 samples and the sample rate is 120 kHz, what is the frequency resolution of the DFT?

The frequency resolution is determined by the number of samples and the sample rate.

$$f = \frac{f_s}{N} = \frac{120 \times 10^3}{120} = 1000\ Hz$$

P2 - Properties of the DFT NO PRINT

- How can I improve the resolution of the DFT to better than 40 Hz?

The frequency resolution can be improved by either taking more samples or by padding the sequence with zeros. The number of samples is determined from the sample rate and the desired resolution.

$$N = \frac{f_s}{f_{res}} = \frac{120kHz}{40} = 3000$$

The FFT requires that there be a power of two number of samples in the sequence to the next largest power of two is 2^12 = 4096 samples. The final frequency resolution is then

$$f_{res} = \frac{120\ kHz}{4096} = 29.3\ Hz$$

P2 - Properties of the DFT NO PRINT

- One way to find the exponent or power of 2 in MATLAB is to use the log to the base 2 and use the next highest integer value

$$N = 3000$$

Find the next closest integer power of 2 to get <u>at least</u> 3000.
Use log base 2 and then the ceiling (ceil) function

$$2^e \geq 3000 \qquad e = CEIL[\log_2 N]$$

```
N = 3000;

%  Find the power of 2 to give a value greater than or equal to N

e = ceil( log2( N ))
```

```
e = 12
```

```
2^e
```

```
ans = 4096
```

P2 - Properties of the DFT NO PRINT

- If the input values to the DFT are multiplied by 2 what happens to the output of the DFT?

The output values are also multiplied by 2 because the DFT is linear and exhibits homogeneity

P3 - Single Pole IIR Filter

- A single pole IIR lowpass filter has a corner frequency of 1.5 kHz when using a sample rate of 100 kHz.

- What is the value of X, the amount of decay between samples?
- What are the recursion coefficients a0 and b1 in the difference equation?
- What is the time constant of the filter?
- For this filter what is the approximate output value for a step decay starting at 1 volt and decaying to 0 volts after one time constant?

P3 - Single Pole IIR Filter NO PRINT

- A single pole IIR lowpass filter has a corner frequency of 1.5 kHz when using a sample rate of 100 kHz.

- What is the value of X, the amount of decay between samples?

Find the corner frequency relative to the sample rate

$$f_c = \frac{f_{c(Hz)}}{f_s}$$

$$f_c = \frac{1.5\ kHz}{100\ kHz} = 0.015$$

$$X = e^{-(2\pi f_c)} = e^{-(2\pi 0.015)} = 0.9101$$

P3 - Single Pole IIR Filter

NO PRINT

- A single pole IIR lowpass filter has a corner frequency of 1.5 kHz when using a sample rate of 100 kHz.

- What are the recursion coefficients a0 and b1 in the difference equation?

y[n] = a0 * x[n] + b1 * y[n-1]

The recursion coefficients are found using the value of X the amount of decay between samples.

$$X = e^{-(2\pi f_c)} = e^{-(2\pi 0.015)} = 0.9101$$

$$b_1 = X = 0.9101 \qquad a_0 = 1 - X = 0.0899$$

P3 - Single Pole IIR Filter

NO PRINT

- A single pole IIR lowpass filter has a corner frequency of 1.5 kHz when using a sample rate of 100 kHz.
- What is the time constant of the filter?

We can express the decay between samples in two ways

$$X = e^{-\frac{1}{d}} \; and \; X = e^{-2\pi f_c}$$

Equating terms and taking the natural log of both sides

$$-\frac{1}{d} = -2\pi f_c \qquad d = \frac{1}{2\pi f_c} = \frac{1}{2\pi \times 0.015} = 10.6 \; samples$$

P3 - Single Pole IIR Filter NO PRINT

- A single pole IIR lowpass filter has a corner frequency of 1.5 kHz when using a sample rate of 100 kHz.
- For this filter what is the approximate output value for a step decay starting at 1 volt and decaying to 0 volts after one time constant?

After one time constant (10.36 samples) the output will have decayed to 36.8% of its initial value for an output of 0.368.

