Digital Signal Processing

Exam 01 Review
In Class Problems

Exam 01 Format/Logistics

- On-Line 10/1 from 07:00 to 23:59 PM
  - Five problems 25, 10, 20, 25, 20 Points each
  - 3-hour time recommendation

- Exam 01 Practice Problems in the assignment section

Exam 01 Format

- Go to Exam 01 Activation Quiz
- Answer the 1 T/F question
- This will open the assignment and an additional quiz
- Complete the problems
- Enter your answers in the Exam 01 Answers Quiz
  - Most questions auto-graded, 1 manually graded
- Scan your document and post in Assignments
  - You must show your work! Partial credit applied

Exam 01 Content

- Statistics
  - Noise, Signal, SNR
- Sampling
  - Nyquist criteria, Aliasing
- ADC
  - Quantization, quantization noise, oversampling and averaging, dithering
- Fixed and Floating point number systems
  - Representing numbers in each system
- Based on the homework, quizzes and in-class problems

Adding Noise Powers

- In the signals below, the signal of interest is the mean of each signal.
- The noise is the standard deviation.

- Compute the signal to noise ratio in dB for each signal in decibels and the combined

  - Signal 1: $\mu = 1.5V, \sigma = .2V$ Signal 2: $\mu = 4V, \sigma = .55V$
  - Signal 1: $\mu = .2V, \sigma = .05V$ Signal 2: $\mu = 1.3V, \sigma = .05V$

Adding Noise Powers

- Signal 1: $\mu = 1.5V, \sigma = .2V$ Signal 2: $\mu = 4V, \sigma = .55V$

$$SNR_1 = 20\log_{10}\left(\frac{\mu_1}{\sigma_1}\right) = 20\log_{10}\frac{1.5v}{0.2V} = 17.5dB$$

$$SNR_2 = 20\log_{10}\left(\frac{\mu_2}{\sigma_2}\right) = 20\log_{10}\frac{4v}{0.55v} = 17.2dB$$

$$\mu_{12} = \mu_1 + \mu_2 = 1.5V + 4V = 5.5V$$

$$\sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{0.2^2 + .55^2} = .585V$$

$$SNR_{12} = 20\log_{10}\left(\frac{\mu_{12}}{\sigma_{12}}\right) = 20\log_{10}\frac{5.5v}{0.585v} = 19.5dB$$

NOPRINT

Adding Noise Powers

- Signal 1: $\mu = .2V, \sigma = .05V$ Signal 2: $\mu = 1.3V, \sigma = .05V$

$$SNR_1 = 20\log_{10}\left(\frac{\mu_1}{\sigma_1}\right) = 20\log_{10}\frac{0.2v}{0.05V} = 12.4dB$$

$$SNR_2 = 20\log_{10}\left(\frac{\mu_2}{\sigma_2}\right) = 20\log_{10}\frac{1.3v}{0.05v} = 28.3dB$$

$$\mu_{12} = \mu_1 + \mu_2 = 0.2V + 1.3V = 1.5V$$

$$\sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{.05^2 + .05^2} = .0707V$$

$$SNR_{12} = 20\log_{10}\left(\frac{\mu_{12}}{\sigma_{12}}\right) = 20\log_{10}\frac{1.5v}{0.0707v} = 26.5dB$$

SNR

- In the signals below, the signal of interest is the standard deviation of each signal.
- The noise level is also given as a standard deviation.
- Compute the signal to noise ratio for each signal in decibels.
  - $\sigma_{signal} = 0.75V,\ \sigma_{noise} = \ .65\text{V}$
  - $\sigma_{signal} = 1.5V, \sigma_{noise} = \ .18V$

NOPRINT

SNR

- Signal 1 -- $\sigma_{signal} = 0.75V,\ \sigma_{noise} = $ .65V

$$SNR = 10\log_{10}\left(\frac{\sigma_{signal}^2}{\sigma_{noise}^2}\right) = 10\log_{10}\frac{(.75V)^2}{(.65V)^2} = 1.243\ dB$$

- Signal 2 -- $\sigma_{signal} = 1.5V, \sigma_{noise} = .18V$
-

$$SNR = 10\log_{10}\left(\frac{\sigma_{signal}^2}{\sigma_{noise}^2}\right) = 10\log_{10}\frac{(1.5V)^2}{(.18V)^2} = 18.42\ dB$$

ADC Resolution and Quantization Noise

- The noise level of a sensor as measured by its standard deviation is 6mV.
- The sensor is being read by an ADC with a full-scale range of 5V and 10 bits of resolutions.
- What is the total combined noise of the input sensor and the ADC?

