Homework 2

EEET-427-01: Controls Systems

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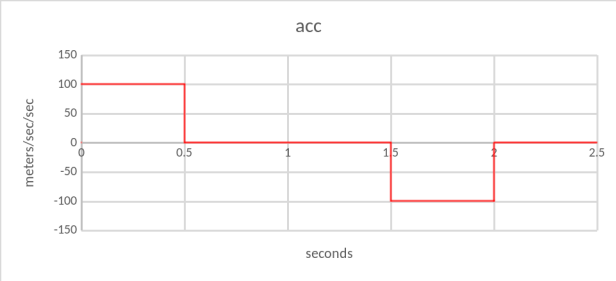
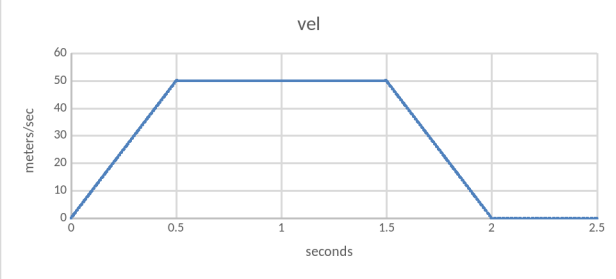
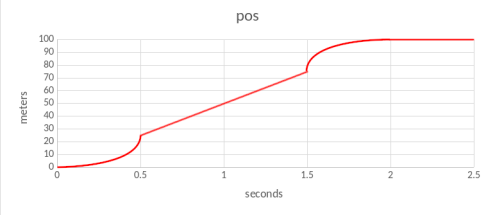
10/02/2021

1 Question 1

Sketch the acceleration and position profiles associated with the given velocity.



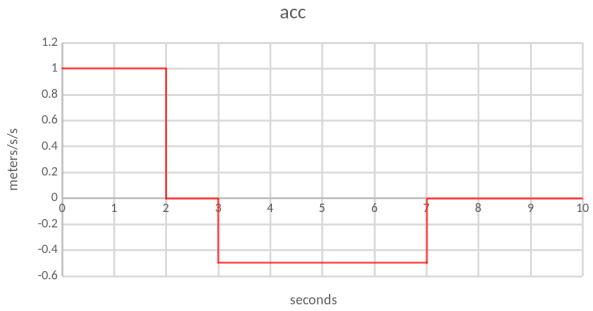


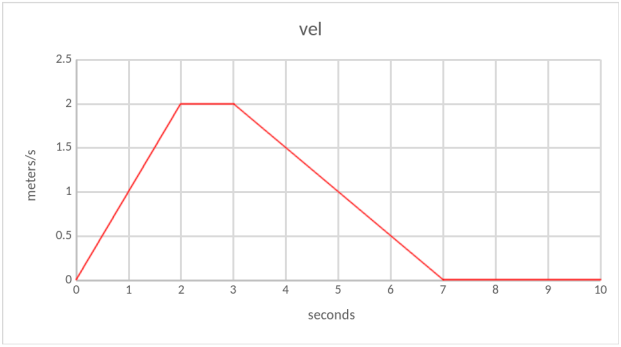


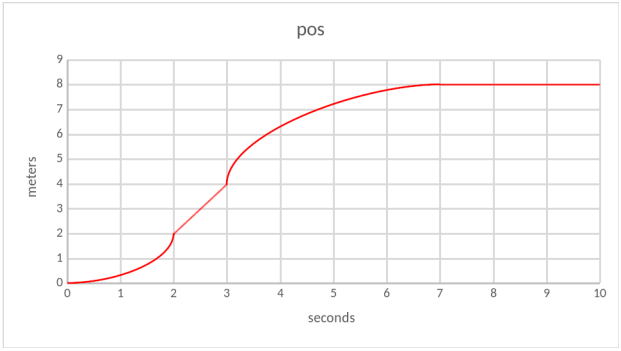
2 Question 2

A force profile is applied that moves a sled from rest with a fixed acceleration of $1.0\frac{m}{s^2}$ to a plateau velocity of $2\frac{m}{s}$, and then back to rest with a fixed acceleration of $-0.5\frac{m}{s^2}$ such that the total distance traveled is $8m$.