P5 - MATLAB Exercise

- The input file Exam02_Practice_Problem8.mat has an impulse response of a filter in the second column of the variable "outData" that is brought into the workspace when the file is loaded.
- Plot the impulse response of the filter
- What is the length of the impulse response?
- If the impulse response is sampled at 8000 Hz what length of FFT is required to have at least 40 Hz of frequency resolution? Recall that the FFT should padded to a number of samples that is a power of 2.
- Plot the frequency response of the filter at this resolution. Be sure to create a vector that represents the frequency in Hz
- What kind of filter is implemented using this frequency response (i.e. LPF, HPF, BPF, BSF)

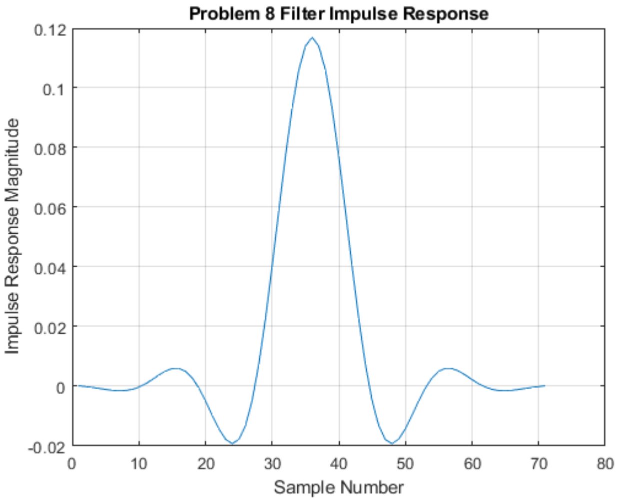
P5 - MATLAB Exercise

NO PRINT

- The input file Exam02_Practice_Problem8.mat has an impulse response of a filter in the second column of the variable “outData” that is brought into the workspace when the file is loaded.

- Plot the impulse response of the filter

- What is the length of the impulse response?



The length of the impulse response is 71 samples

P5 - MATLAB Exercise

NO PRINT

- If the impulse response is sampled at 8000 Hz what length of FFT is required to have at least 40 Hz of frequency resolution? Recall that the FFT should padded to a number of samples that is a power of 2.

Find the number of samples required for a frequency resolution of 40 Hz.

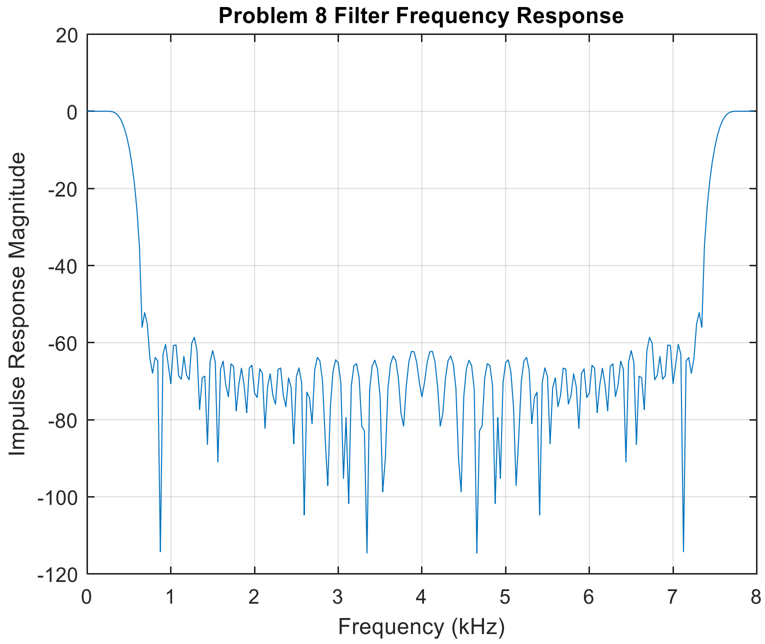
$$N = \frac{8kHz}{40Hz} = 200$$

The next power of two that would give at least this resolution is 256.