ADC Resolution and Quantization Noise

NOPRINT

- Compute the quantization noise of the ADC

$$\sigma_q = 0.29 \times 1\, CV$$

- 1 Code value is found from the ADC range and number of bits

$$1\, CV = \frac{V_{fs}}{2^N - 1} = \frac{5V}{2^{10} - 1} = 4.88\, mV$$

- The quantization noise in equivalent voltage is

$$\sigma_q = 0.29 \times 1\, CV = 0.29 \times 4.88\, mV = 1.415\, mV$$

ADC Resolution and Quantization Noise

NOPRINT

- The input noise expressed in code values is

$$\sigma_{signal} = \frac{6mV}{1CV} = \frac{6mV}{4.88mV} = 1.23\ CV$$

ADC Resolution and Quantization Noise

NOPRINT

- Then the combined noise is units of volts is

$$\sigma_{total} = \sqrt{\sigma_{signal}^2 + \sigma_q^2}$$

$$\sigma_{total} = \sqrt{6^2 mV + 1.415^2 mV} = 6.17\ mV$$

ADC Resolution and Quantization Noise (2)

- The noise level of a sensor as measured by its standard deviation is 2mV. The sensor is being read by an ADC with a full-scale range of 3V, but I have a choice of several ADCs with different resolution. They are 8 bits, 10 bits and 12 bits. I need to keep the quantization noise low enough that I don't increase my input signal by more than 10%. Which ADC should I choose?

ADC Resolution and Quantization Noise

NOPRINT

- The noise levels will add in quadrature. Compute how much noise quantization can be added while keeping the increase to less than 10%

$$\sigma_{total} = \sqrt{\sigma_{signal}^2 + \sigma_q^2}$$

$$\sigma_q = \sqrt{\sigma_{total}^2 - \sigma_{signal}^2}$$

$$\sigma_{total} \leq \sigma_{signal}(1.1)$$

$$\sigma_q \leq \sqrt{\sigma_{total}^2 - \sigma_{signal}^2}$$

$$\sigma_q \leq \sqrt{2.2^2 - 2^2} = 0.917mV$$

ADC Resolution and Quantization Noise

NOPRINT

- Find the ADC resolution (number of bits) to keep the quantization noise less than .917 mV

$$\sigma_q = 0.29 \times 1\ CV \qquad 1\ CV = \frac{V_{fs}}{2^N - 1}$$

$$\sigma_q = 0.29 \times \frac{V_{fs}}{2^N - 1} \leq 0.917\ mV$$

Solve for N

$$N = \log_2\left(\frac{0.29 \times V_{fs}}{0.917\ mV} + 1\right) = 9.89\ bits$$

Use 10 bits

Oversampling Averaging

- The noise level of a sensor as measured by its standard deviation is 15 mV. The sensor is being read by an ADC with a full-scale range of 5V and 8 bits of resolutions. Dither noise with a standard deviation of 15 mV is being added to the input
- I only need to sample the input at a rate of 1kHz, however the ADC system can be sampled as fast as 1 MHz
- If I use oversampling and averaging how fast should I sample the signal to make sure that the total noise of my signal is less than an equivalent value of 1 mV.

Oversampling Averaging

- Compute the total noise input from the sensor, quantization and dither noise.

$$\sigma_q = 0.29 \times 1\,CV \qquad 1\,CV = \frac{V_{fs}}{2^N - 1} = \frac{5\text{V}}{2^8 - 1} = 19.61\text{ mV}$$

$$\sigma_q = 0.29 \times 19.61\,mV = 5.69mV$$

Dither noise $\longrightarrow$ $\sigma_d = 15mV$

$$\sigma_{total} = \sqrt{\sigma_{signal}^2 + \sigma_d^2 + \sigma_q^2} = \sqrt{15^2mV + 15^2mV + 5.69^2mV}$$

$$\sigma_{total} = 21.96\,mV$$

NOPRINT

Oversampling Averaging

- Oversampling and averaging will reduce the standard deviation of my sample average by

$$\sigma_{ave} = \frac{\sigma_{total}}{\sqrt{N}}$$

- I want the σ of the sample averages to be less than 1 mV

$$N = \left(\frac{\sigma_{total}}{\sigma_{ave}}\right)^2 = \left(\frac{21.96\ mV}{1\ mV}\right)^2 = 482.2 \Rightarrow 483\ samples$$

I would need to sample <u>at least</u> 483 kHz and average those samples to achieve the noise requirements

Sampling Theory

- I'm sampling a sine wave of 7 kHz at a sampling rate of 10 kHz. After sampling at what positive frequencies will the first 4 copies of the sinusoids be located?