Sketch the acceleration,velocity, and position profiles.





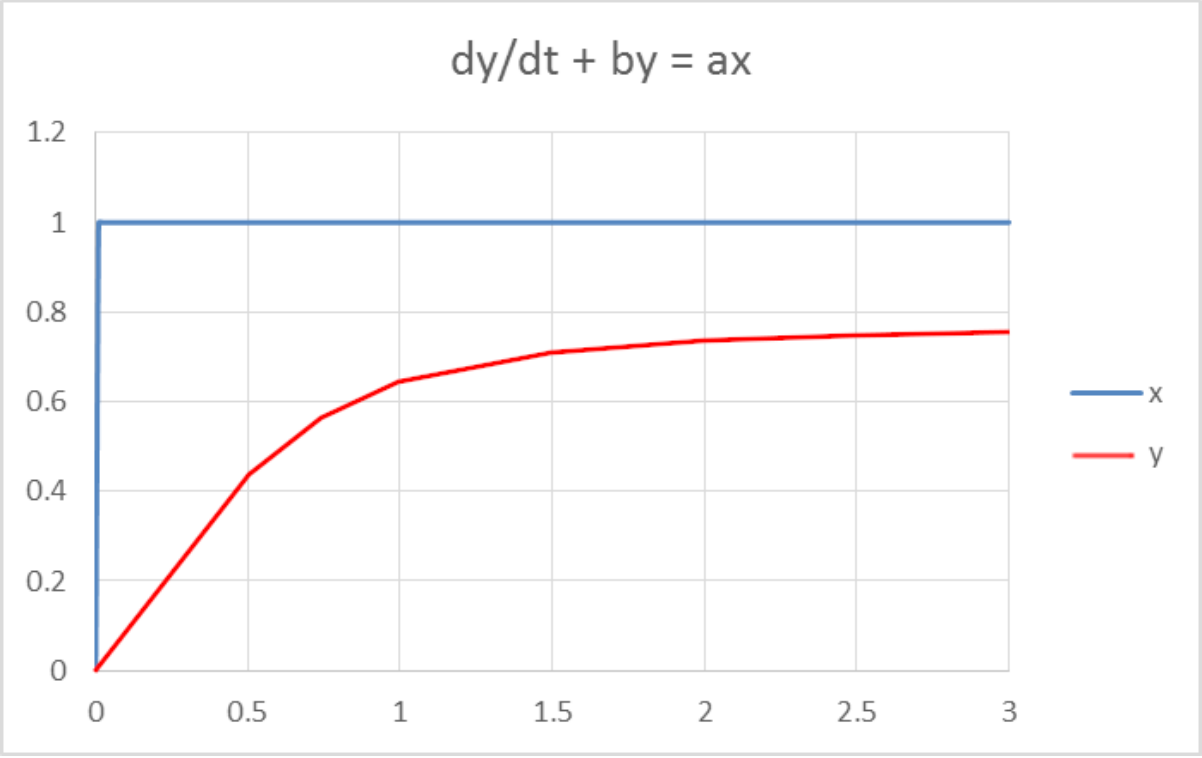


3 Question 3

Sketch the response of $\frac{dy}{dt} + by = ax$ given the inputs and initial conditions.