P5 - MATLAB Exercise

NO PRINT

- Plot the frequency response of the filter at this resolution. Be sure to create a vector that represents the frequency in Hz
- What kind of filter is implemented using this frequency response (i.e. LPF, HPF, BPF, BSF)



The filter is a low pass filter

P6 – MATLAB Exercise

- Loading the file “Exam02_Practice_Problem10.mat” into MATLAB. This will bring two variables into the workspace, “inputSignal” and “impResponse”. Each variable has two columns, the first is the sample number. The second column is the inputSignal and impResponse data respectively. The signal is sampled at a rate of 44.1 kHz a common audio sampling rate.
- Plot the input signal.
- What is the length of the input signal? Find the length of the FFT needed to plot the frequency spectrum of the signal at a resolution of at least 50 Hz. Plot the magnitude of the frequency spectrum of the signal using the FFT.
- Plot the impulse response of the filter
- What is the length of the input signal? Find the length of the FFT needed to plot the frequency response of the filter at a resolution of at least 50 Hz. Plot the magnitude of the frequency response of the signal using the FFT.

P6 – MATLAB Exercise

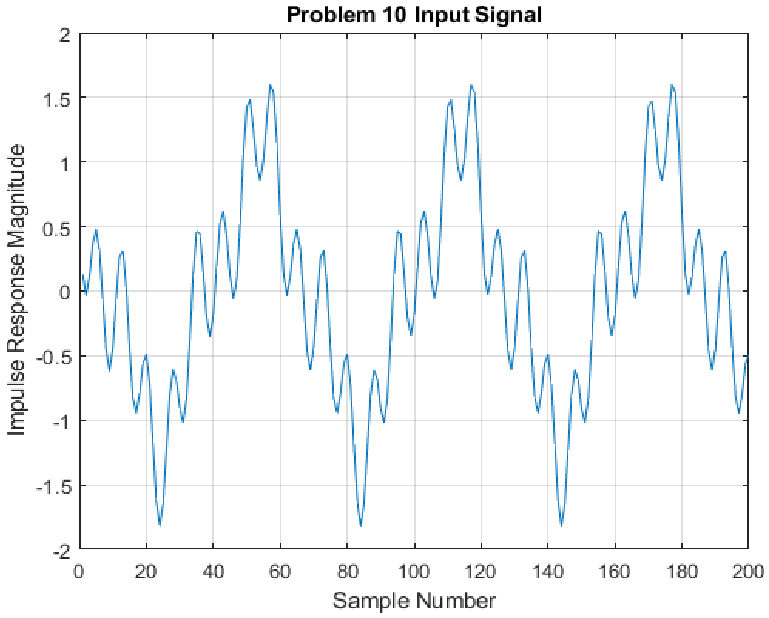
- If the two signals are convolved in the time domain what would be the length of the resulting convolution?

- Convolve the two signals in the frequency domain. Don’t forget to pad each signal to the correct length so that circular convolution is avoided. Plot the resulting convolution.

P6 – MATLAB Exercise

NO PRINT

- Loading the file "Exam02_Practice_Problem10.mat" into MATLAB. This will bring two variables into the workspace, "inputSignal" and "impResponse". Each variable has two columns, the first is the sample number. The second column is the inputSignal and impResponse data respectively. The signal is sampled at a rate of 44.1 kHz a common audio sampling rate.
- Plot the input signal.



P6 – MATLAB Exercise

NO PRINT

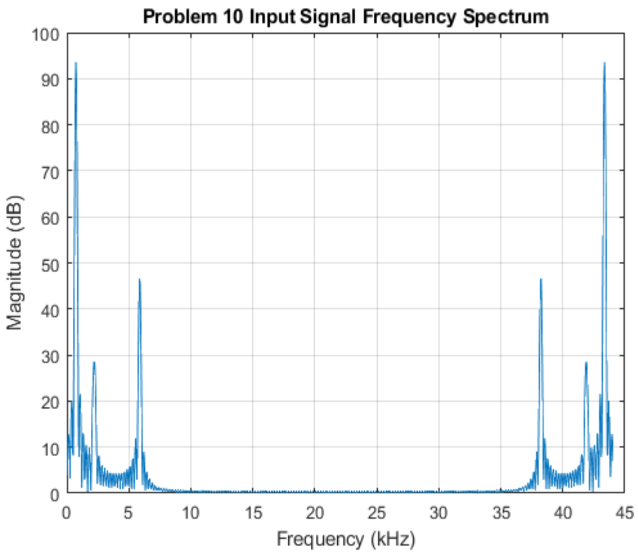
- What is the length of the input signal? Find the length of the FFT needed to plot the frequency spectrum of the signal at a resolution of at least 50 Hz. Plot the magnitude of the frequency spectrum of the signal using the FFT.
- Plot the impulse response of the filter

