Homework # 4

EEET–427–01:Controls Systems

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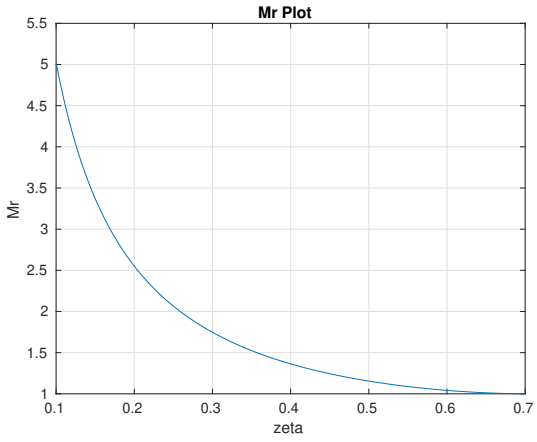
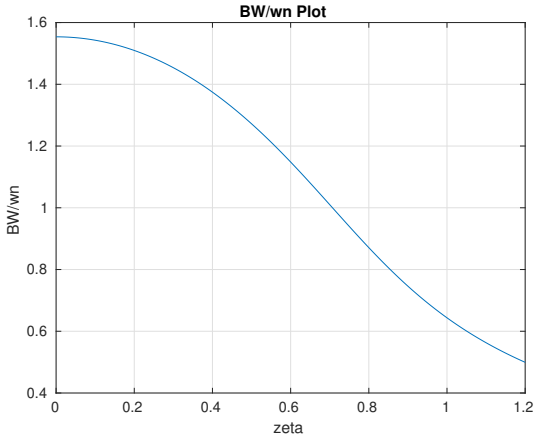
$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \tag{1}$$

$$\frac{BW}{\omega_n} = \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \tag{2}$$

1. Plot a fully labeled M_r as the damping ratio varies from $0.1 - 0.7$.



2. Plot and fully label $\frac{BW}{\omega_n}$ as the damping ratio varies from $0 - 1.2$.



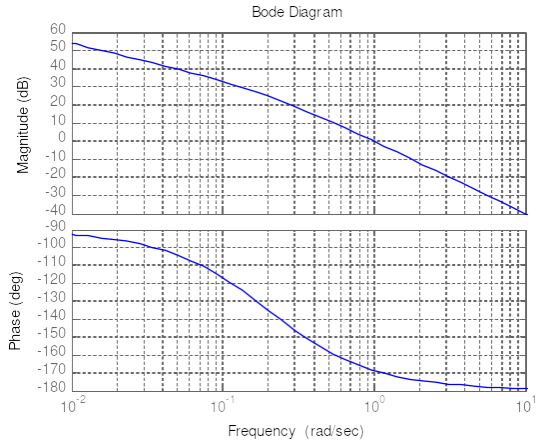


Figure 1: Open Loop Gain Plot

3. What is the phase margin in Figure 1?
   $\approx 10°$.

4. What is the approximate damping ratio, given Figure 1?
   ≈ 0.1.

5. What is the approximate steady state error in Figure 1, assuming $H = 1.0$?

$$\begin{aligned} Steady\ State &= E \\ E &= R - CH \\ E &= R - EGH \\ E + EGH &= R \\ E(1 + GH) &= R \\ \frac{E}{R} &= \frac{1}{1 + GH} \\ GH = 55dB = 10^{\frac{55dB}{20}} &\approx 562.3 \\ \frac{E}{R} &= \frac{1}{1 + 562.3} \\ \frac{E}{R} &\approx 0.001775 \end{aligned} \tag{3}$$

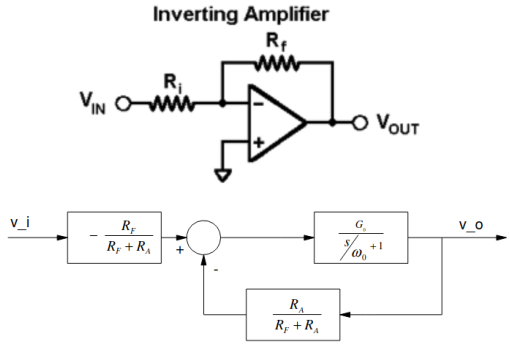
6. What is the closed loop bandwidth of this system in Figure 1, assuming $H = 1.0$?

$$\begin{aligned} BW_{CL} &= 1 \\ \omega &= 1 \end{aligned} \tag{4}$$

7. If you reduce the loop gain by $20dB$, what will the phase margin be? What will the damping ratio be? What will the magnitude peaking ratio be in the closed loop frequency response?

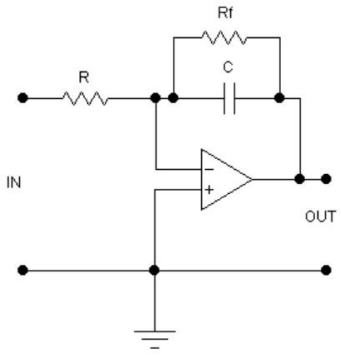
$$\begin{aligned} PM &\approx 5° \\ \zeta &= 0.05 \\ \frac{E}{R} &= \frac{1}{1 + GH} \\ GH = 35dB = 10^{\frac{35}{20}} &\approx 56.23 \\ \frac{E}{R} = \frac{1}{1 + 56.23} &\approx 0.01747 \end{aligned} \tag{5}$$

8. Draw a control system block diagram for simple inverting amplifier op-amp circuit (shown below).

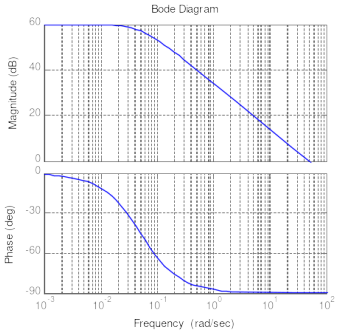


Full disclosure, this is taken from a paper found online from Arizona State University. I can't remember how Op-Amps work, and I ran out of time to try to re-learn the material.

9. Sketch a bode plot of the transfer function of the inverting practical integrator amplifier circuit shown, assuming an ideal op-amp. Use $R_f = 10M\Omega$; $C_f = 2\mu F$; $R_{in} = 10k\Omega$. After you have the sketch, plot the bode plot in MATLAB with the frequency ranging from $10^{-3} - 10^{3}$.



10. At $1 rad/sec$, how much phase does this circuit have, and what is the gain?



$$Gain = 37.5dB;\ Phase - 87.5° \tag{6}$$

11. For the practical Op-Amp differentiator shown, assuming an ideal op-amp, sketch the bode plot over the frequency range $10^{-3} - 10^{3}$. Use $R_f = 100k\Omega$; $C_a = 20\mu F$; $R_a = 1k\Omega$. After you sketch the plot by hand, use matlab to confirm your results.