Digital Signal Processing

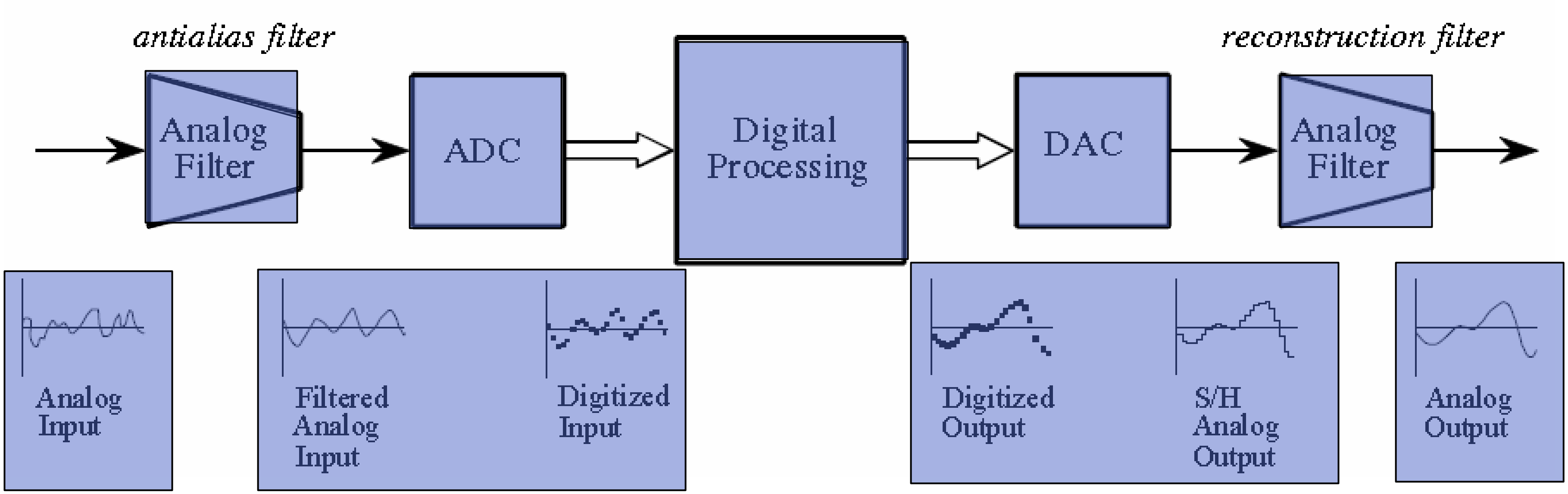
Analog to Digital Conversion

Sampling

Key Points For Today

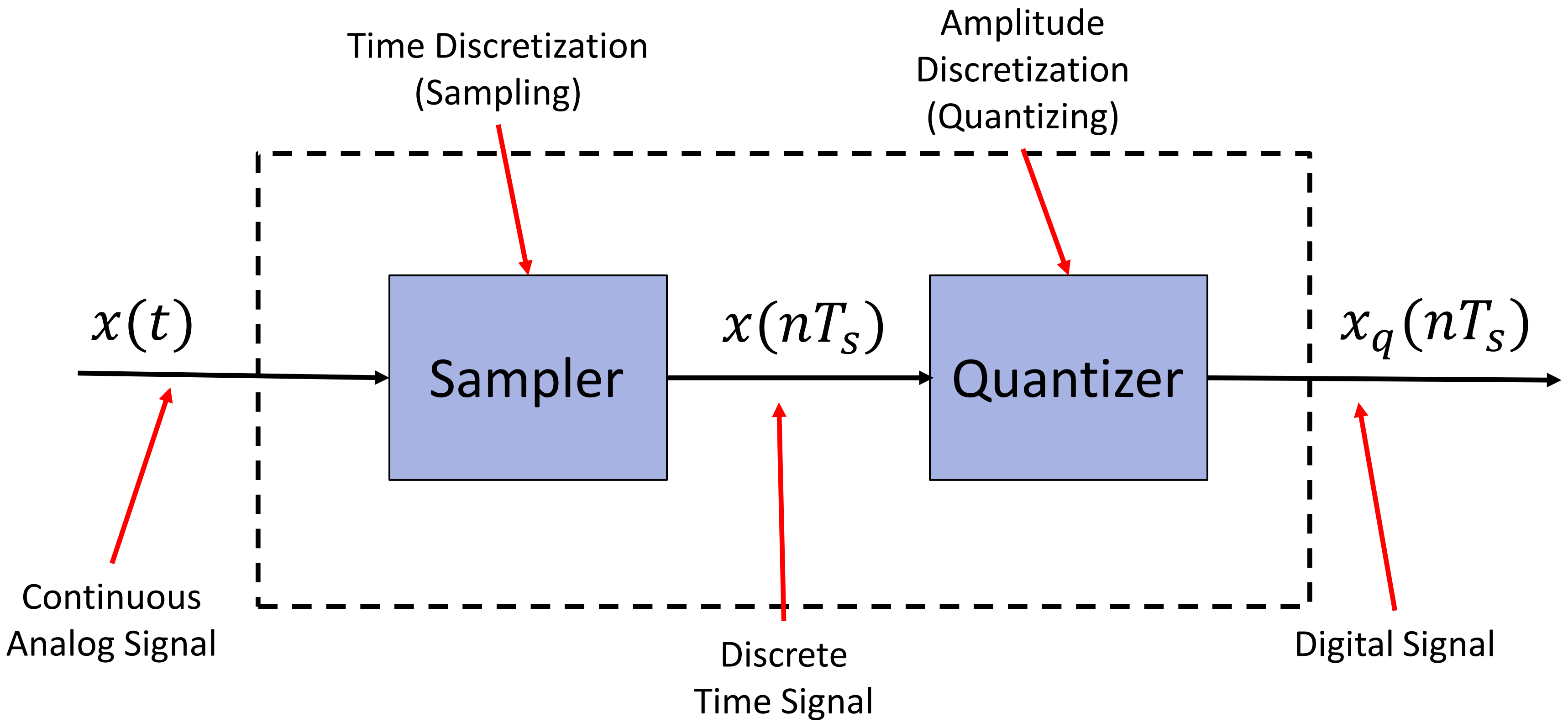
- Analog to digital conversion consists of sampling and quantizing
- Sampling must be done properly to retain the information in the signal
- Sampling at too low of a rate can lose or distort information in the signal (aliasing)

Overview of ADC/DSP/DAC System



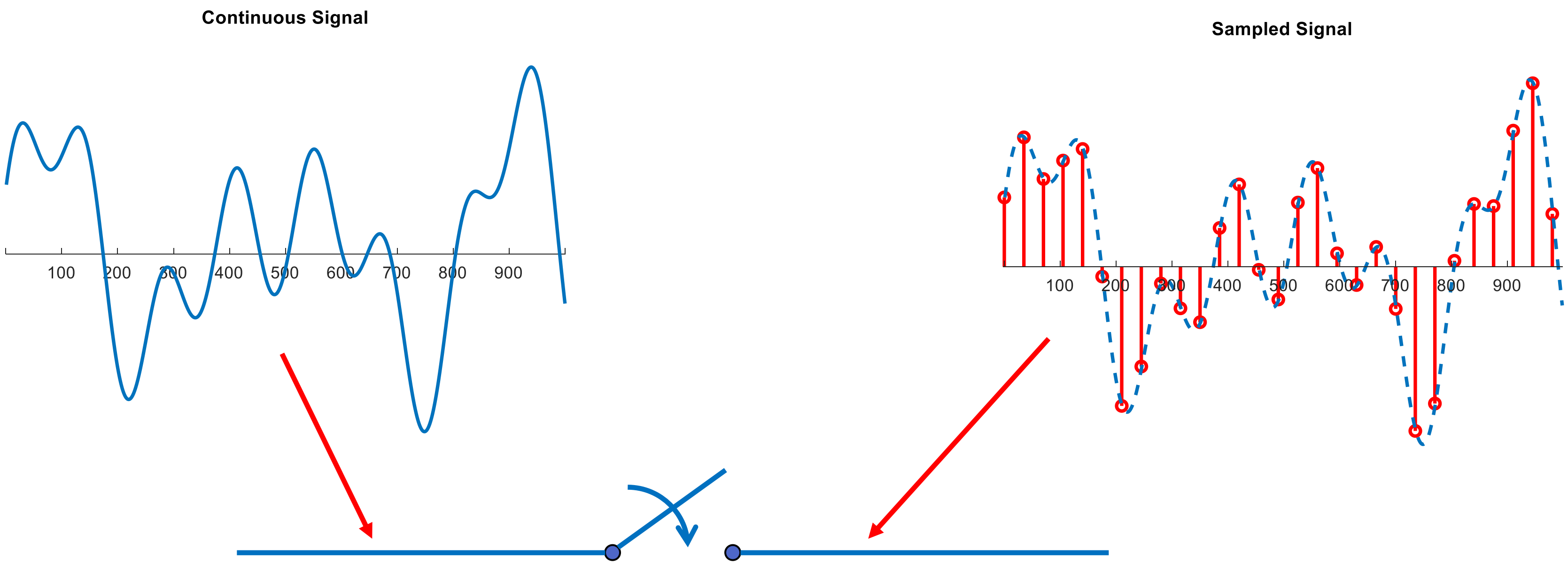
Analog to Digital Conversion

- Conversion from analog to digital requires two forms of discretization, Time and Amplitude



Sampling

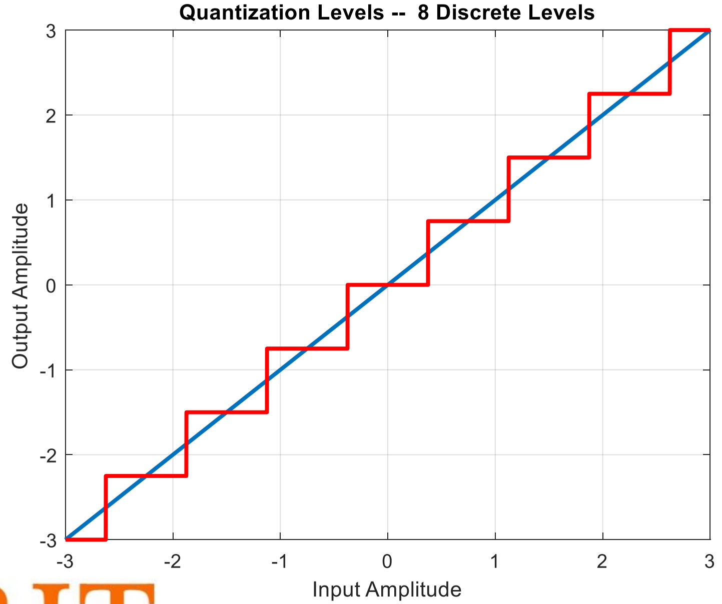
- The continuous analog signal is “viewed” or “sampled” at a periodic time T_s
- T_s is the sampling interval. $F_s = 1/T_s$ is the sample rate

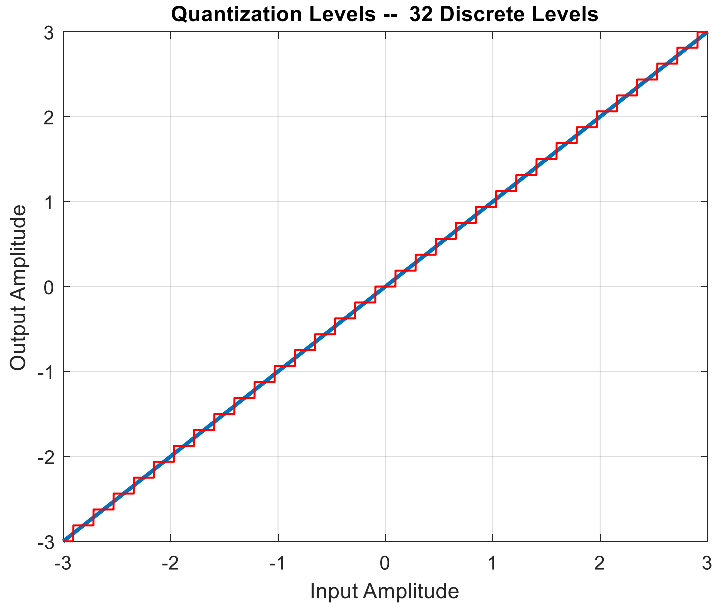


$$F_s = 1/T_s$$

Quantizing

- When represented digitally the amplitude of the signal is limited to discrete levels
- Amplitudes in between the discrete levels are rounded to the nearest discrete level





ADC Conversion Impairments

- If sampling is done properly, then the sampled signal contains all of the original information
  - The continuous signal can be reproduced exactly

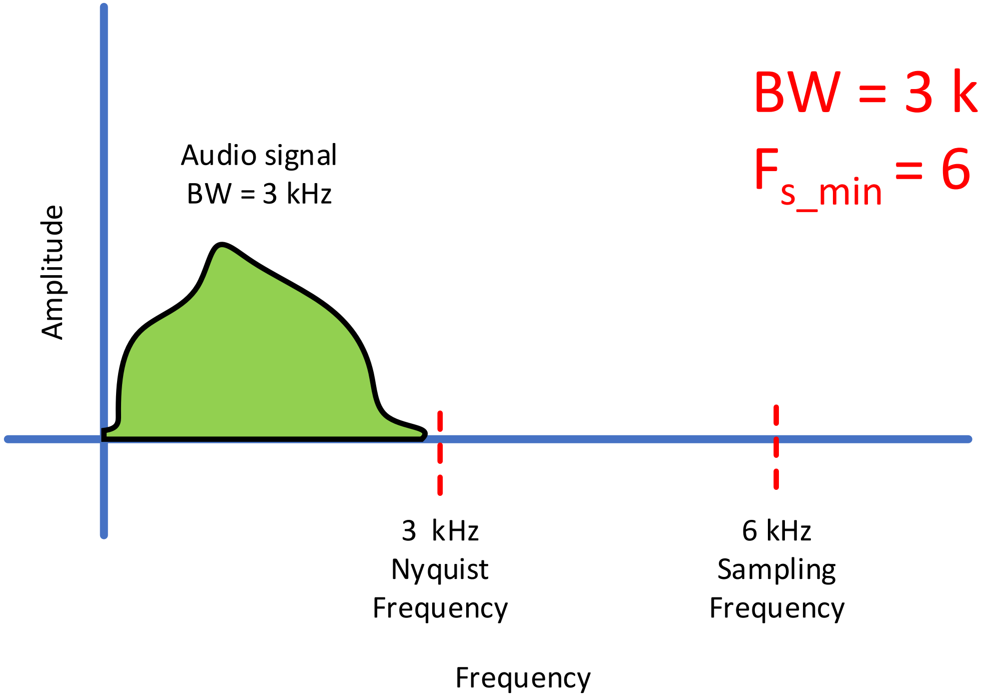
- Quantizing can cause some loss in information.
  - Usually in the form of additive noise

What is Proper Sampling?

- If a signal has a bandwidth B, then in order to retain all of the information it must be sampled at a rate of at least $2 \times B$
- Example – An audio signal has a bandwidth of 3 kHz
- To retain the original information is must be sampled at least 6 kHz. That is 6000 samples per second

What is Proper Sampling?