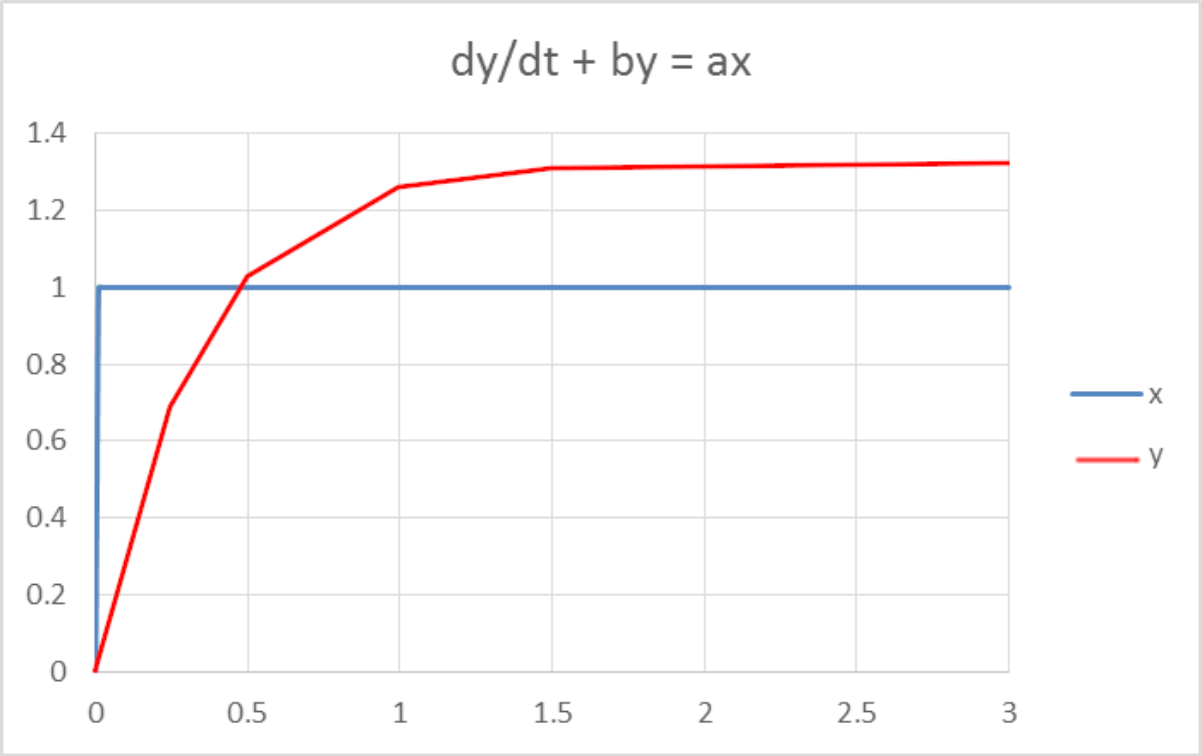
A. $a = 1.5$, $b = 2$. Solve for $t = 0.75$

$$
\begin{aligned}
Y &= \frac{1.5}{s+2}X \\
x(t) &= u(t) \\
X &= \frac{1}{s} \\
Y &= \frac{1.5}{s(s+2)} \\
\frac{1.5}{s(s+2)} &= \frac{A}{s} + \frac{B}{s+2} \\
1.5 &= A(s+2) + Bs \\
2A &= 1.5 \\
A &= \frac{3}{4} \\
B &= -A \\
B &= -\frac{3}{4} \\
y(t) &= \frac{3}{4}(u(t) - e^{-2t}u(t)) \\
y(0.75) &= \frac{3}{4}(1 - e^{-2(\frac{3}{4})}) \\
y(0.75) &\approx 0.58
\end{aligned}
\tag{1}
$$