The length of the input signal is 200 samples

Find the number of samples required for a frequency resolution of 50 Hz.

$$N = \frac{44.1kHz}{50Hz} = 882$$

The next power of two that would give at least this resolution is 1024.



P6 – MATLAB Exercise

NO PRINT

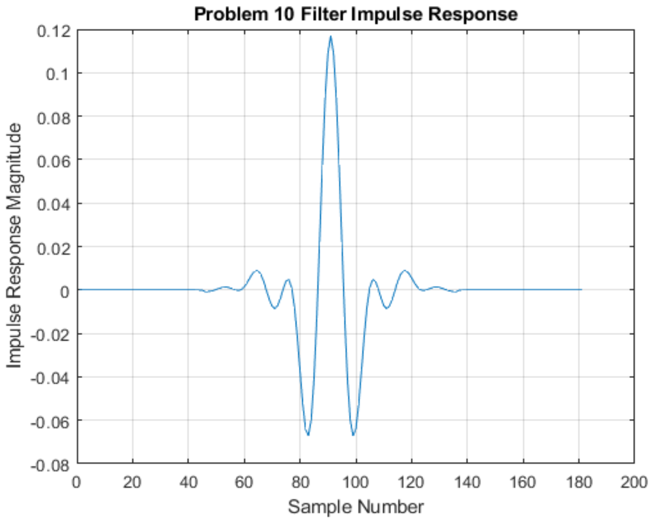
- Plot the impulse response of the filter
- What is the length of the input signal? Find the length of the FFT needed to plot the frequency response of the filter at a resolution of at least 50 Hz. Plot the magnitude of the frequency response of the signal using the FFT.

The length of the impulse response is 181 samples

Find the number of samples required for a frequency resolution of 50 Hz.

$$N = \frac{44.1kHz}{50Hz} = 882$$

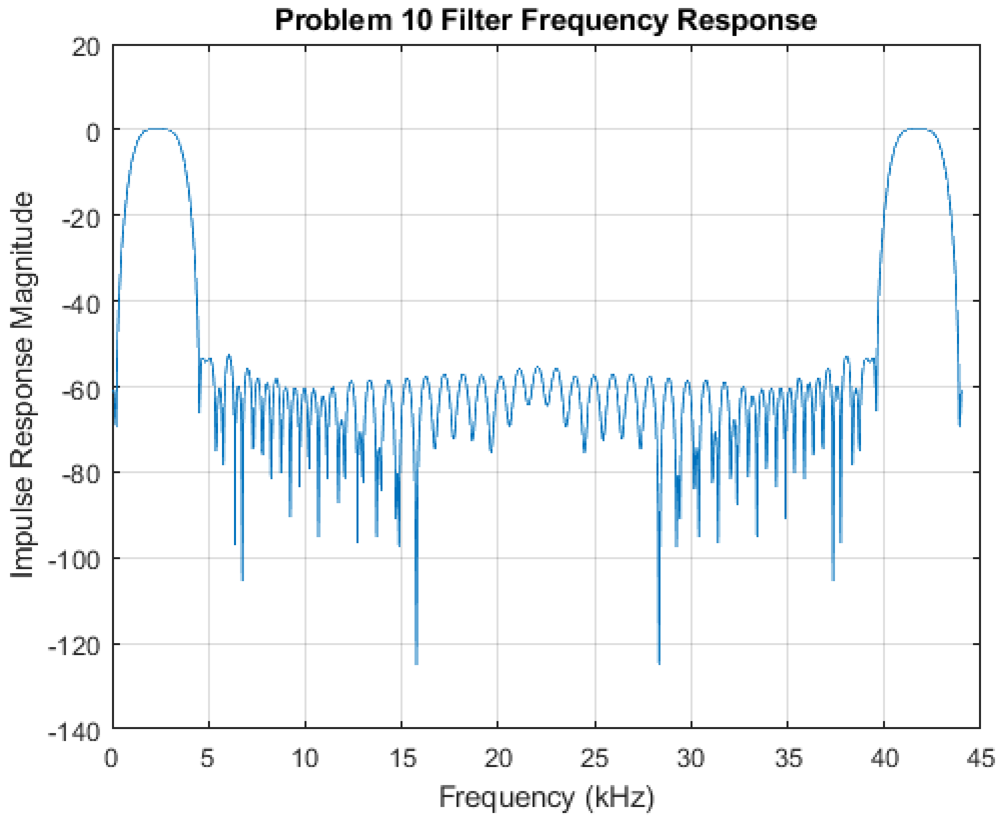
The next power of two that would give at least this resolution is 1024.