- Theoretical minimum sampling frequency to sample the original signal without distortion is >= 2X the bandwidth

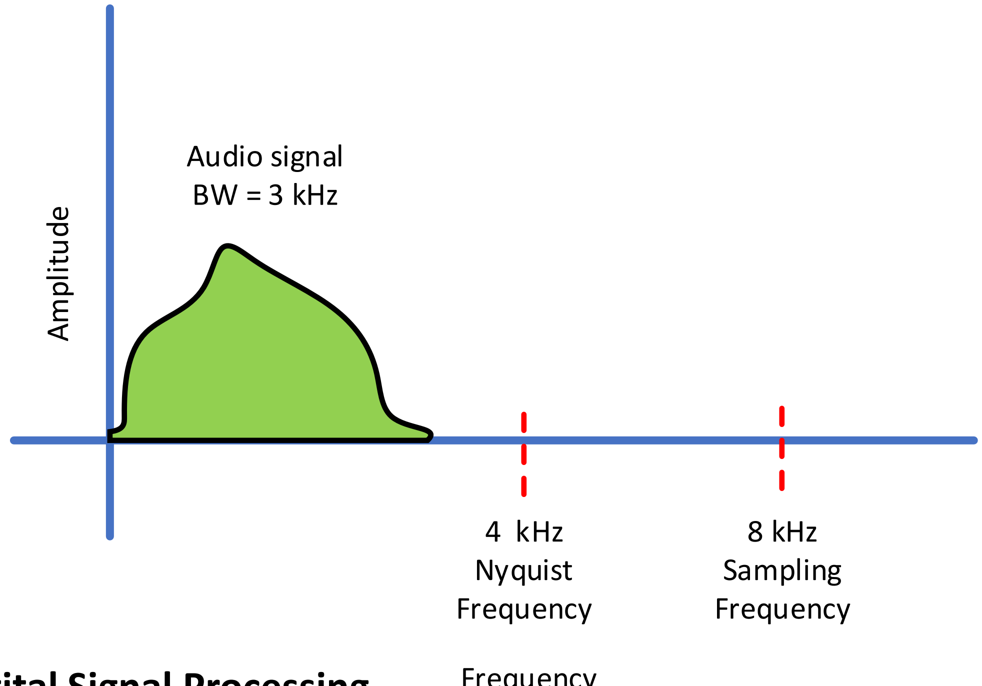


BW = 3 kHz

F_{s_min} = 6 kHz

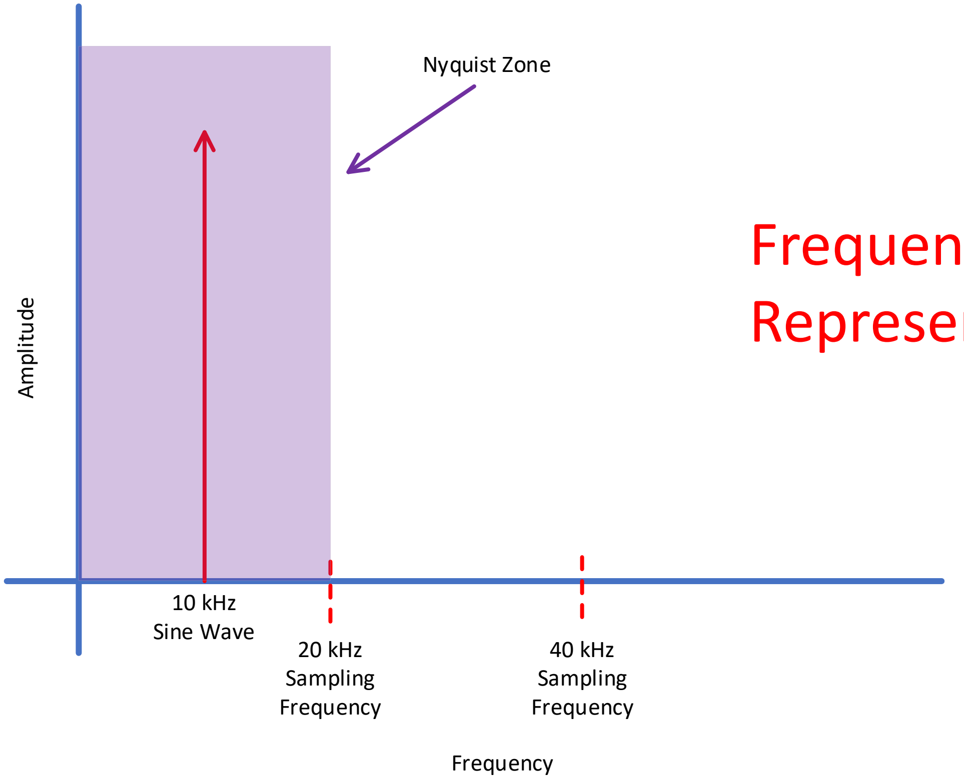
What is Proper Sampling?

- Typically the sample frequency is greater than that required for easier signal processing
  - Digital Filtering of the signal
  - Easier analog filtering to prevent aliasing



Example – 10 kHz Sine Wave

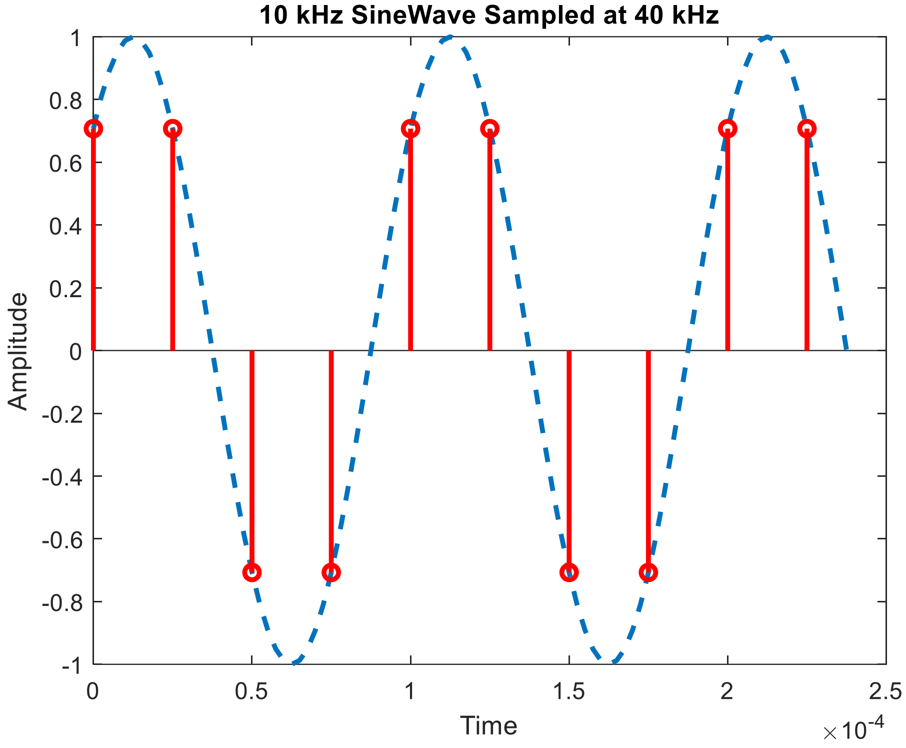
- I have a 10 kHz sine wave. Sample the signal at a rate of 40 kHz



Frequency Domain Representation

Example – 10 kHz Sine Wave

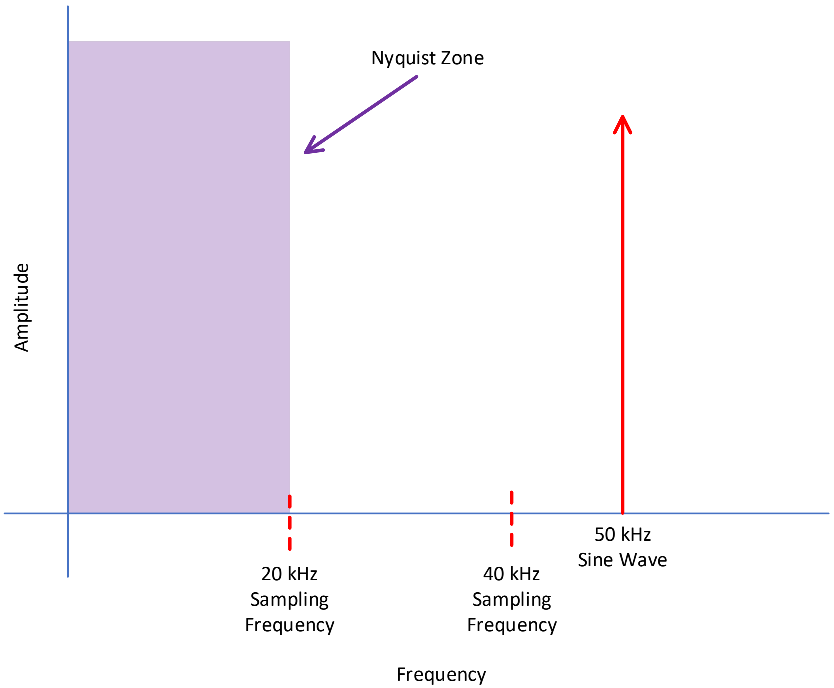
- I have a 10 kHz sine wave. Sample the signal at a rate of 40 kHz



Time Domain Representation

50 kHz Sine Wave
Sample Rate = 40 kHz

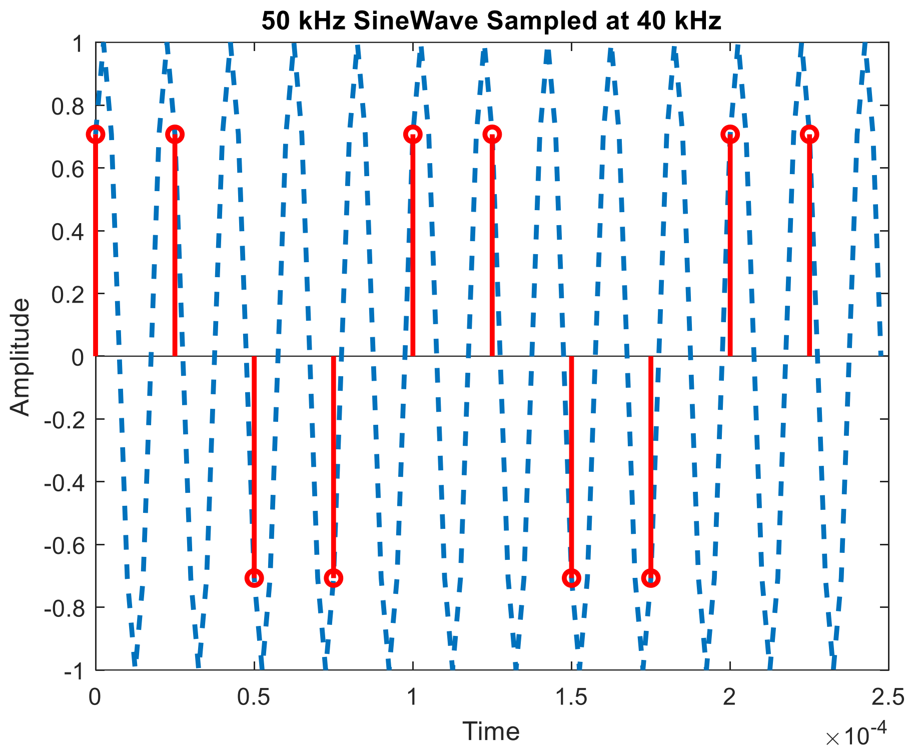
- Here is a 50 kHz Sine Wave Sampled at 40 kHz



- What do you think will happen?

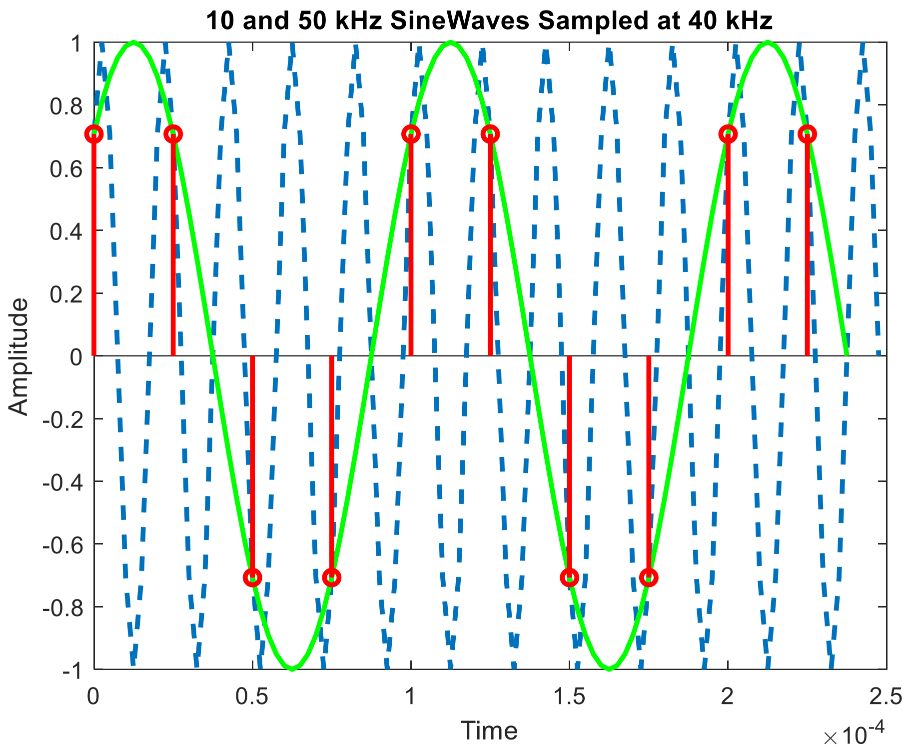
50 kHz Sine Wave
Sample Rate = 40 kHz

- Here is a 50 kHz Sine Wave Sampled at 40 kHz



Compare the Two

- What do you notice about the samples for each sine wave?



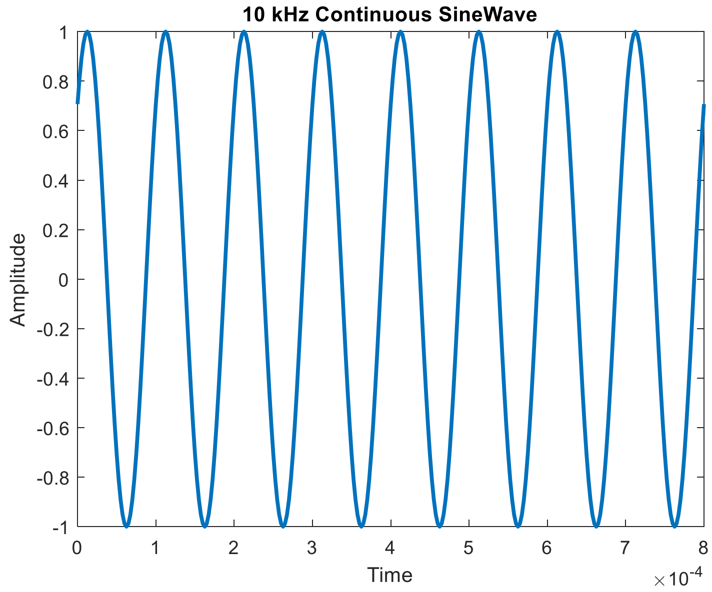
The samples for the 10 kHz and the 50 kHz sinewaves are the same when sampled at 40 kHz

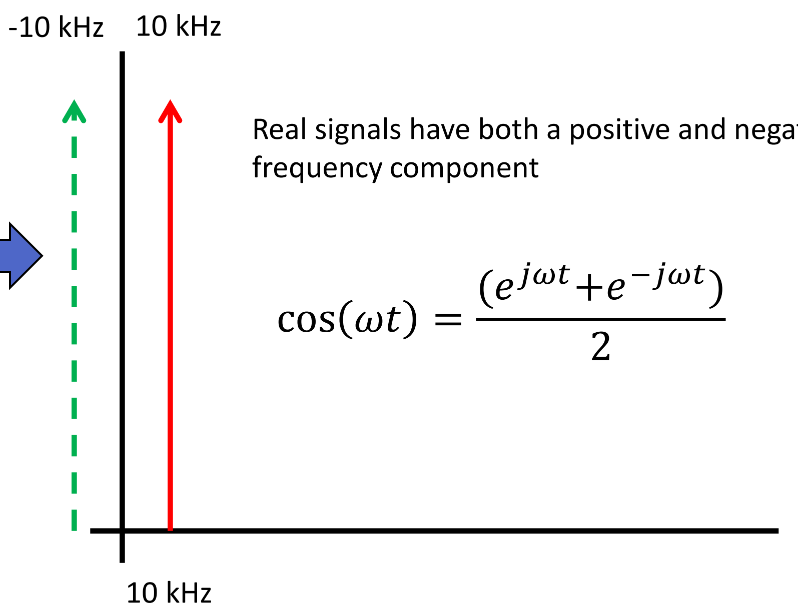
I've lost some information about the 50 kHz sine wave when sampled incorrectly