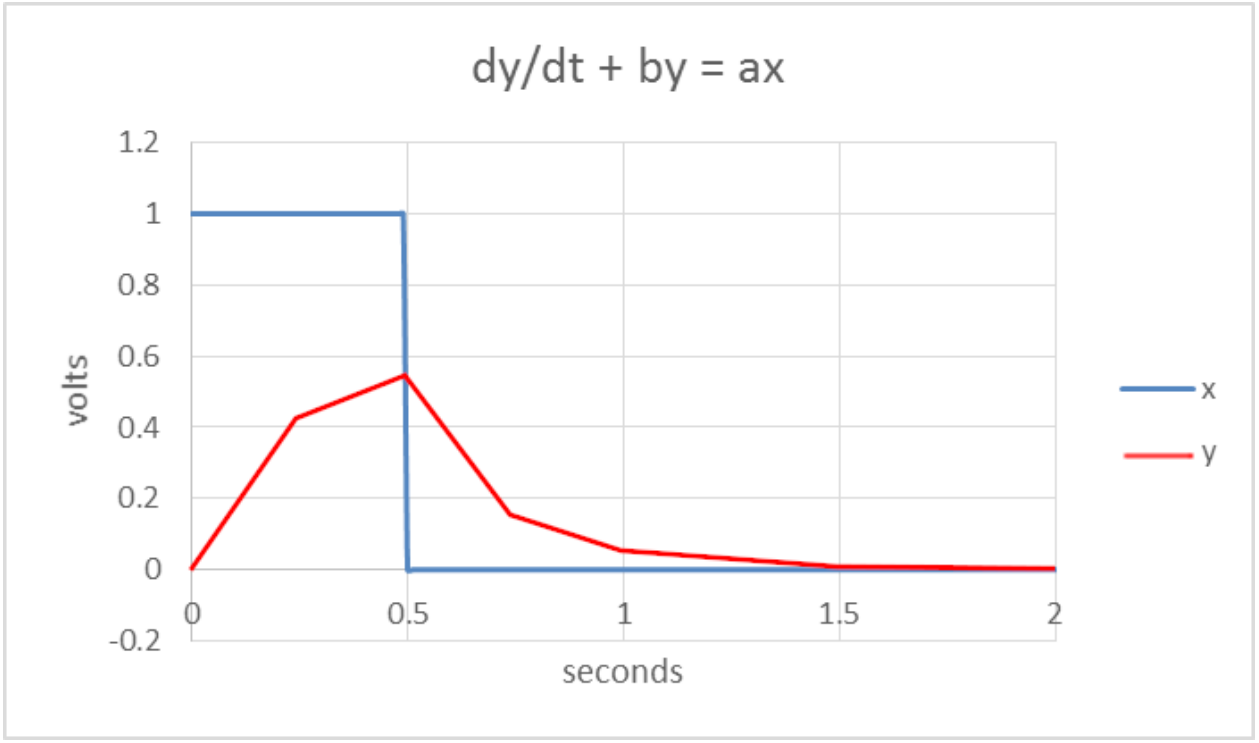
B. $a = 4$, $b = 3$; Solve for $t = 0.25$

$$
\begin{aligned}
Y &= \frac{4}{s+3}X \\
x(t) &= u(t) \\
X &= \frac{1}{s} \\
Y &= \frac{4}{s(s+3)} \\
\frac{4}{s(s+3)} &= \frac{A}{s} + \frac{B}{s+3} \\
4 &= A(s+3) + Bs \\
3A &= 4 \\
A &= \frac{4}{3} \\
B &= -A \\
B &= -\frac{4}{3} \\
y(t) &= \frac{4}{3}(u(t) - e^{-3t}u(t)) \\
y(0.25) &= \frac{4}{3}(1 - e^{-3(\frac{1}{4})}) \\
y(0.25) &\approx 0.7035
\end{aligned}
\tag{2}
$$



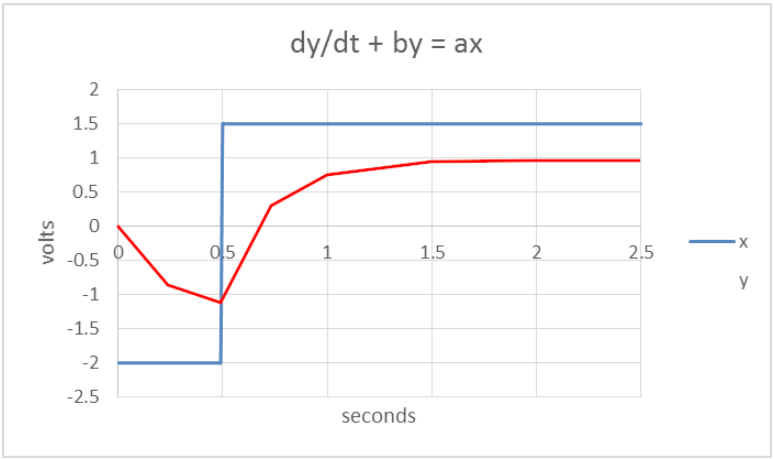
C. $a = 3$, $b = 5$; Solve for $t = 0.25$, $t = 0.75$