P6 – MATLAB Exercise

NO PRINT

- Plot the frequency response



P6 – MATLAB Exercise

NO PRINT

- If the two signals are convolved in the time domain what would be the length of the resulting convolution?

- Convolve the two signals in the frequency domain. Don't forget to pad each signal to the correct length so that circular convolution is avoided. Plot the resulting convolution.

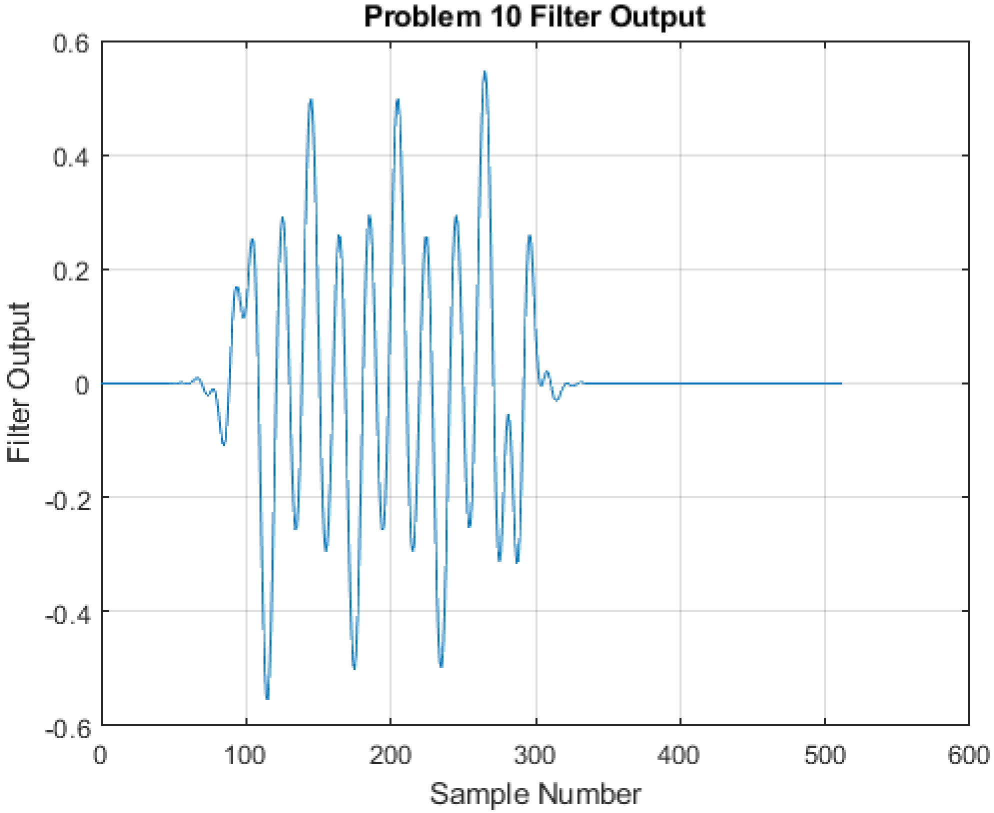
The length of the input signal is 200 samples, the length of the impulse response is 181 samples. Then the length of the convolution of the two signals in the time domain would be

$$M + N - 1 = 200 + 181 - 1 = 381\ samples$$

The length of the convolution in the time domain is 381. The next power of two greater than this is 512. Find the FFT of the input signal and the impulse response at a length of 512 samples. Then multiply them point by point and take the inverse FFT. Plot the result of the convolution.