This is ALIASING

Consider the Frequency Domain

- Sampling creates copies of the signal at intervals of the sample rate





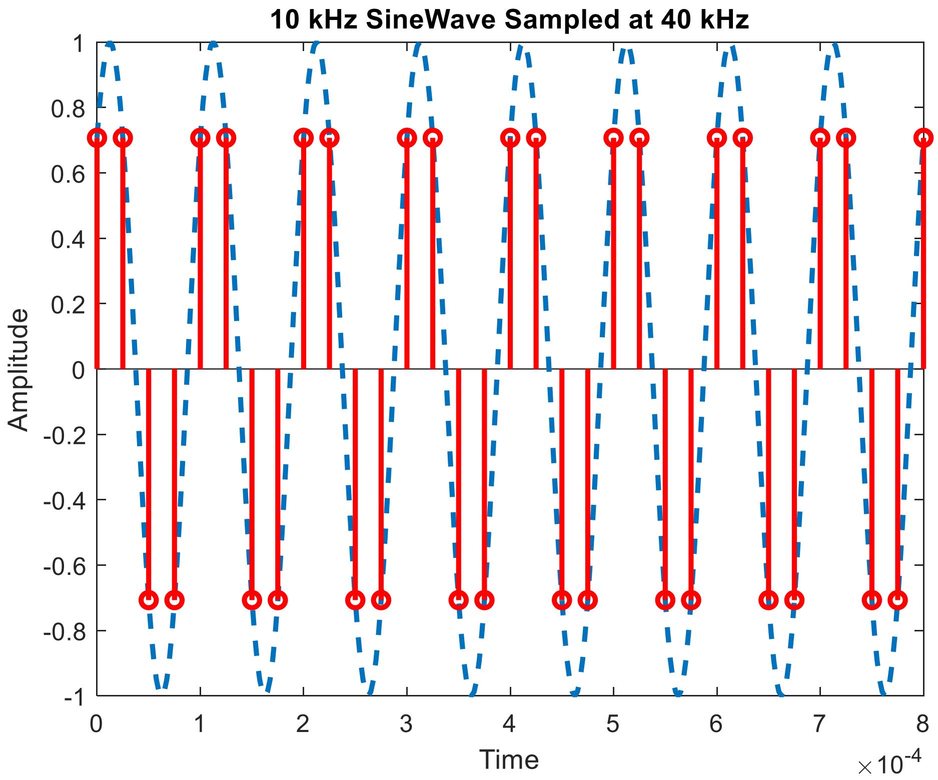
Real signals have both a positive and negative frequency component

$$\cos(\omega t) = \frac{(e^{j\omega t} + e^{-j\omega t})}{2}$$

Spectrum of continuous Signal

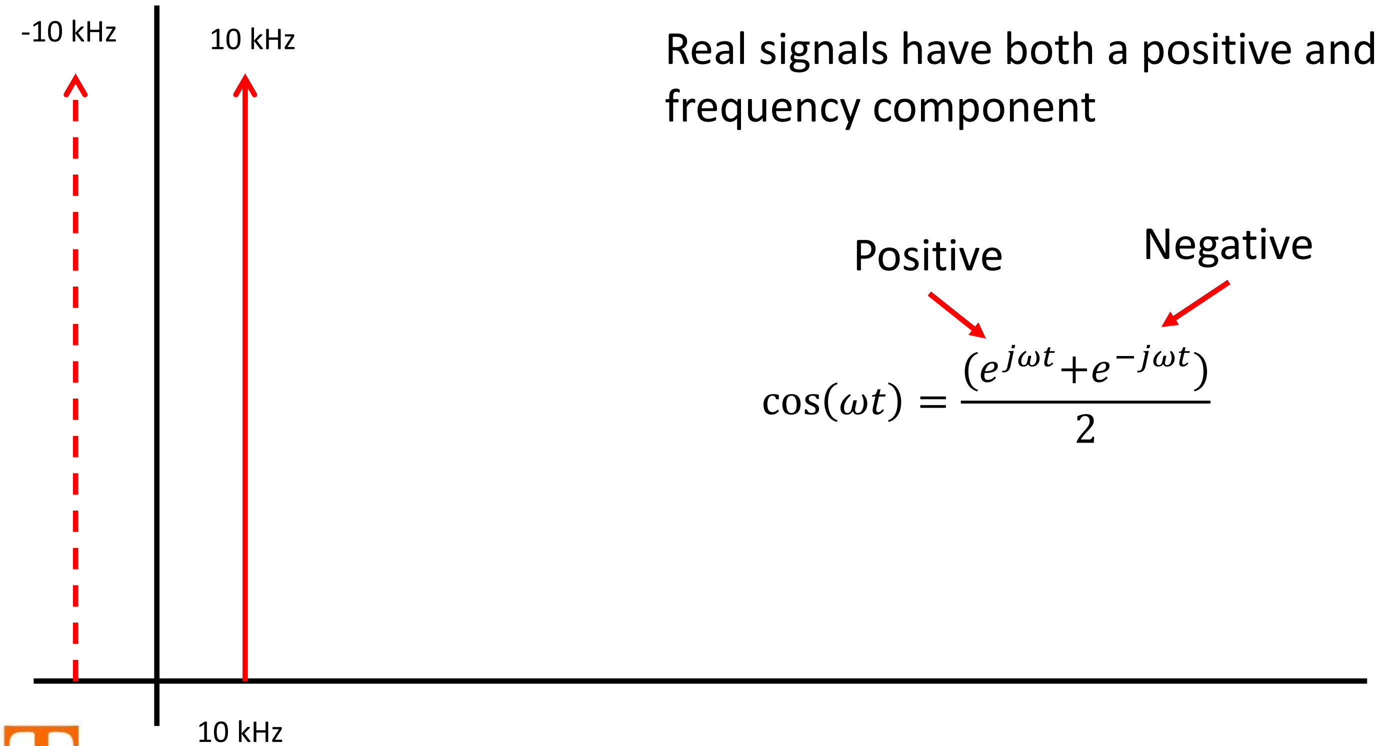
Consider the Frequency Domain

- Sampling creates copies of the signal at intervals of the sample rate



Where do the copies exist?

- A continuous signal at 10 kHz.
- A negative frequency component exists at -10 kHz
- Sometimes referred to as the Upper and Lower sideband



Real signals have both a positive and negative frequency component

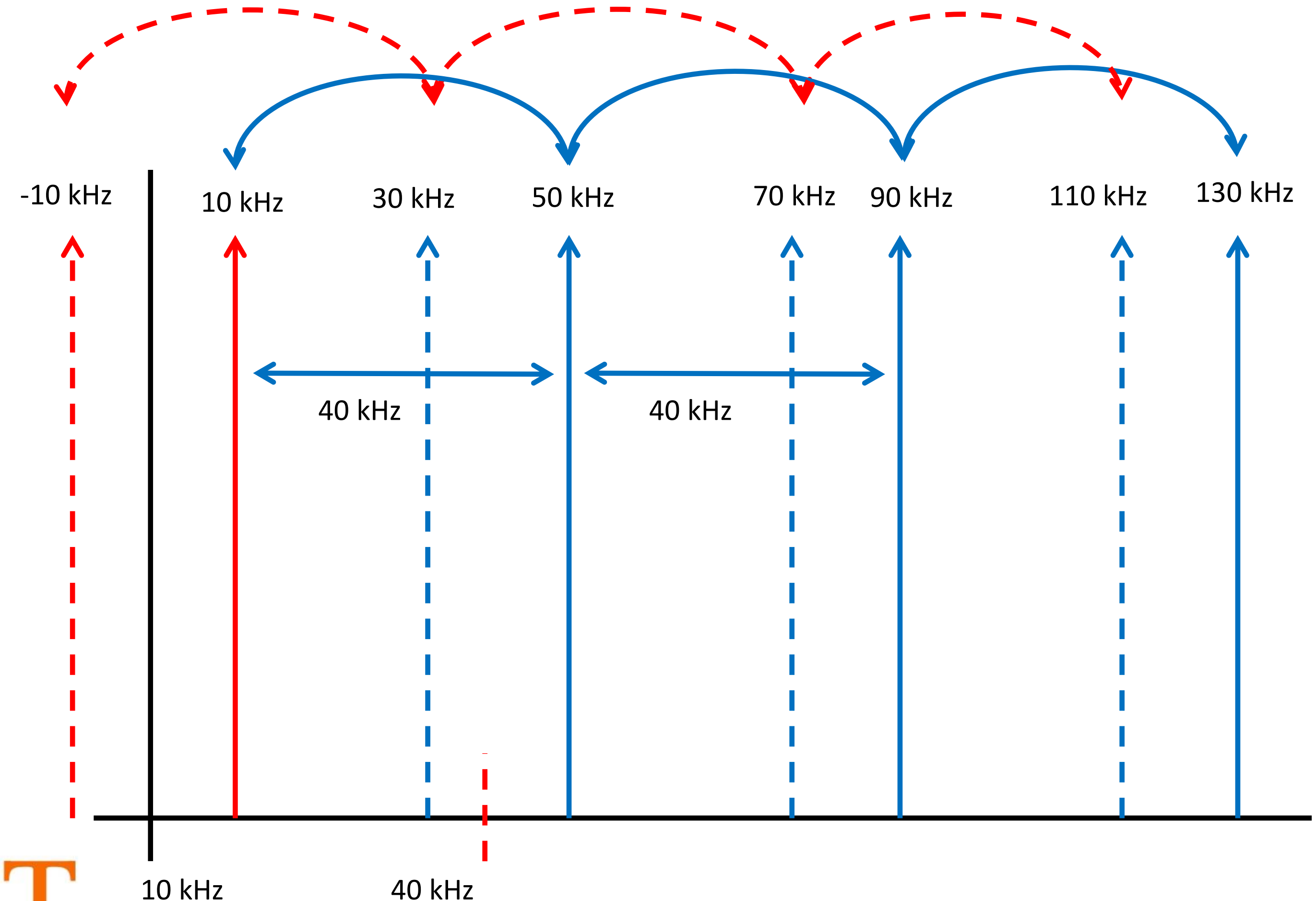
Positive

Negative

$$\cos(\omega t) = \frac{(e^{j\omega t} + e^{-j\omega t})}{2}$$

Where do the copies exist when sampled at $F_s = 40\ kHz$

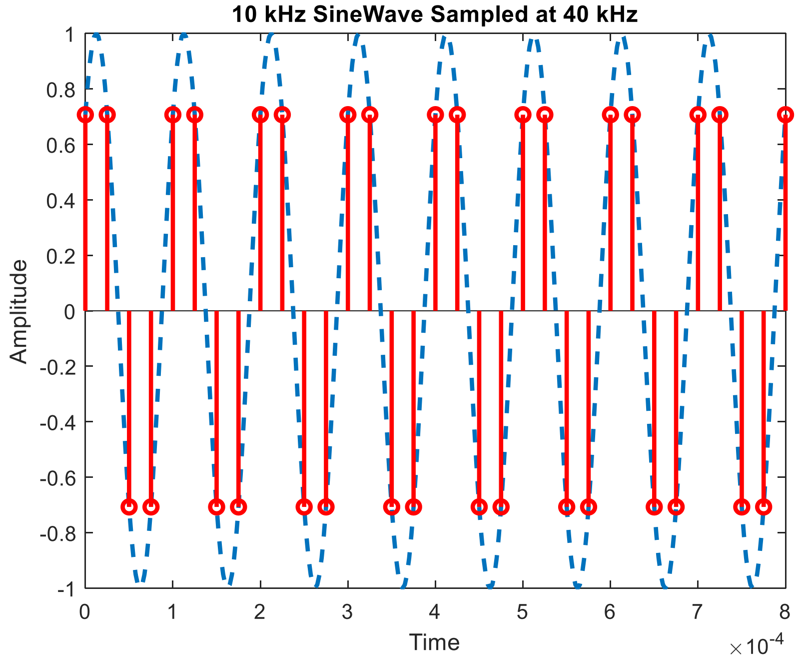
- Signals are duplicated every $F_s = 40\ kHz$

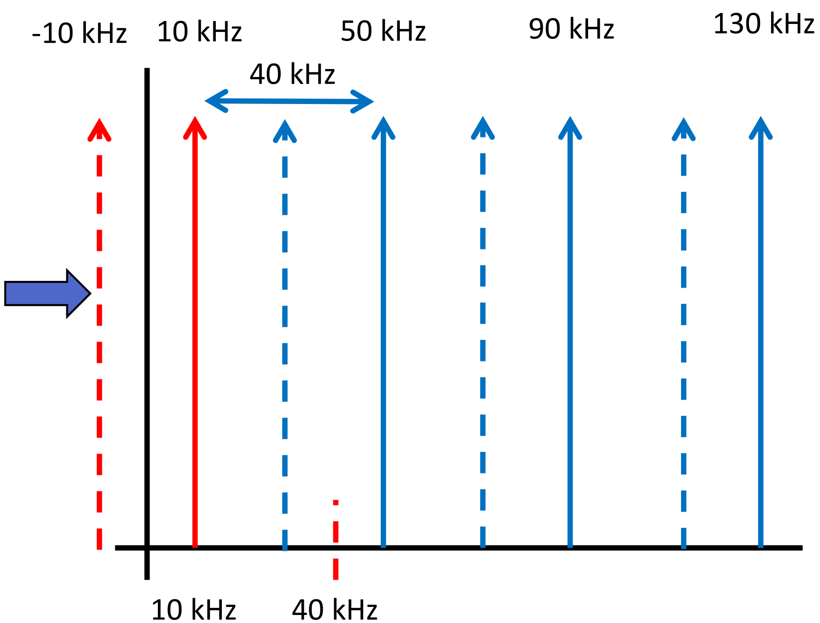


Consider the Frequency Domain

- Spectrum of the sampled signal

Copies in BLUE

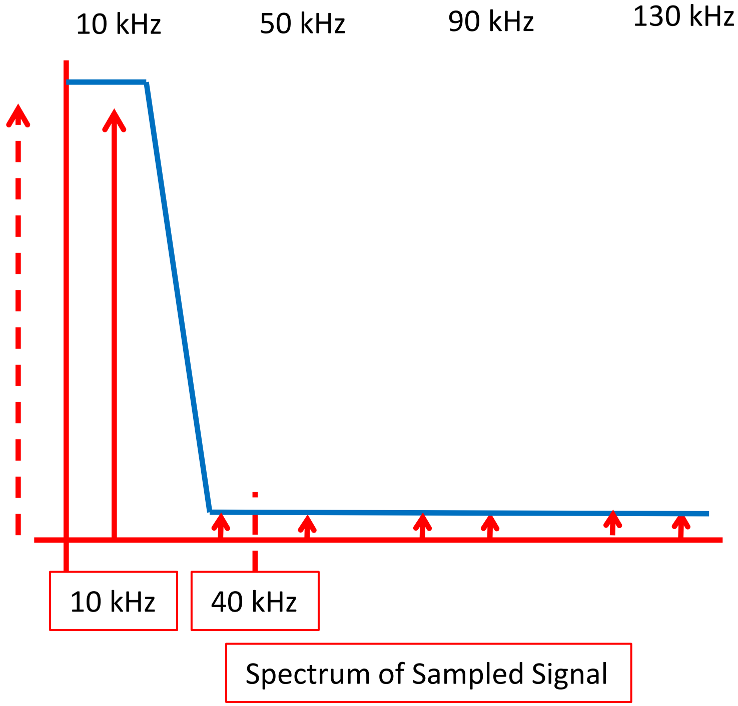


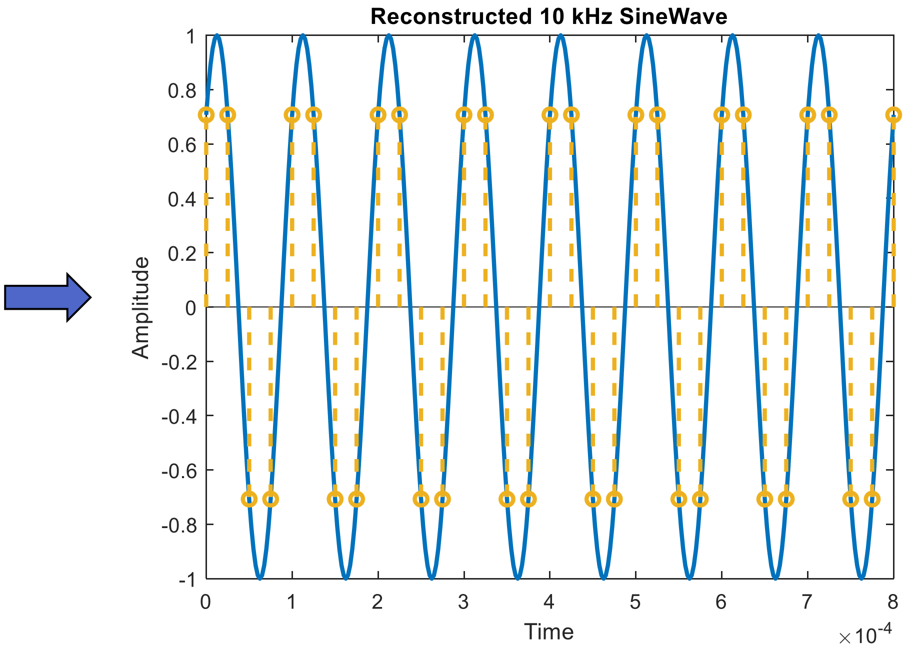


Spectrum of Sampled Signal

Reconstructing the Signal

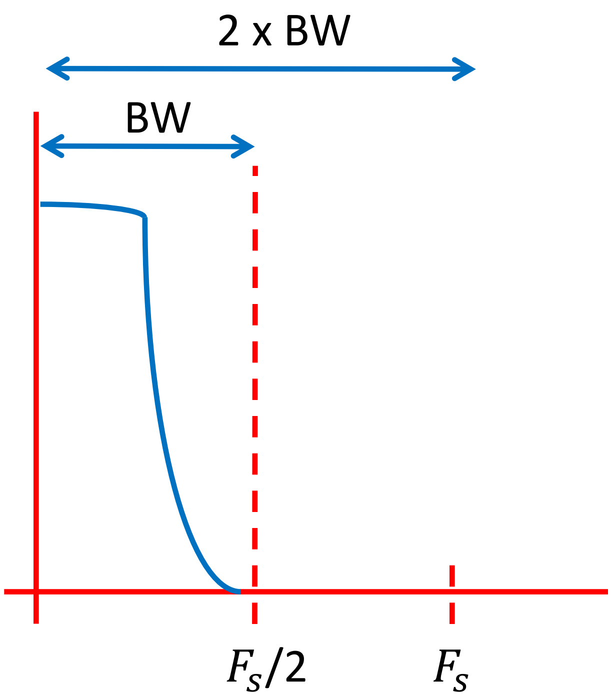
- Placing a low pass filter after the signal removes the copies of the signal and reproduces the sine wave.
- No loss of information





The Nyquist Rate

- If I am sampling at a rate that is <u>at least twice</u> the highest frequency in my signal, then I can reconstruct the signal with no loss.