$$
\begin{aligned}
Y &= \frac{3}{s+5}X \\
x(t) &= u(t) \\
X &= \frac{1}{s} \\
Y &= \frac{3}{s(s+5)} \\
\frac{3}{s(s+5)} &= \frac{A}{s} + \frac{B}{s+5} \\
3 &= A(s+5) + Bs \\
5A &= 3 \\
A &= \frac{3}{5} \qquad (3) \\
B &= -A \\
B &= -\frac{3}{5} \\
y(t) &= \frac{3}{5}(u(t) - e^{-5t}u(t)) - \frac{3}{5}(u(t-0.5) - e^{-5t}u(t-0.5)) \\
y(0.25) &= \frac{3}{5}(1 - e^{-5(\frac{1}{4})}) \\
y(0.25) &\approx 0.7035 \\
y(0.75) &= \frac{3}{5}((1-1) - (e^{-5t} - e^{-5(t-0.5)})) \\
y(0.75) &= \frac{3}{5}(e^{-5(t-0.5)} - e^{-5t})
\end{aligned}
$$



D. $a = 3$, $b = 5$, Solve for $t = 0.25$, 0.75, given $y(0) = 0$

$$
\begin{aligned}
Y &= \frac{3}{s+5}X \\
x(t) &= -2u(t) + 3.5u(t-0.5) \\
X_1 &= \frac{-2}{s} \\
X_2 &= \frac{3.5}{s} \\
Y_1 &= \frac{-6}{s(s+5)} \\
-6 &= A_1(s+5) + B_1 s \\
5A &= -6 \\
A_1 &= -\frac{6}{5} \\
B_1 &= -A_1 \\
B_1 &= \frac{6}{5} \\
Y_2 &= \frac{10.5}{s(s+5)} \\
10.5 &= A_2(s+5) + B_2 s \\
5A_2 &= 10.5 \\
A_2 &= \frac{21}{10} \\
B_2 &= -A_2 \\
B_2 &= -\frac{21}{10} \\
y(t) &= \frac{6}{5}(e^{-5t}u(t) - u(t)) + \frac{21}{10}(u(t-0.5) - e^{-5(t-0.5)}u(t-0.5)) \\
y(0.25) &= \frac{6}{5}(1 - e^{-5(0.25)}) \\
y(0.25) &= -0.8562 \\
y(0.75) &= \frac{6}{5}(e^{-5(0.75)} - 1) + \frac{21}{10}(1 - e^{-5(0.75-0.5)}) \\
y(0.75) &= 0.3266
\end{aligned}
\tag{4}
$$



E. $a = 3$, $b = 5$, Solve for $t = 0.25$, 0.75, given $y(0) = STEADY\ STATE$

Copied From Part D :

$$
\begin{aligned}
y(t) &= \frac{6}{5}(e^{-5t}u(t) - u(t)) + \frac{21}{10}(u(t-0.5) - e^{-5(t-0.5)}u(t-0.5)) \\
y(t) &= \frac{6}{5} + \frac{21}{10}(u(t-0.5) - e^{-5(t-0.5)}u(t-0.5)) \\
y(0.25) &= \frac{6}{5} \\
y(0.75) &= \frac{6}{5} + \frac{21}{10}(1 - e^{-5(0.75-0.5)}) \\
y(0.75) &= 0.2983
\end{aligned}
\tag{5}
$$