P6 – MATLAB Exercise

NO PRINT



P7 - FIR Filter Design

- Using the FIR_Designer tool design a lowpass filter to pass frequencies of 15 BPM and below with little attenuation, but attenuating frequencies above 25 BPM

- Plot the impulse response of the filter

- Plot the frequency response of the filter. Insure that the gain of the filter at DC is 1.0

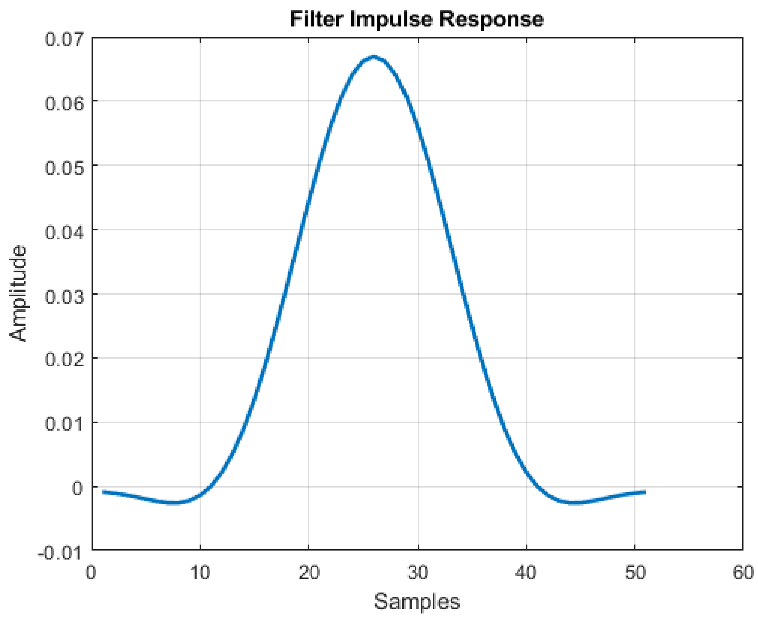
P7 - FIR Filter Design

- Using the FIR_Designer tool design a lowpass filter to pass frequencies of 15 BPM and below with little attenuation, but attenuating frequencies above 25 BPM

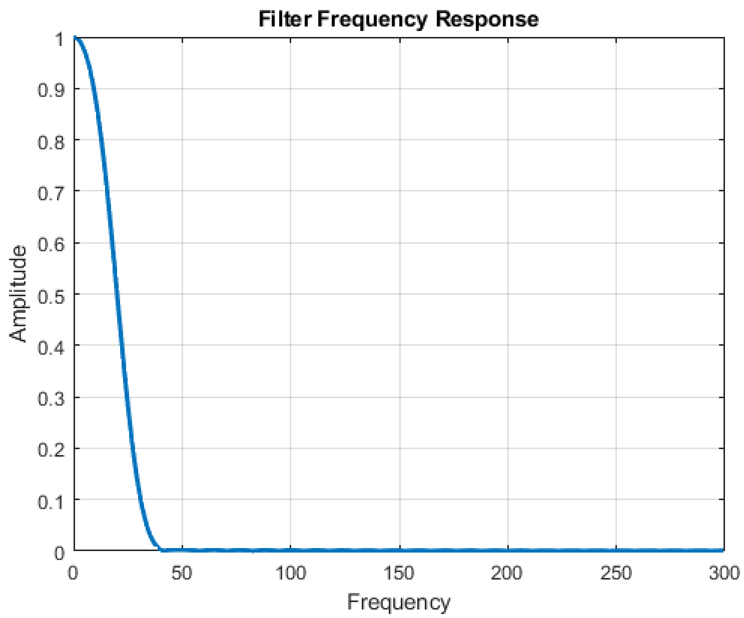
P7 - FIR Filter Design

```
clearvars¶
¶
%··Set·up·the·FIR·Designer·tool·to·create·an·LPF.··Adjust·the·order·until·the·
attenuation·is·achieved¶
%··Set·the·corner·frequency·above·the·frequencies·to·pass·and·below·the¶
%··frequencies·to·attenuate¶
¶
fCorner·=·20;··%·BPM¶
filterOrder·=·51;¶
¶
%··Let·the·tool·print·the·responses,·but·suppress·the·header·and·the·MATLAB¶
%··output·normally·sent·to·the·command·window¶
¶
h·=·FIR_Designer('nOrder',·filterOrder,·'cutBPM',·fCorner,·'PrintHeader',·
false,·'PlotResponses',true,'FxdPoint',true,·'PrintMATLAB',·false·);¶
```