The highest frequency in my signal determines its bandwidth

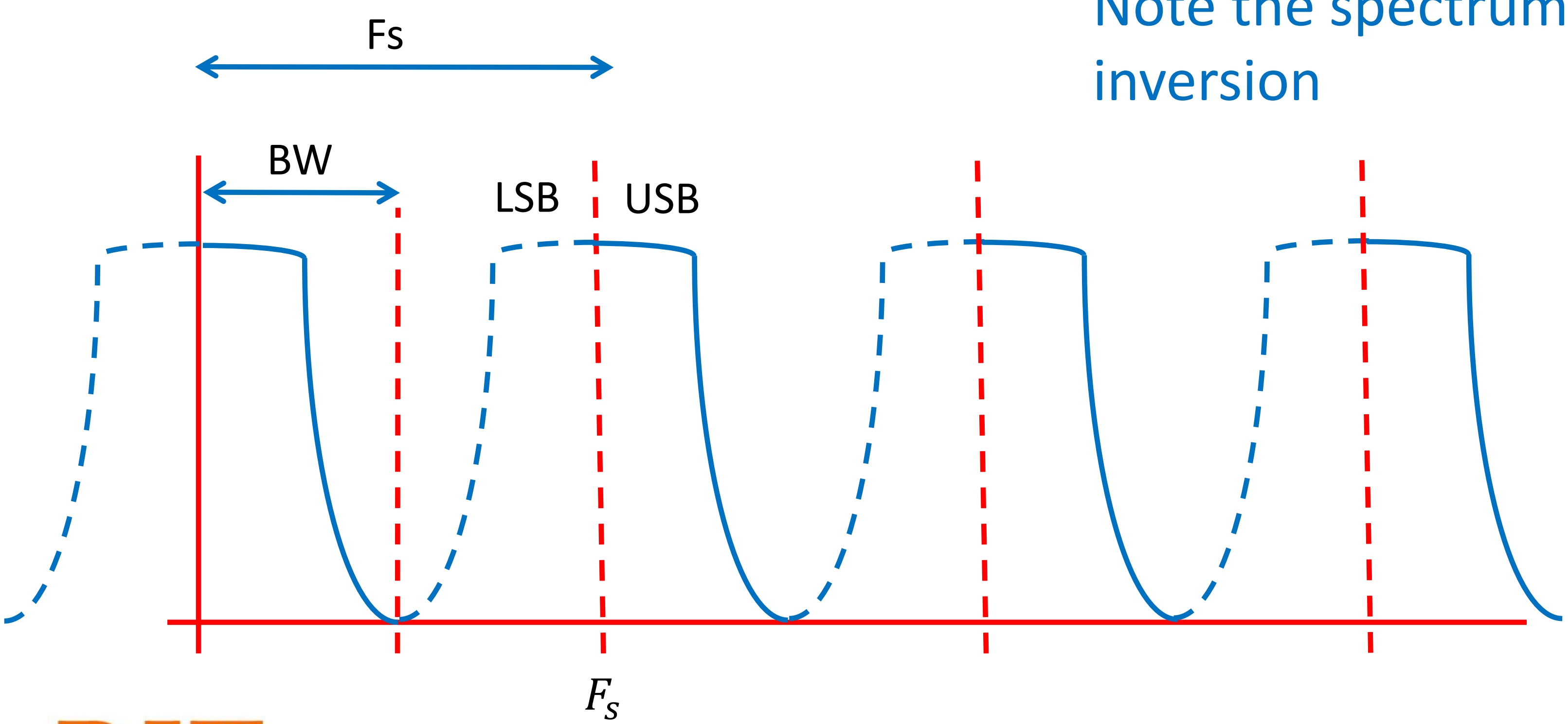
Must sample at least twice the BW to not lose information!

$$F_s \geq 2 \times BW$$

The Nyquist Rate is <u>½</u> sample rate

Sampling at 2X the Nyquist Rate

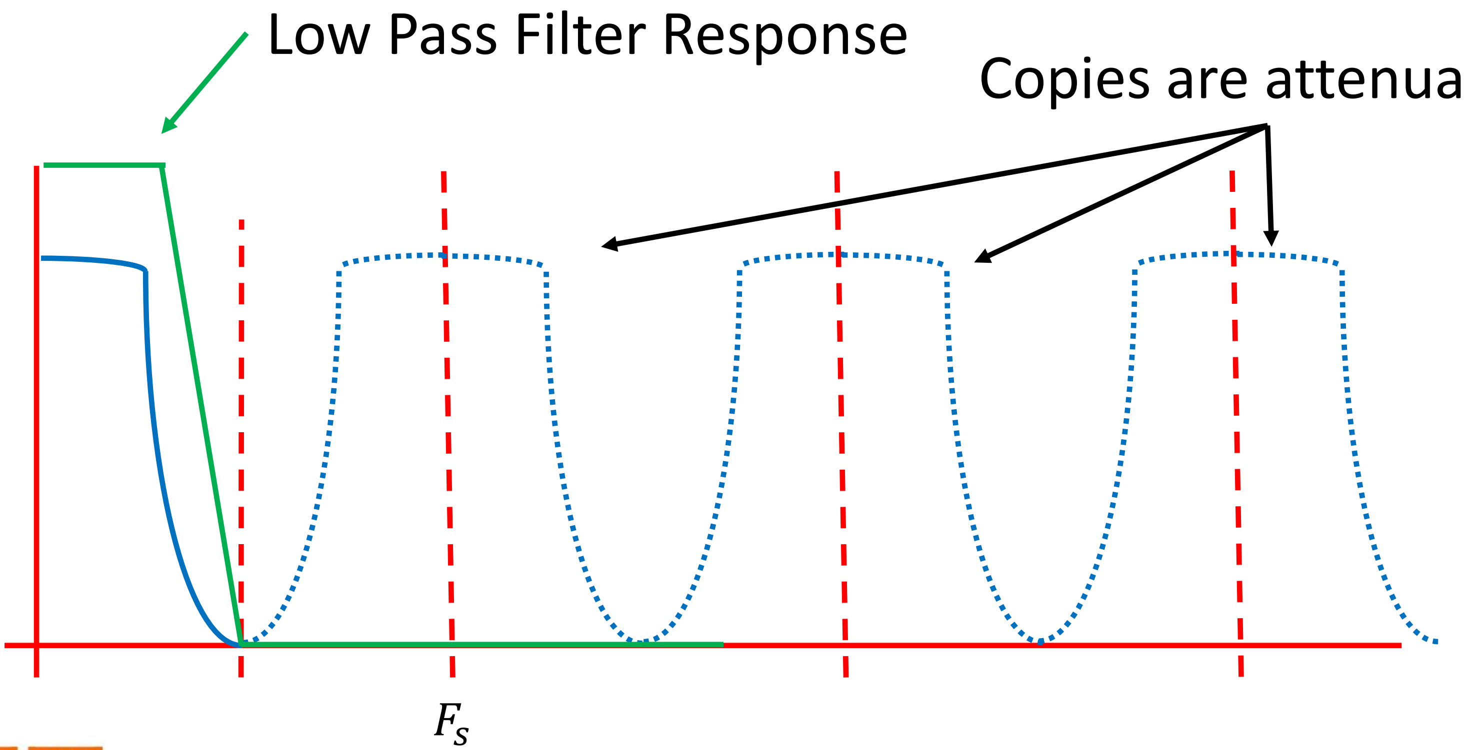
- When the signal is sampled the spectrum is duplicated at intervals of the sample rate



Note the spectrum inversion

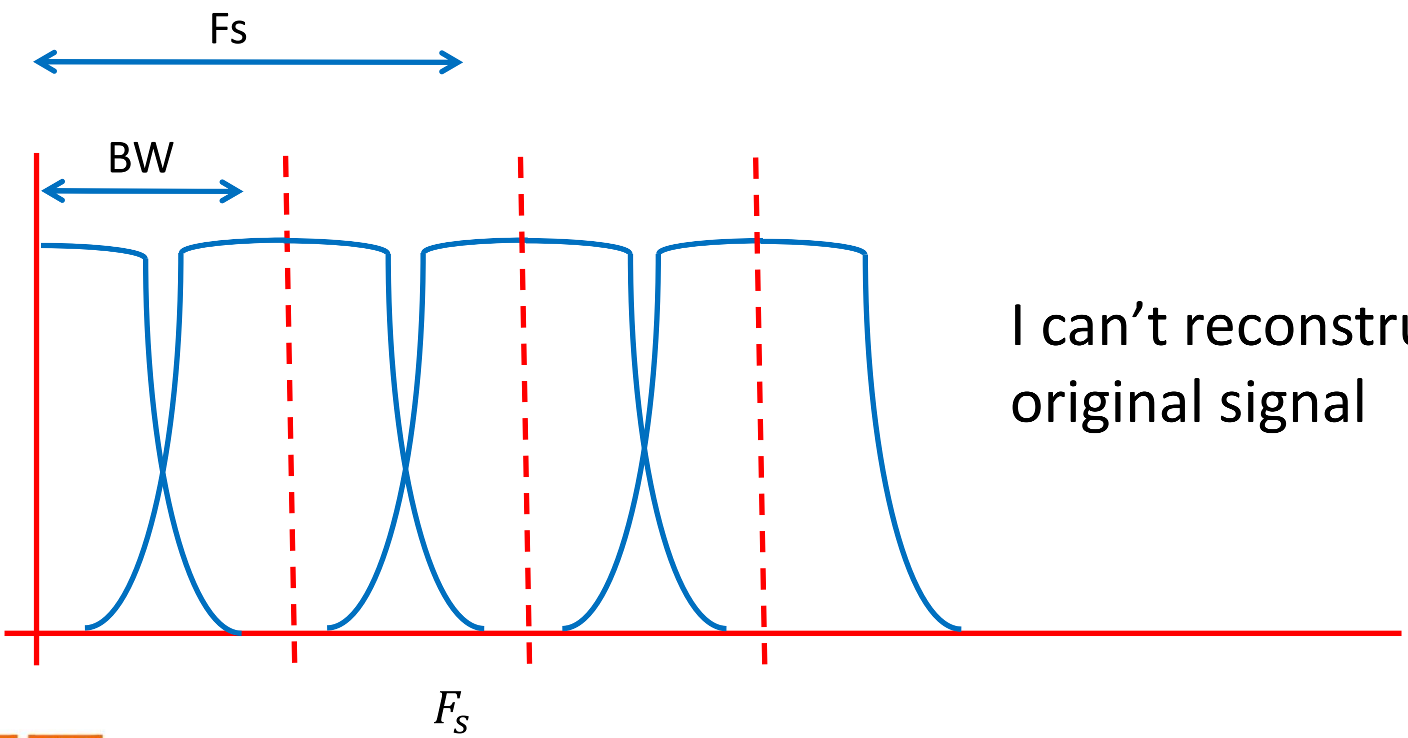
Sampling at 2X the Nyquist Rate

- Filtering the sampled signal reconstructs the original signal without loss of information



Sampling at less than 2X the Nyquist Rate

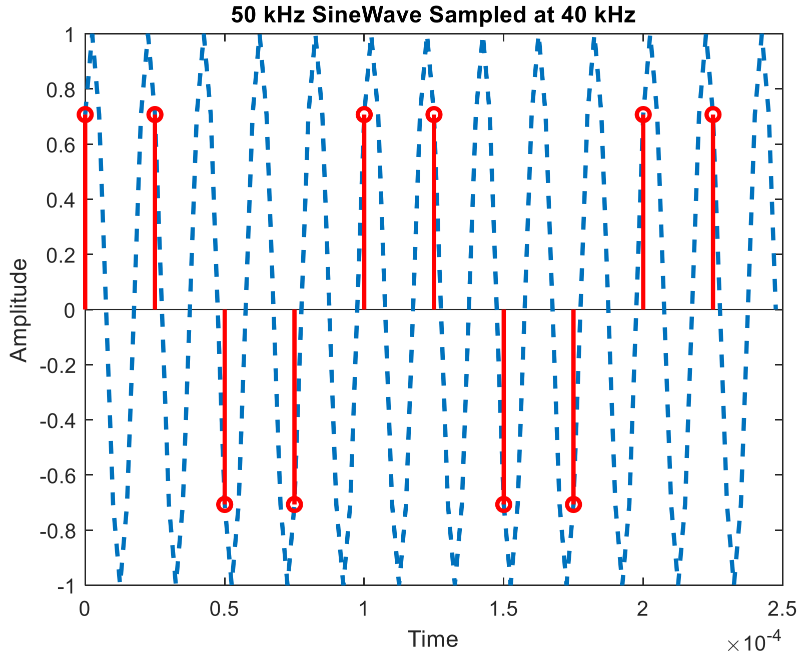
- If the sample rate is too low ($F_s < 2 \times BW$) then information is lost. This is called aliasing.

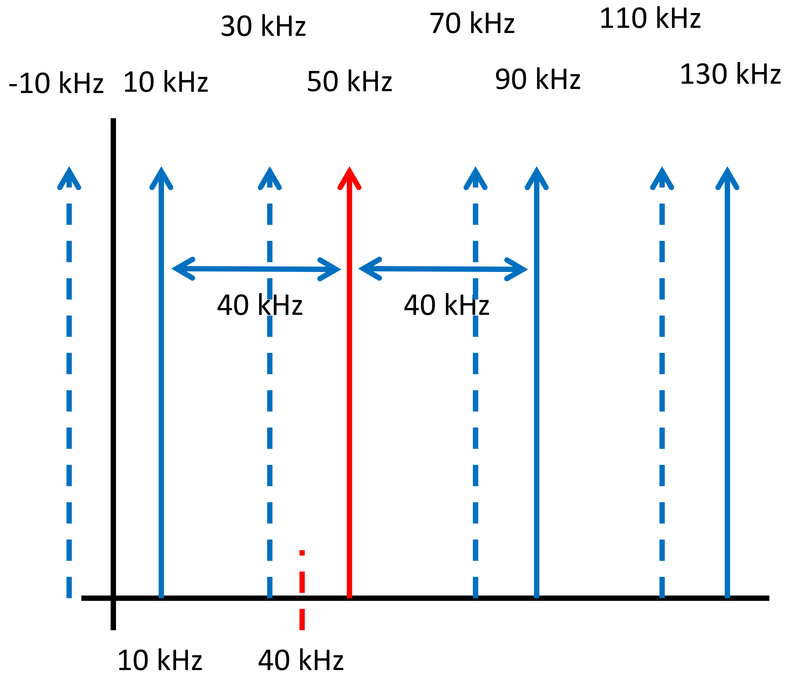


I can't reconstruct the original signal

A Sine Wave Example

- Assume that I am sampling a 50 kHz sine wave at 40 kHz
- Does this meet the criteria for proper sampling?