Sampling Theory

- A sine wave of 7 kHz will have frequency components at both $\pm 7\ kHz$. These tones will repeat every 10 kHz. Adding and subtracting multiples of 10 kHz from each

Aliases of 7 kHz	Aliases of -7 kHz
37	23
27	13
17	3
7	-7
-3	-17
-13	-27
-23	-37

Signal aliased in-band

Floating Point Numbers

- You are coding with single precision floating point numbers. Recall that single precision floating point numbers have 8 bits in the exponent and 23 bits in the mantissa.
- What is the binary representation of 88?
- What is the next largest number that can be represented?
- What is the binary representation of -0.01953125?

NOPRINT

Floating Point Numbers

- What is the binary representation of 88?

$$v = (-1)^S \times M \times 2^{E-127}$$

The sign bit is 0

The exponent of 2 is $exponent = floor(\log_2|v|)$

$$exponent = floor(\log_2|88|) = floor(6.459) = 6$$

Find the value of E

$$E = exponent + 127 = 6 + 127 = 133$$

NOPRINT

Floating Point Numbers

- The Mantissa is

$$M = \frac{abs(v)}{2^{exponent}} = \frac{abs(88)}{2^6} = 1.375$$

- The bits in the mantissa represent the fractional portion of the mantissa using decreasing powers of 2 starting with -1 (i.e. $2^{-1}, 2^{-2}, etc..$).

$$M = 1 + m_{22}(2^{-1}) + m_{21}(2^{-2}) + \cdots$$

$$M = 1 + 0(2^{-1}) + 1(2^{-2}) + 1(2^{-3}) + \cdots$$

NOPRINT

Floating Point Numbers

- The binary value is then

$$\underset{S\,=\,0}{0}\ \underbrace{1000\ 0101}_{E\,=\,133}\ \underbrace{01100000000000000000000}_{M\,=\,1+.375} \qquad 88$$

- The next largest value will occur when the mantissa is incremented by 1 (one LSB)

0 1000 0101 01100000000000000000001

NOPRINT

Floating Point Numbers

- The next largest value will occur when the mantissa is incremented by 1 (one LSB)

0 1000 0101 01100000000000000000001

- 1 LSB is worth 2^{-23}

2^{-23}

$$v = (-1)^S \times M \times 2^{E-127}$$

$$v = (-1)^0 \times (1.375 + 2^{-23}) \times 2^{133-127}$$

$$v = 88.000007629394$$

NOPRINT

Floating Point Numbers

- What is the binary representation of -0.01953125?

$$v = (-1)^S \times M \times 2^{E-127}$$

The sign bit is 1

The exponent of 2 is $exponent = floor(\log_2|v|)$

$$exponent = floor(\log_2|.01953125|) = floor(-5.678) = -6$$

Find the value of E

$$E = exponent + 127 = -6 + 127 = 121$$

NOPRINT

Floating Point Numbers

- The Mantissa is

$$M = \frac{abs(v)}{2^{exponent}} = \frac{|-0.01953125|}{2^{-6}} = 1.25$$

- The bits in the mantissa represent the fractional portion of the mantissa using decreasing powers of 2 starting with -1 (i.e. $2^{-1}, 2^{-2}, etc..$).

$$M = 1 + m_{22}(2^{-1}) + m_{21}(2^{-2}) + \cdots$$

$$M = 1 + 0(2^{-1}) + 1(2^{-2}) + 0(2^{-3}) + \cdots$$

Floating Point Numbers

- The binary value is then

$$\underbrace{1}_{S=1}\ \underbrace{0111\ 1001}_{E=121}\ \underbrace{01000000000000000000000}_{M=1+.25} \qquad -0.01953$$