P7 - Fc = 20, N = 51



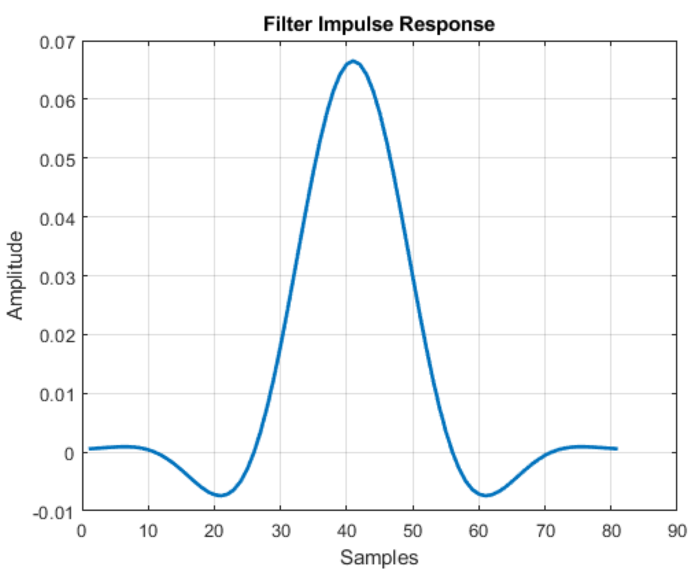
Filter Impulse Response
Amplitude
Samples
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
10
20
30
40
50
60

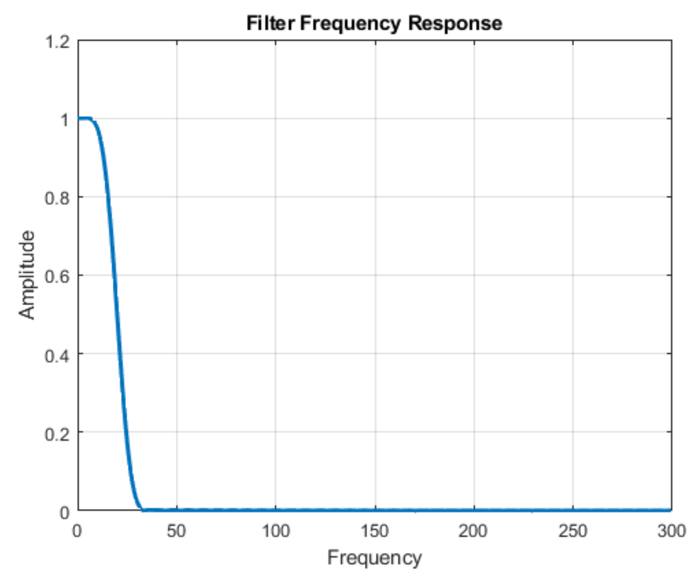


Filter Frequency Response
Amplitude
Frequency
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
300