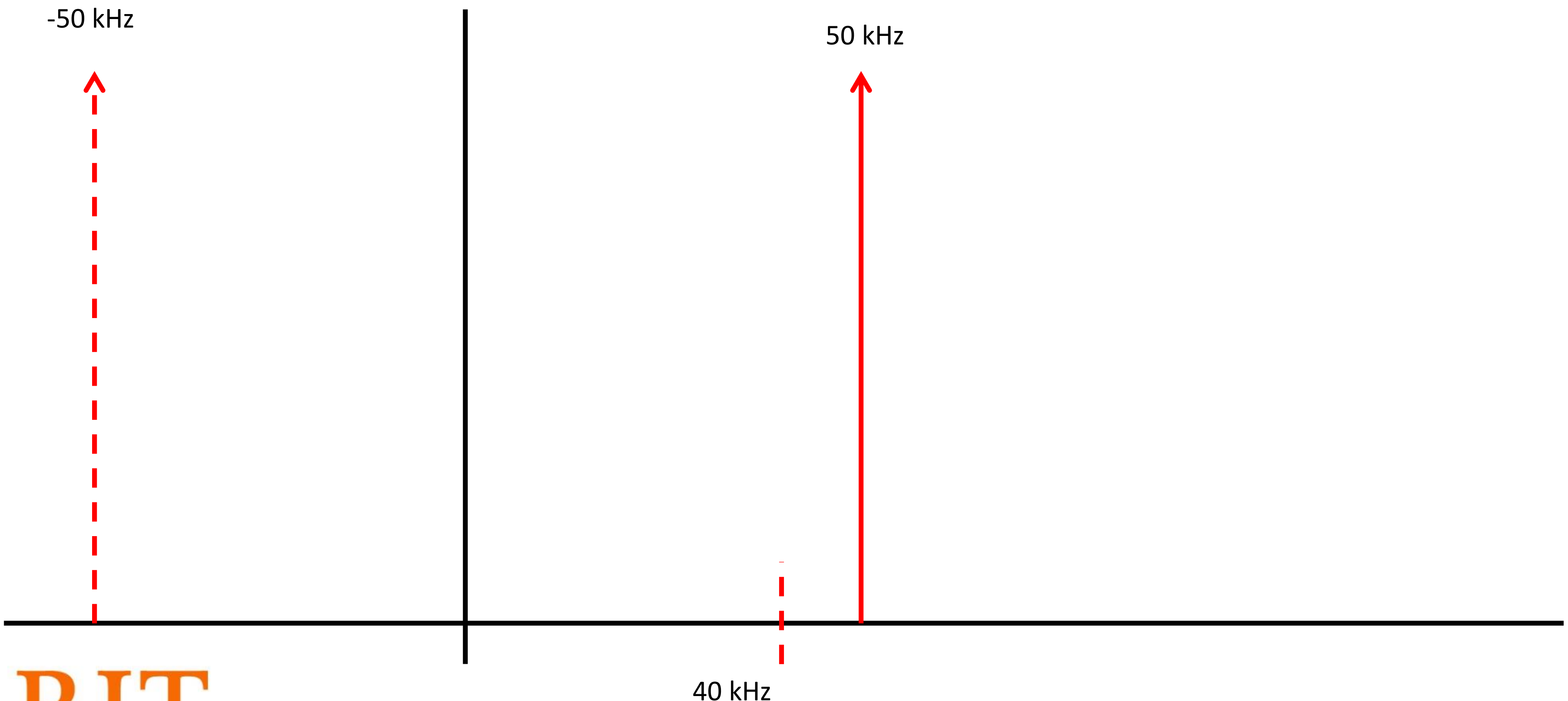




Spectrum of Sampled Signal

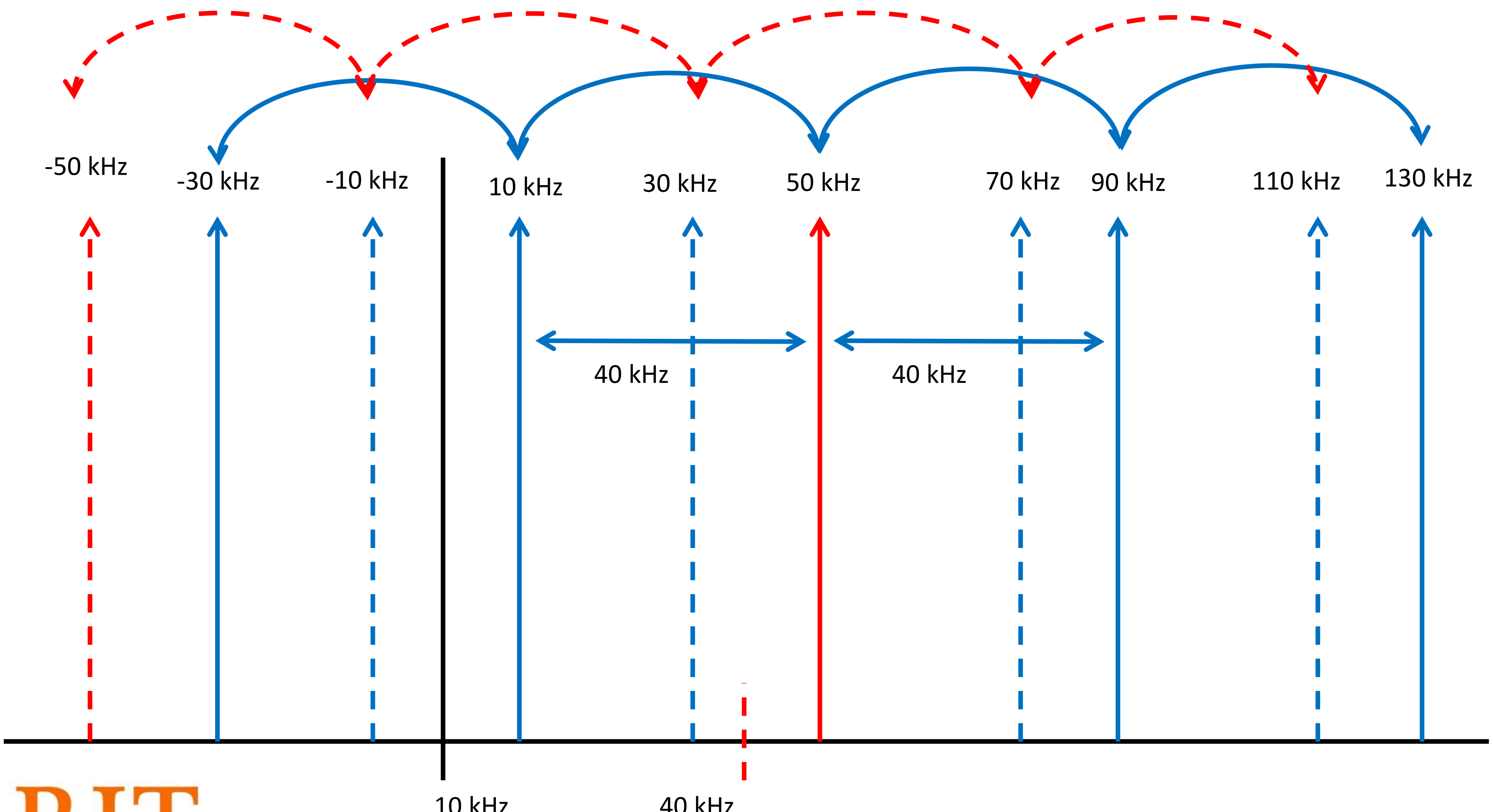
Where do the copies exist?

- Starting with my continuous signal at 50 kHz. There is a negative frequency component at -50 kHz



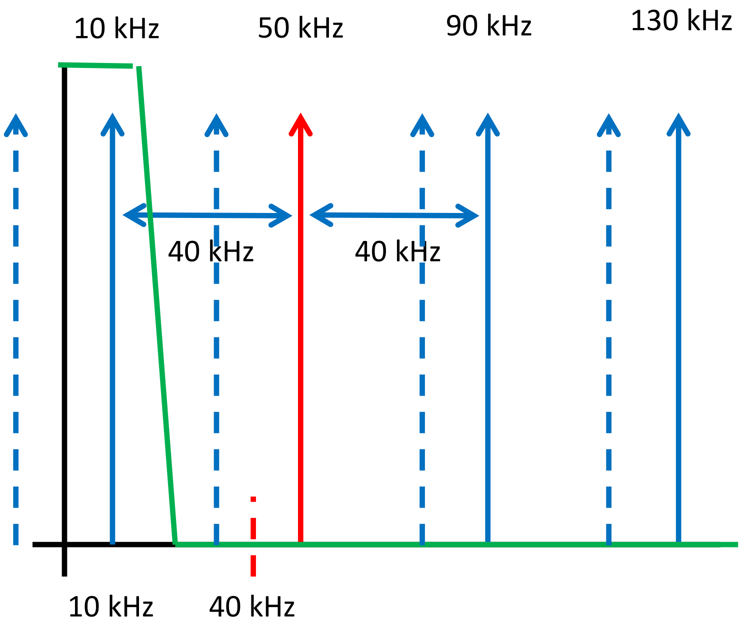
Where do the copies exist?

- Signals are duplicated every $F_s = 40\ kHz$

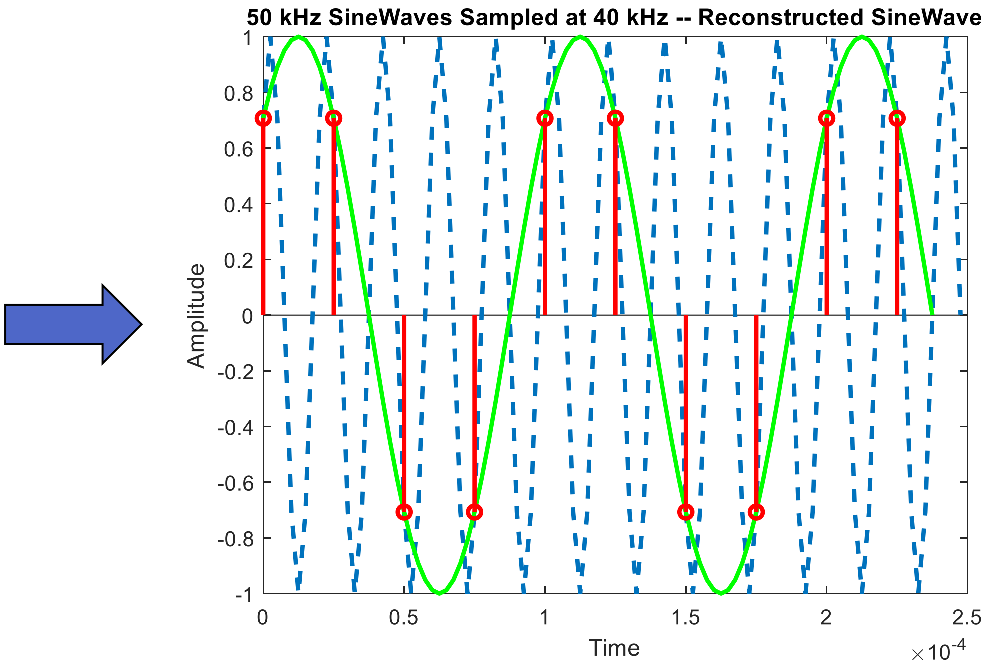


A Sine Wave Example

- Filtering the signal with a low pass would result in a 10 kHz signal



Spectrum of Sampled Signal



ICP Sampling Sinusoids

- Consider a single sinusoid

$$x(t) = 10\sin(2\pi \times 150\mathrm{t})$$

- What is the minimum sampling frequency for this signal?
- Draw the locations of the frequency when the signal is sampled at 400 Hz

ICP Sampling Sinusoids

- Consider a single sinusoid

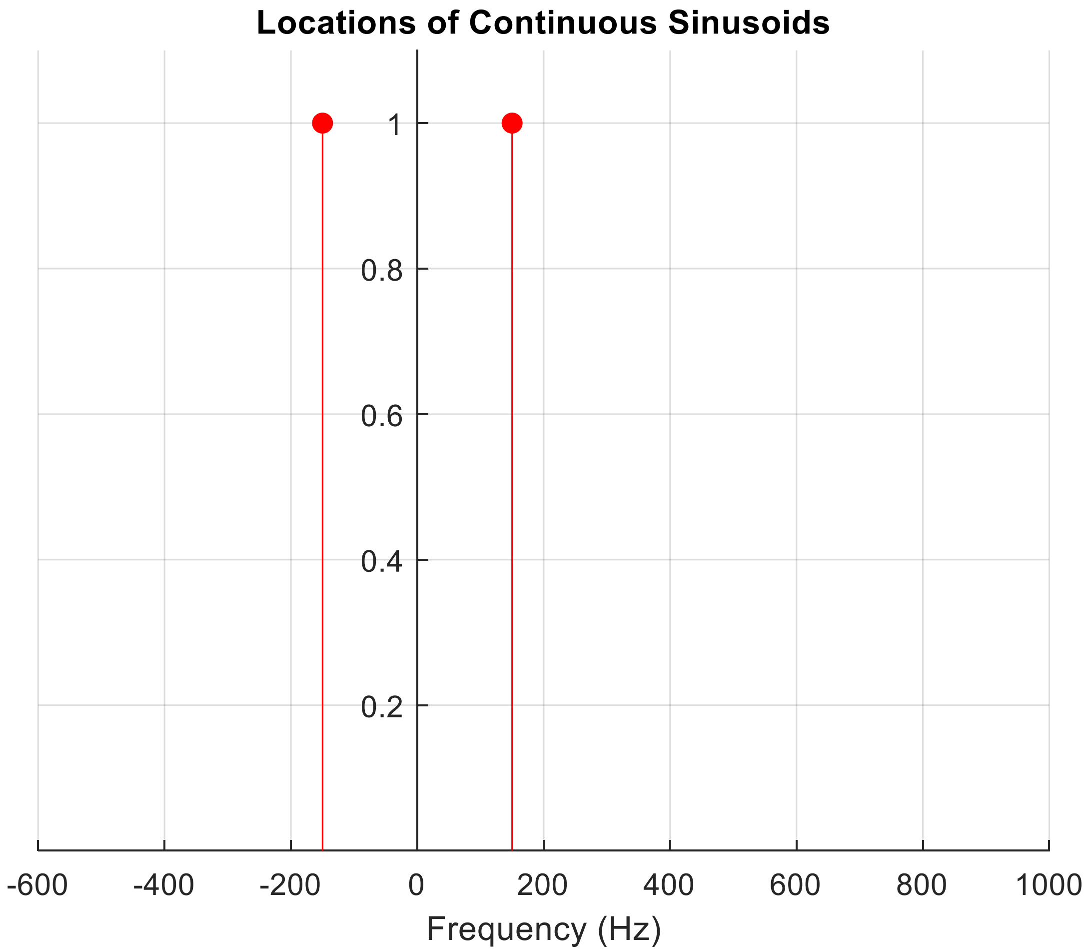
$$x(t) = 10\sin(2\pi \times 150\mathrm{t})$$

- What is the minimum sampling frequency for this signal?

The maximum frequency is 150 Hz. The signal must be sampled at least at 300 Hz

- Draw the frequency spectrum when the signal is sampled at 400 Hz

Continuous Sinusoid Locations

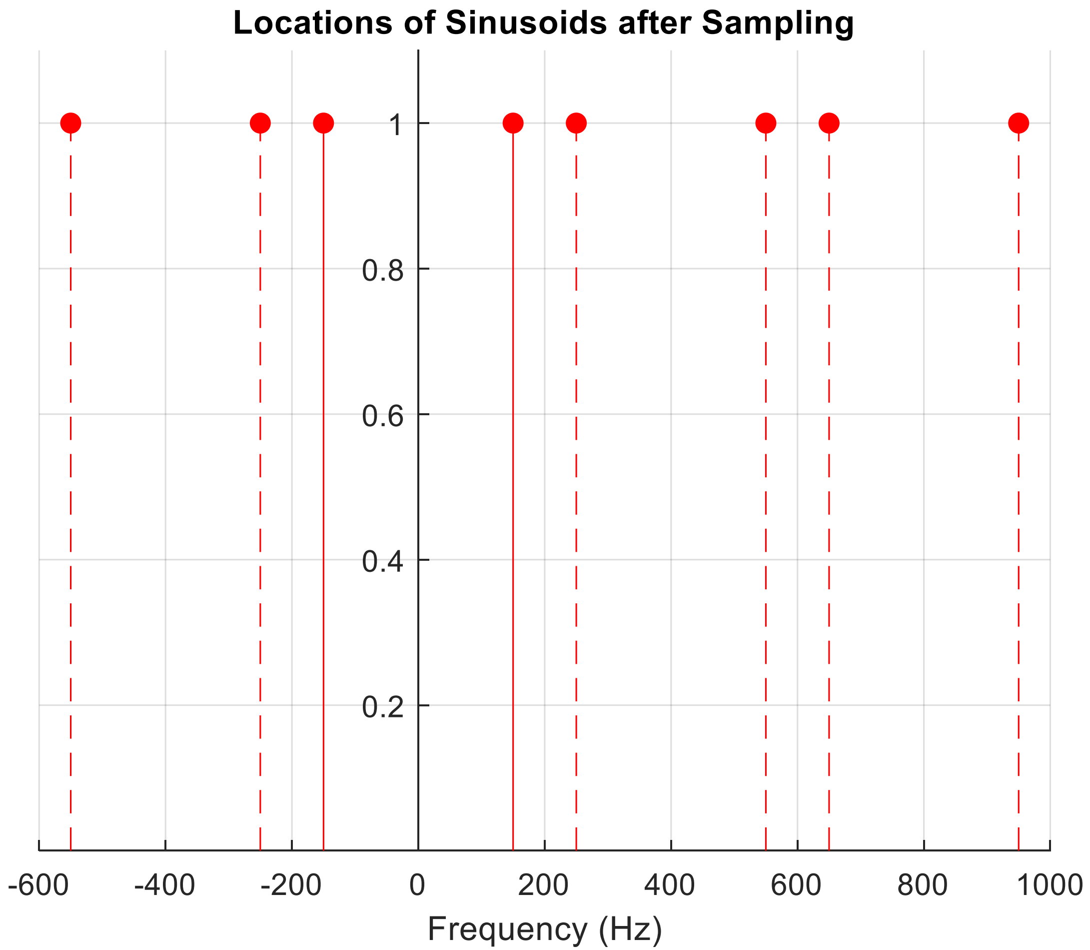


F1 = 150 Hz

F2 = -150 Hz

NOPRINT

Sampled Sinusoid Locations

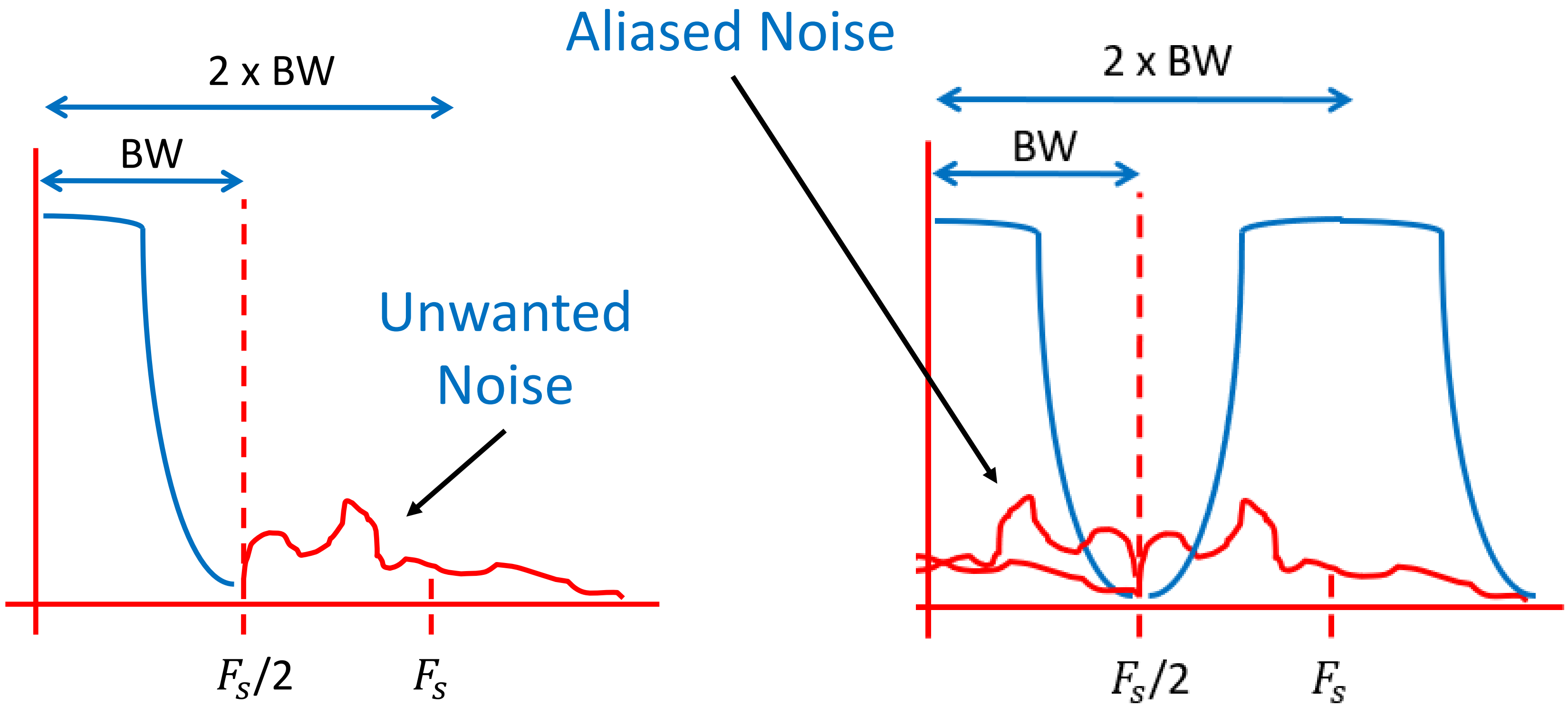


-550
-250
-150
150
250
550
650
950

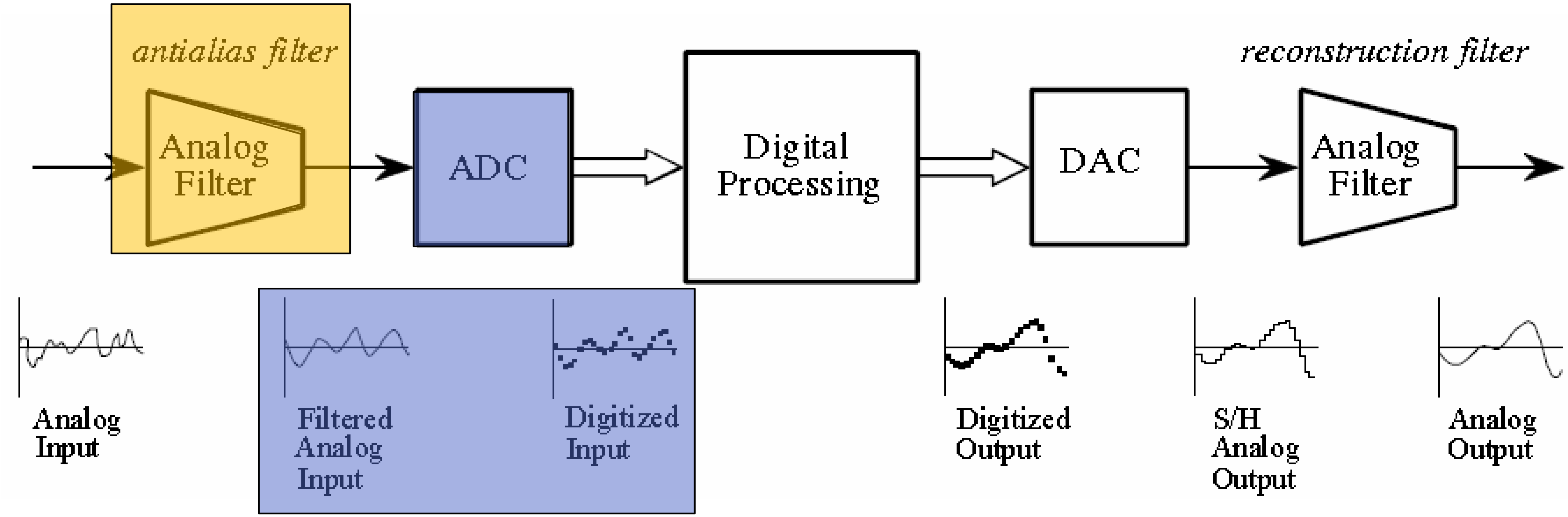
NOPRINT

Aliasing Noise

- If there is unwanted information above the Nyquist rate, then that energy will alias into the desired signal



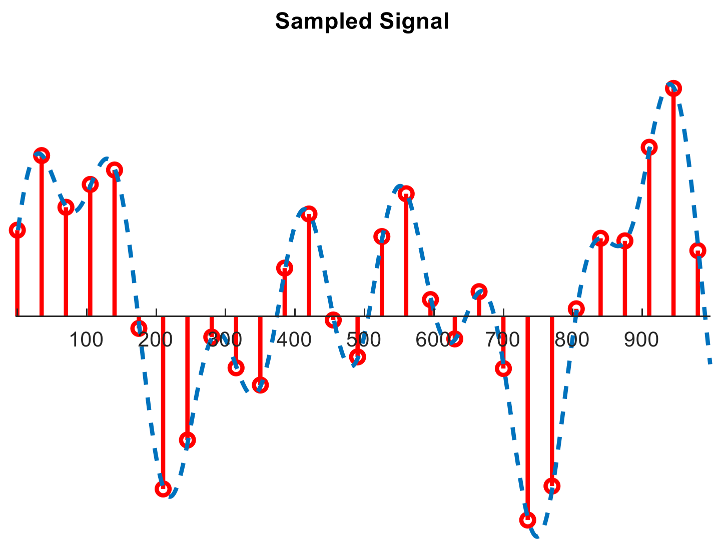
Preventing Aliasing



- To prevent aliasing due to sampling from occurring an analog filter is placed in front of the ADC
- This filter, generally a low pass attenuates energy that is outside of the Nyquist region
- This is called an anti-alias filter

Signal Reconstruction

- How do we convert the samples back to the continuous signal

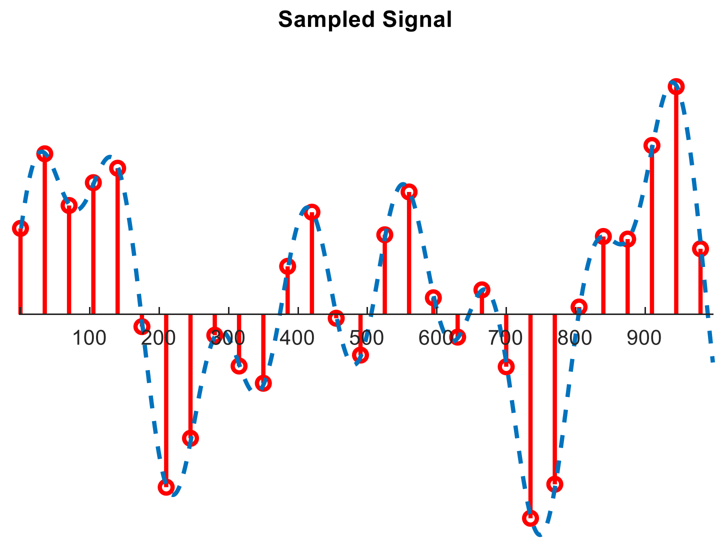


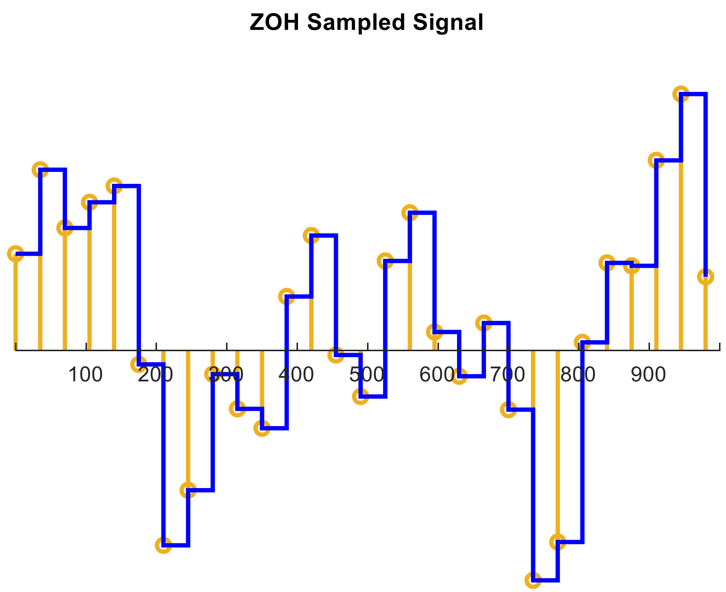
- The numerical values need to be converted into a voltage. This is done by a Digital to Analog Converter (DAC)

- Somehow we need to "connect the dots" between each sample point.

Signal Reconstruction

- We could hold the value of the sample over 1 sample time – Called a Zero Order Hold (ZOH)

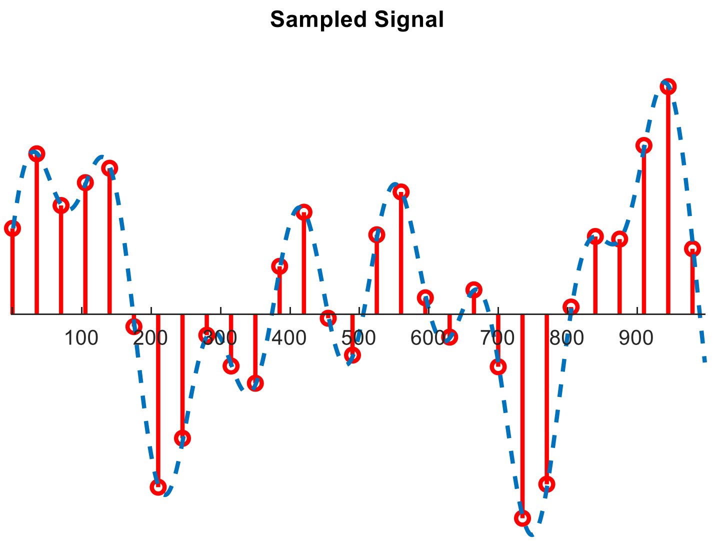


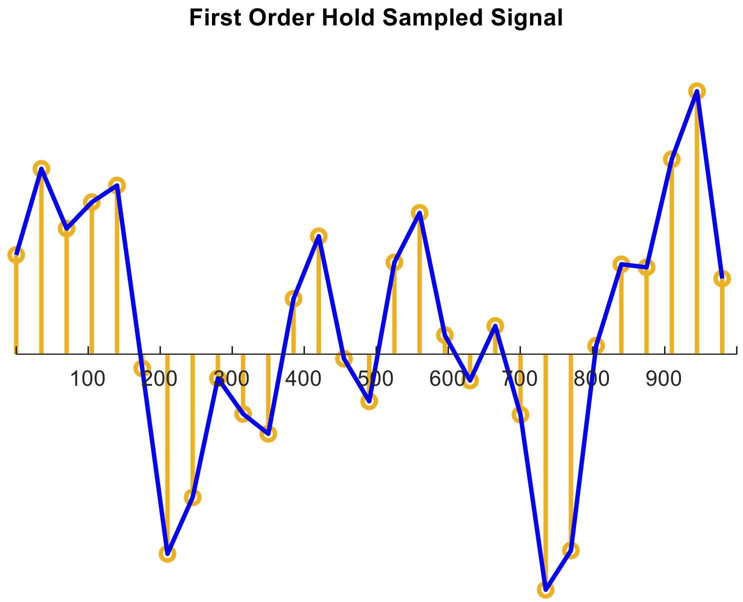


- This is what a DAC does at its output

Signal Reconstruction

- We could run a straight line between the points (First Order Hold)

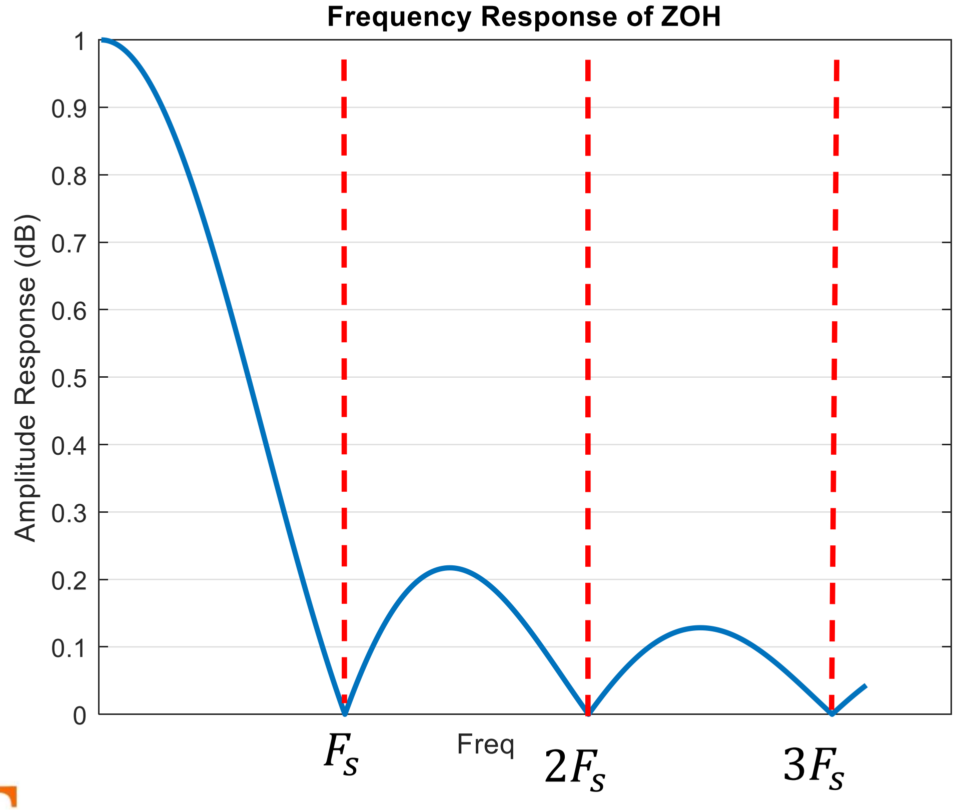




- Not practical to achieve with DAC output

What is the impact of the ZOH?