P7 - Fc = 20, N = 81

```
¶
¶
%·This·first·filter·didn't·meet·the·requirements·so·increase·the·order.¶
%·Normally·you·could·do·this·just·with·the·same·line·of·code,·but·shown¶
%·here·to·demonstrate·the·process¶
¶
filterOrder·=·81;¶
¶
h·=·FIR_Designer('nOrder',·filterOrder,·'cutBPM',·fCorner,·'PrintHeader',·
false,·'PlotResponses',true,'FxdPoint',true,·'PrintMATLAB',·false·);¶
```





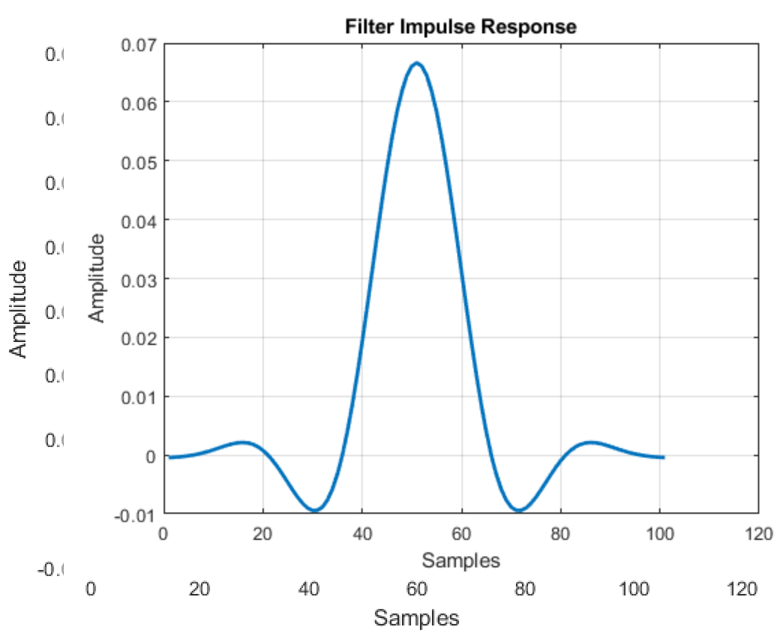
P7 - Fc = 20, N = 101

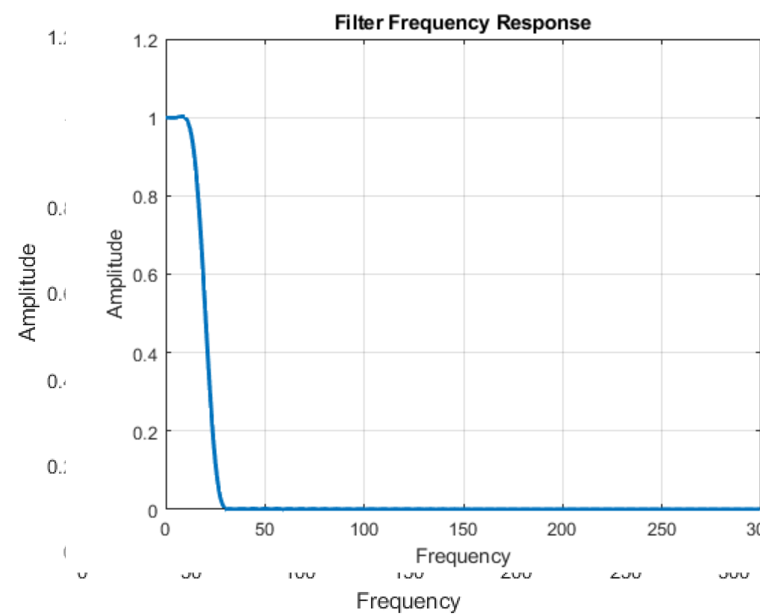
```matlab

% This second filter didn't meet the requirements so increase the order.
% Normally you could do this just with the same line of code, but shown
% here to demonstrate the process

filterOrder = 101;

h = FIR_Designer('nOrder', filterOrder, 'cutBPM', fCorner, 'PrintHeader', 
false, 'PlotResponses',true,'FxdPoint',true, 'PrintMATLAB', false );
```





P7 - If you copy from Command Window

- Filter coefficients are fixed point (though this can be turned off)

- If they are fixed point they are scaled by HFXPT

- You need to normalize by HFXPT to get a DC gain of 1.0