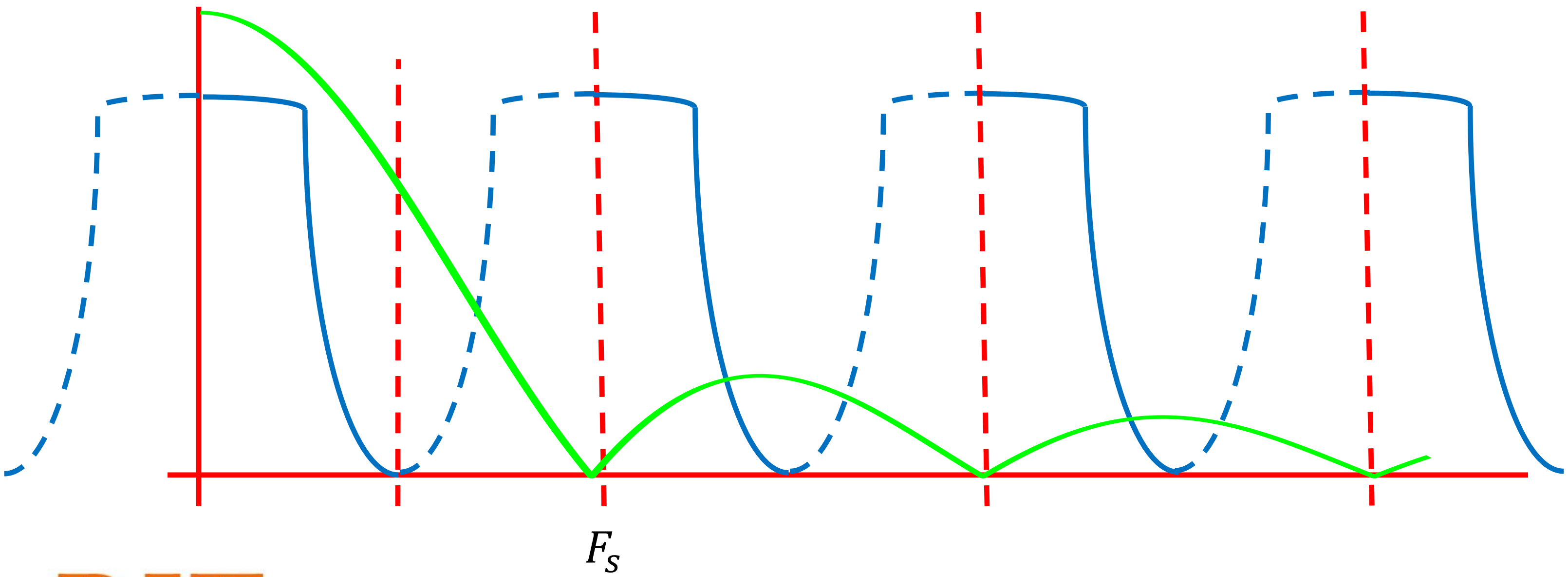
- The frequency response of the Zero Order Hold is a SINC function in the frequency domain



$$Sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

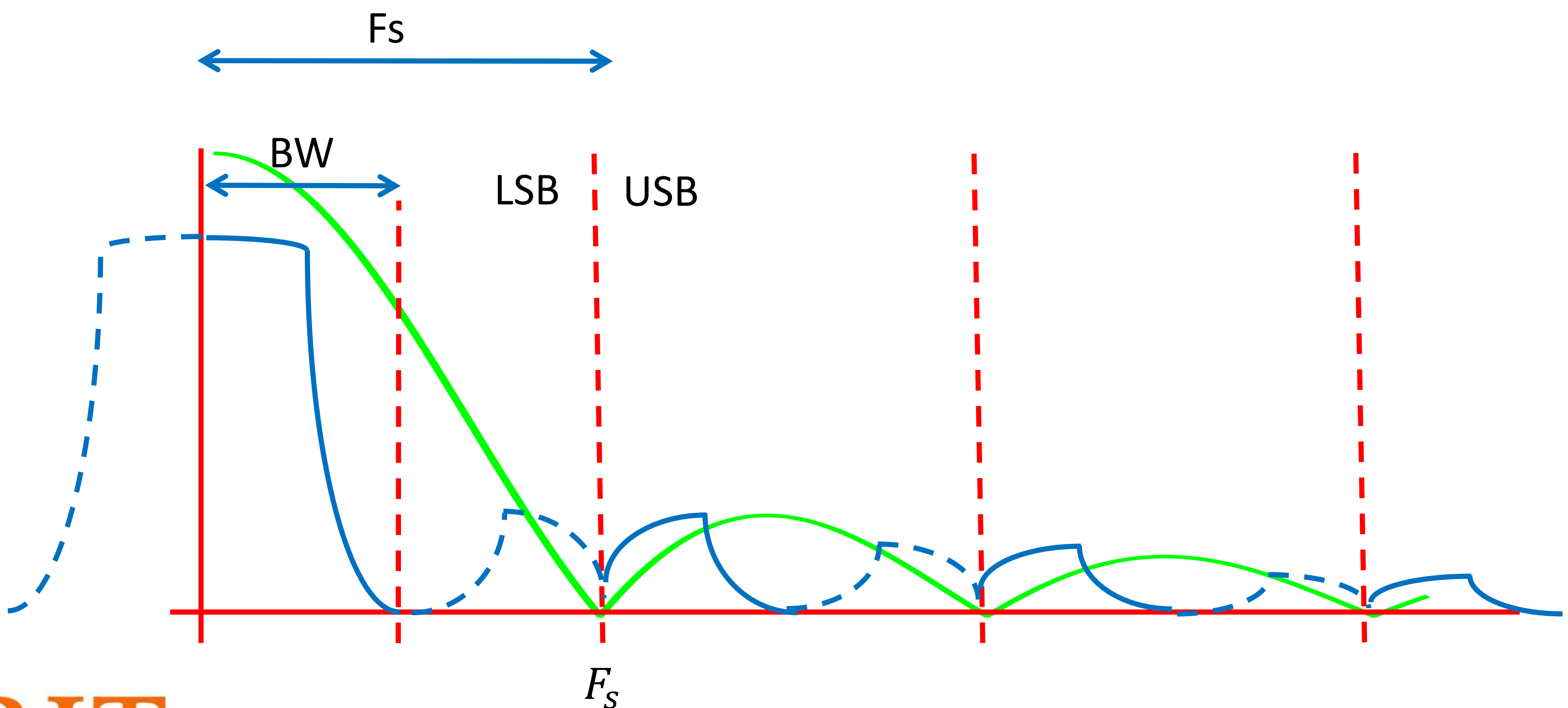
Sampling at 2X the Nyquist Rate

- The ZOH frequency response is not “flat” across the range from DC to $F_s/2$ it creates distortion in the signal.



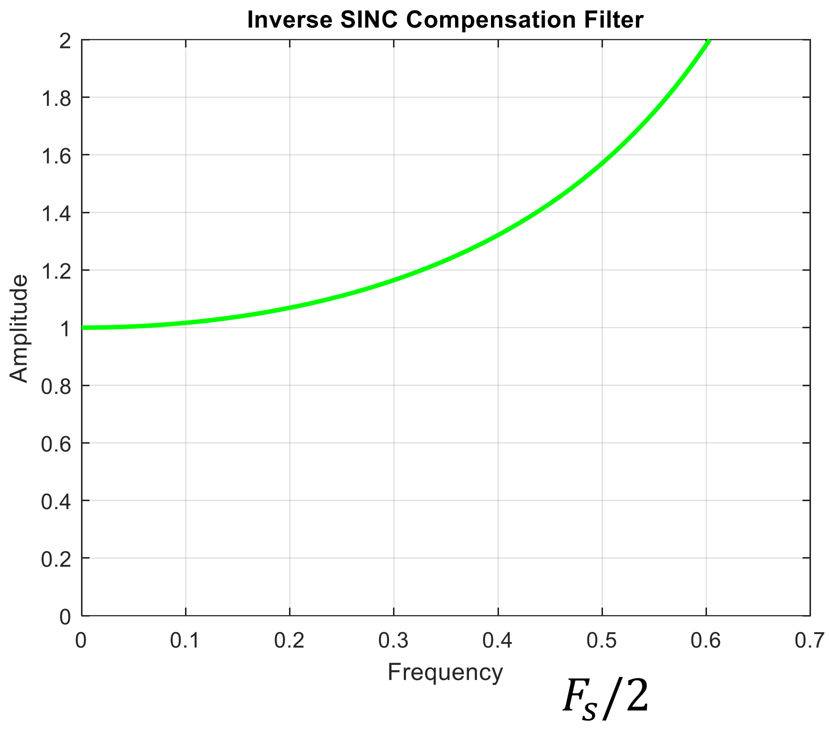
Reconstruction Filter

- After the ZOH, there is still energy left as sampling artifacts



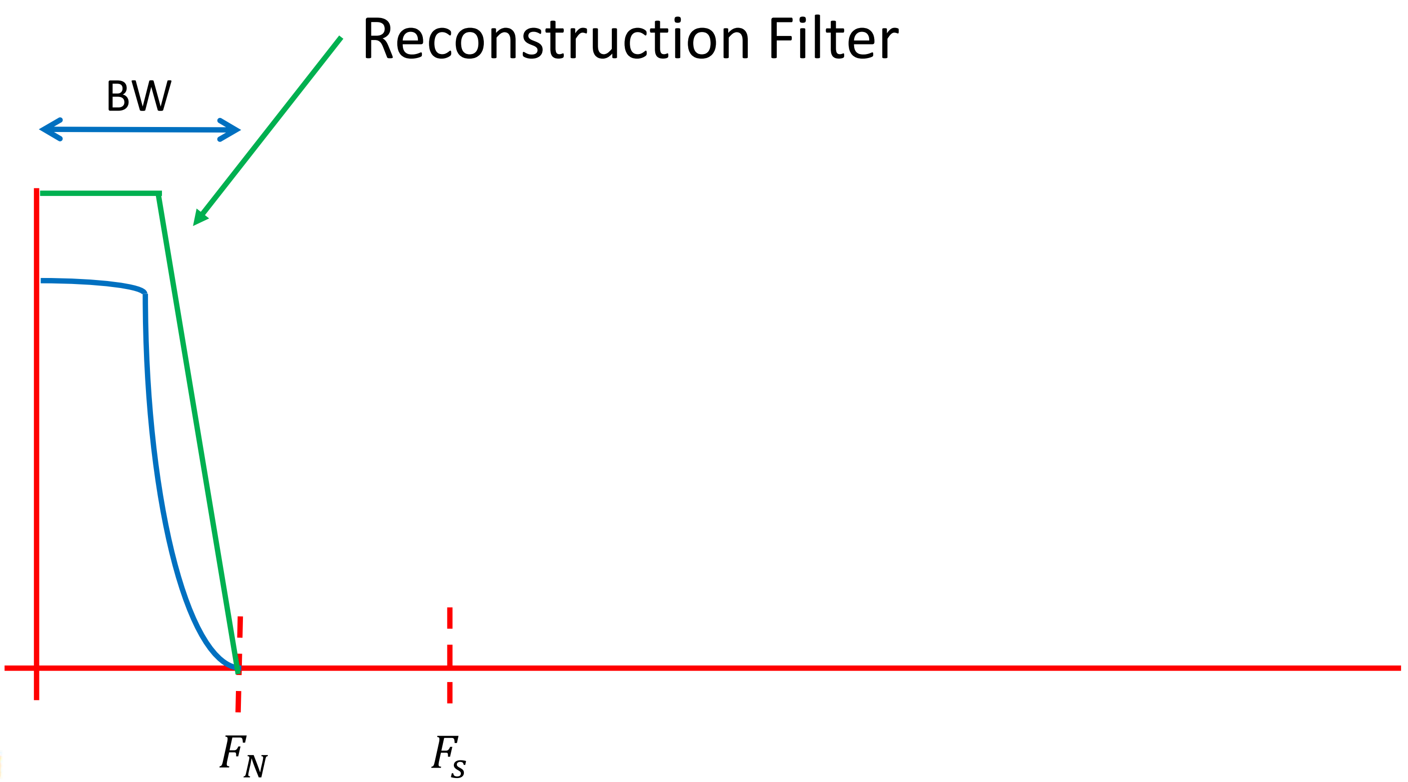
Sampling at 2X the Nyquist Rate

- To compensate for the distortion a reconstruction filter with a 1/Sinc(x) characteristic can be used to cancel out the distortion.



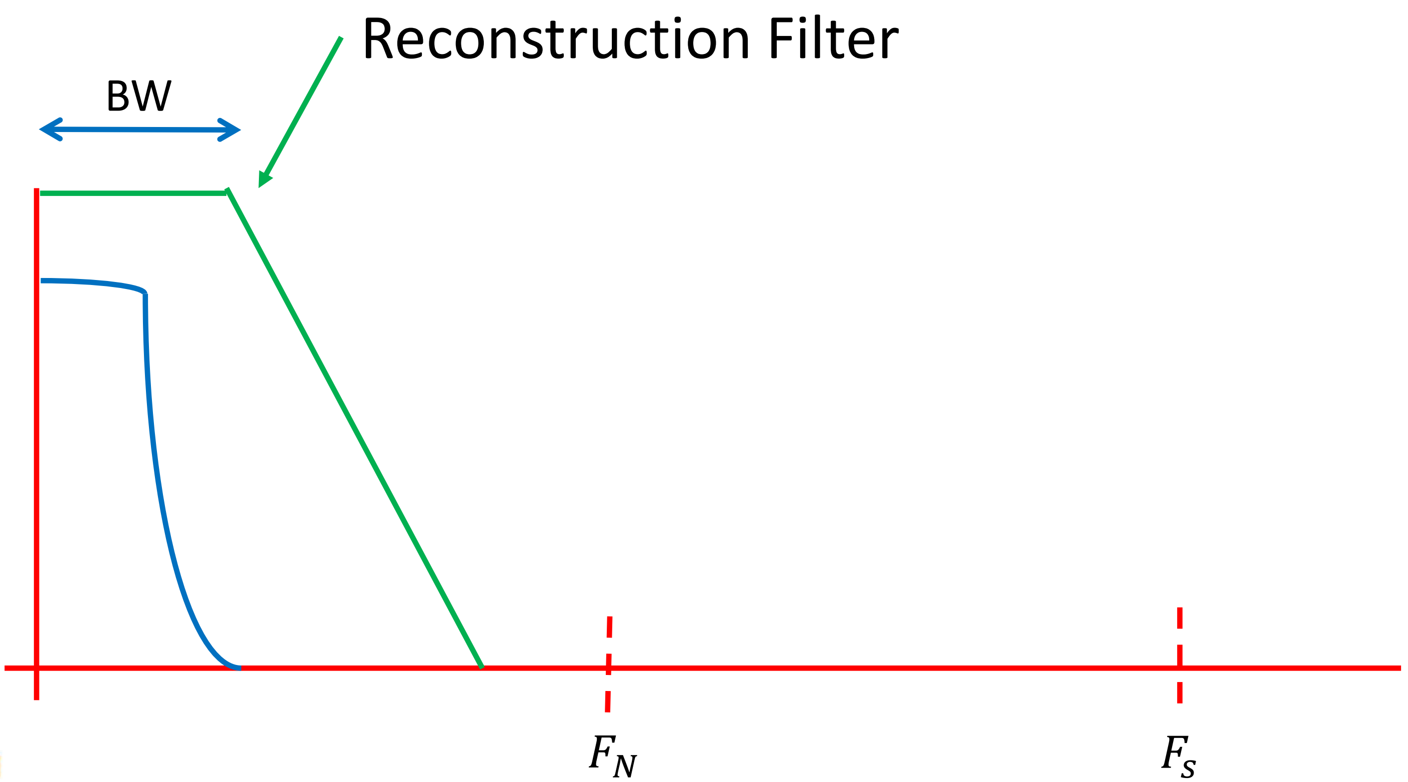
Reconstruction Filter

- Apply an additional reconstruction filter to reduce the artifacts

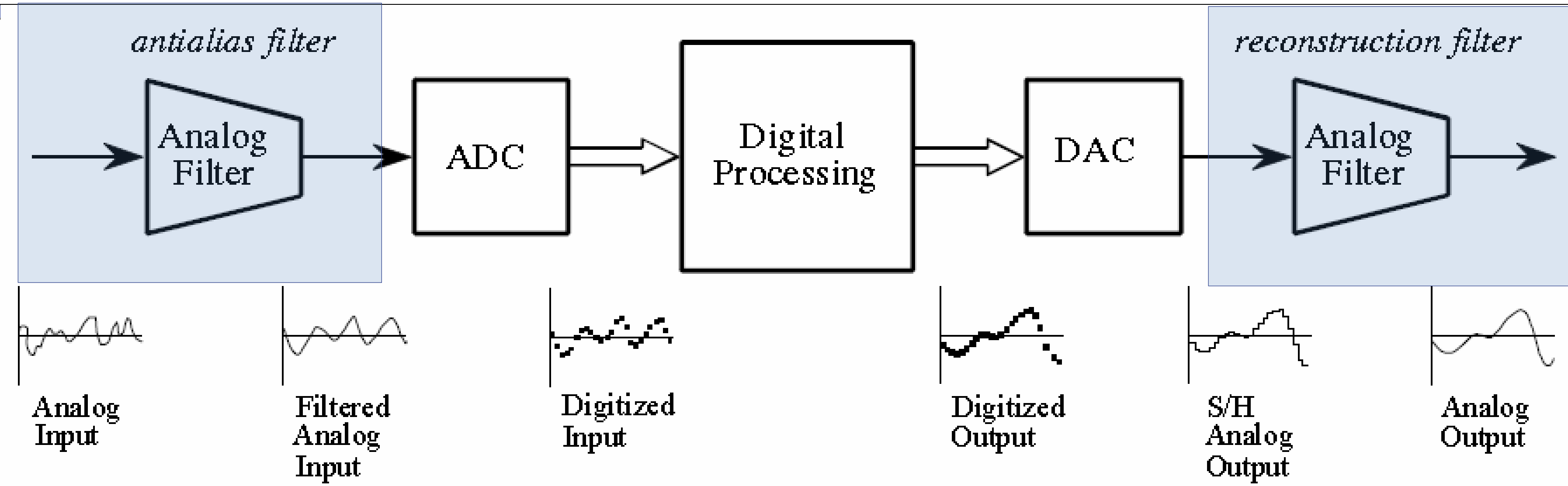


Reconstruction Filter

- Sampling at a rate higher than 2X Nyquist eases the requirements on the analog reconstruction filter



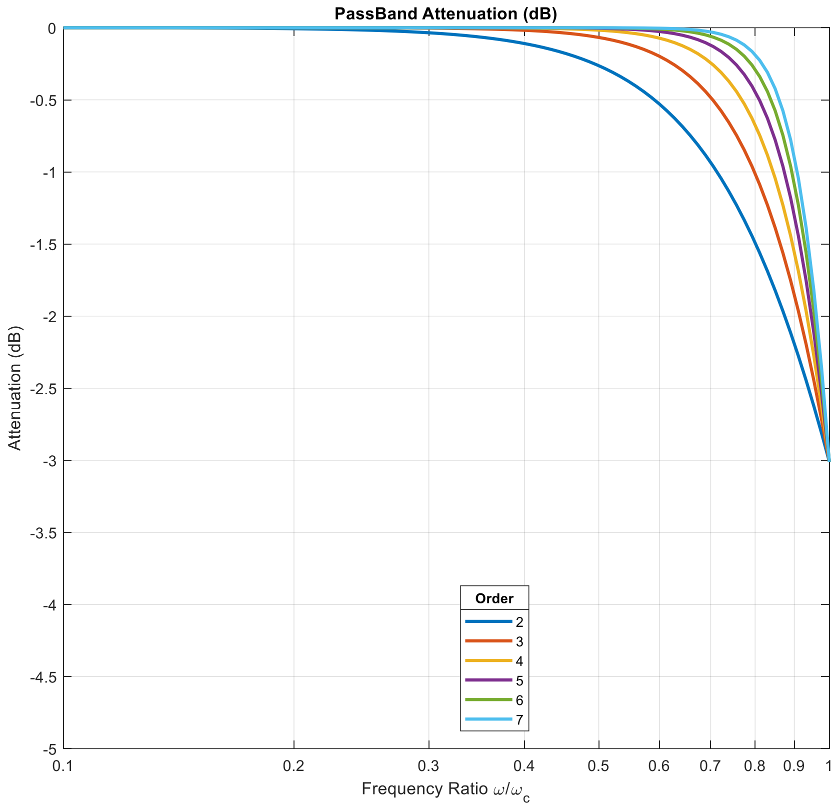
Anti-Alias and Reconstruction Filters

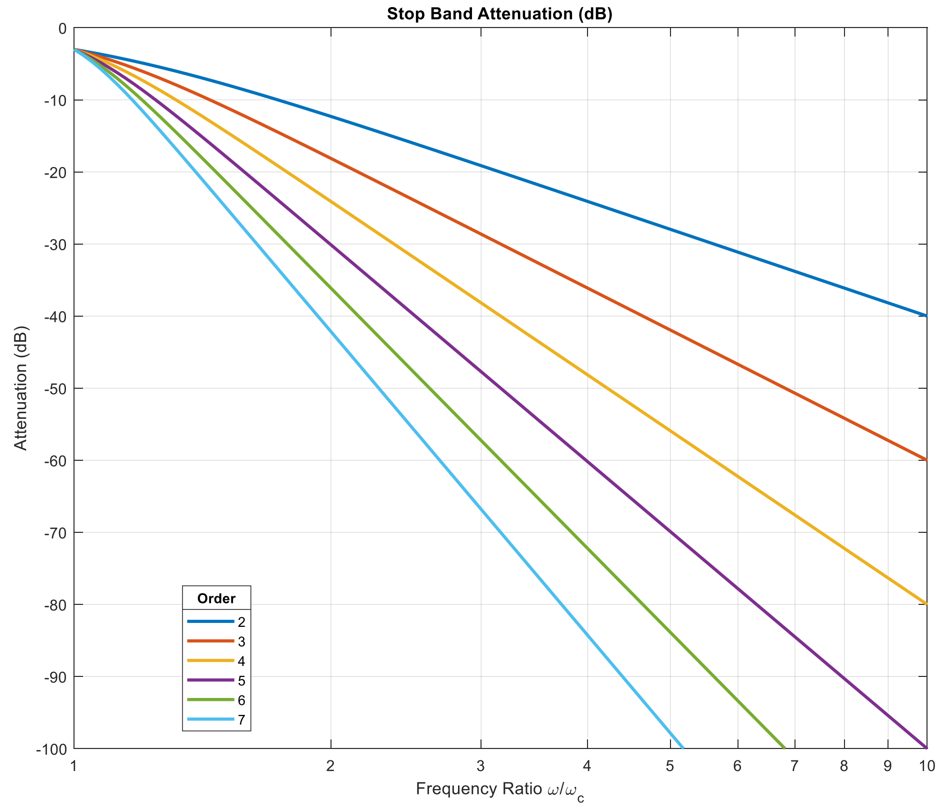


- Three basic types of filters:
  - Chebyshev
    - fastest roll-off in frequency, frequency domain
  - Butterworth
    - maximally flat, frequency domain
  - Bessel
    - Best step response, time domain

Butterworth Filters

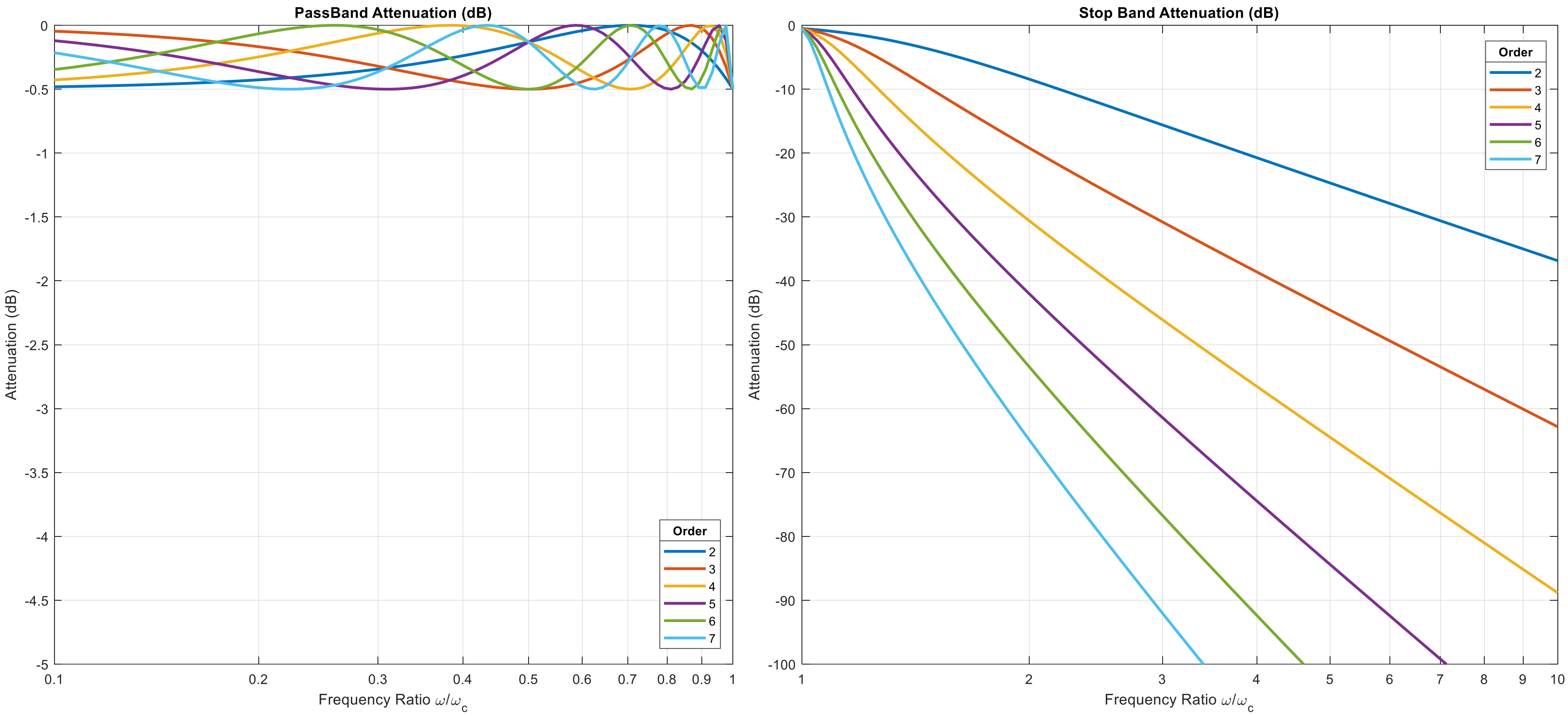
- Passband and Stopband Curves



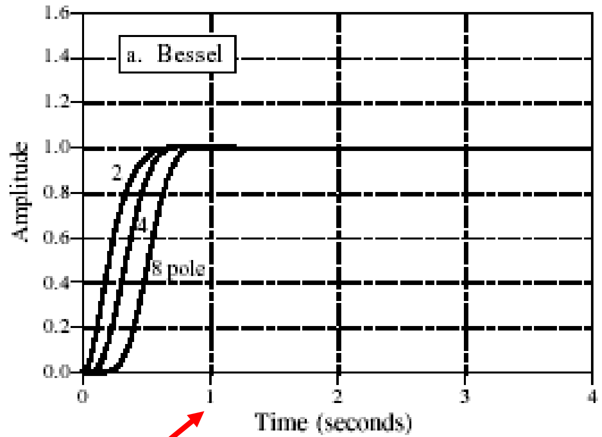


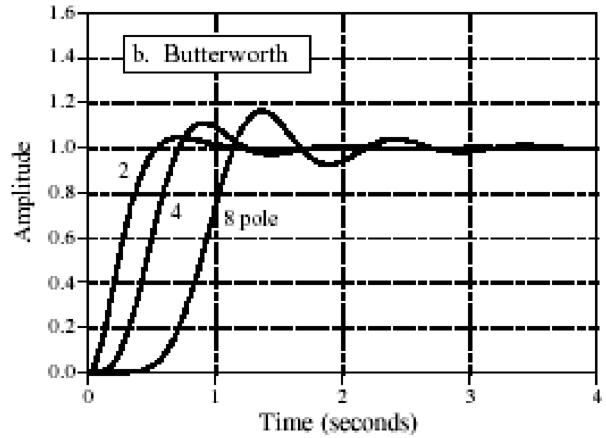
Chebyshev Filters

- Passband and Stopband Curves



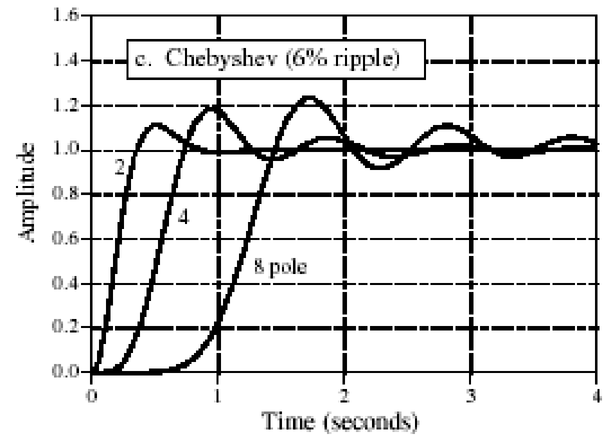
Comparison of Filter Step Response



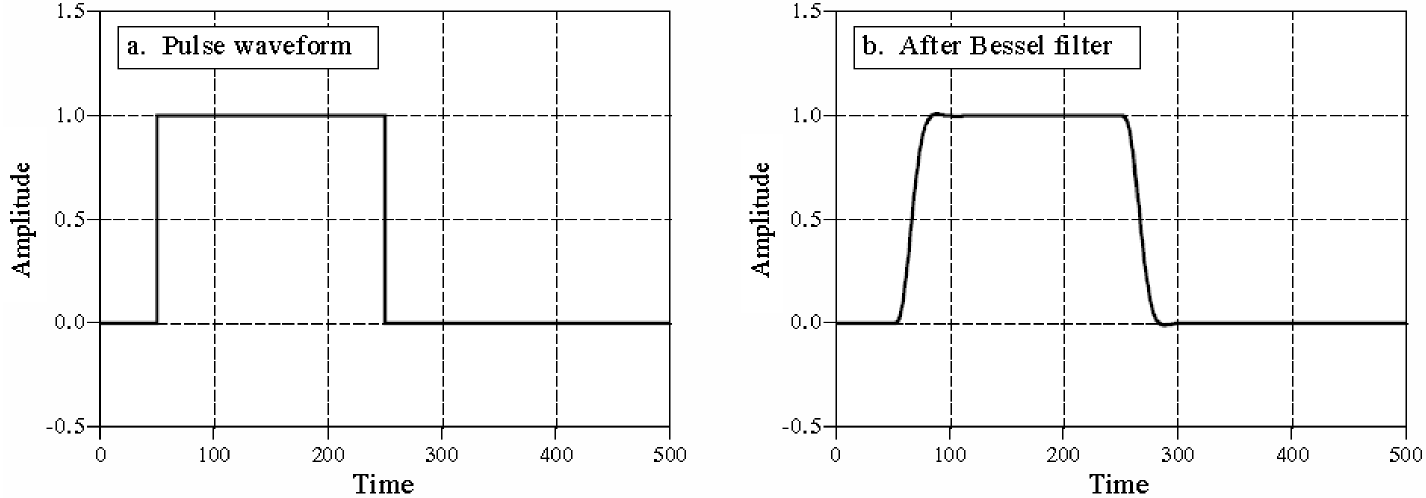


Bessel has good step response –

No significant ringing or overshoot!



Pulse Response



- Depending on the type of signal you may choose one type of filter over the other
- Choose Bessel if you are concerned about overshoot
- Choose Chebyshev if you need more attenuation