MATH 181: HOMEWORK 6 SOLUTIONS

3.1: 7, 19, 15, 21, 35, 37, 49, 55, 67, 71
3.2: 3, 15, 17, 23, 27, 29, 33, 43, 49

3.1.

7:

$$\begin{aligned} df &= d(2t^3 - 3t^2 - 4t) \\ &= 2d(t^3) - 3d(t^2) - 4dt \\ &= 6t^2\,dt - 6t\,dt - 4dt \end{aligned}$$

so

$$\frac{df}{dt} = 6t^2 - 6t - 4$$

15:

$$\begin{aligned} dR &= d((3a+1)^2) \\ &= 2(3a+1)d(3a+1) \\ &= 2(3a+1)3da \end{aligned}$$

so $dR/da = 6(3a+1)$

19:

$$\begin{aligned} dy &= d(3e^x + 4x^{-1/3}) \\ &= 3d(e^x) + 4d(x^{-1/3}) \\ &= 3e^x\,dx - \frac{4}{3}x^{-4/3}dx \end{aligned}$$

so $\frac{dy}{dx} = 3e^x - \frac{4}{3}x^{-4/3}$

35:

$$\begin{aligned} dy &= d(x + 2x^{-1}) \\ &= dx + 2d(x^{-1}) \\ &= dx - 2x^{-2}dx \end{aligned}$$

so $\frac{dy}{dx} = 1 - 2x^{-2}$

37: $dy = 4x^3\,dx + 2e^x\,dx$. Localized at the point $(0,2)$, this gives

$$dy = 2\,dx$$

so the tangent line is $y - 2 = 2(x - 0)$.

49: During a small time dt, the particle undergoes a displacement $ds = 3t^2\,dt - 3\,dt$. So the velocity is

$$v = \frac{ds}{dt} = 3t^2 - 3.$$

The during the same interval dt, the velocity undergoes an impulse of $dv = 6t\,dt$, so the acceleration during that interval is $dv/dt = 6t$.

55: We have $dy = (6x^2+6x-12)\,dx$. The tangent line is horizontal if $dy = 0$, so $x^2 + x - 2 = 0$, or $x = -2, 1$. Finally, the points on the curve are $(x, y) = (-2, 21)$, $(x, y) = (1, -6)$

67: Let $P(t) = At^2 + Bt + C$. Then:
- $P(2) = 5$ gives $4A + 2B + C = 5$
- $P'(2) = 3$ gives $4A + B = 3$
- $P''(2) = 2$ gives $2A = 2$.

Solving this system by back-substitution, we find $A = 1$, $B = -1$, $C = 3$

71: No, the left and right derivatives at $x = 1$ do not agree.

3.2. 3, 15, 17, 23, 27, 29, 33, 43, 49

3:

$$\begin{aligned} df &= d(3x^2 - 3x)\,e^x + (3x^2 - 3x)\,d(e^x) \\ &= (6x\,dx - 3\,dx)e^x + (3x^2 - 3x)e^x\,dx \\ &= \left((6x - 3)e^x + (3x^2 - 3x)e^x\right)\,dx. \end{aligned}$$

15:

$$\begin{aligned} dy &= d\left(\frac{t^3 + 3t}{t^2 - 4t + 3}\right) \\ &= \frac{d(t^3 + 3t)(t^2 - 4t + 3) - (t^3 + 3t)d(t^2 - 4t + 3)}{(t^2 - 4t + 3)^2} \\ &= \frac{(3t^2 + 3)(t^2 - 4t + 3)dt - (t^3 + 3t)(2t\,dt - 4\,dt)}{(t^2 - 4t + 3)^2} \\ &= \frac{(3t^2 + 3)(t^2 - 4t + 3) - (t^3 + 3t)(2t - 4)}{(t^2 - 4t + 3)^2}dt \end{aligned}$$

17:

$$\begin{aligned} dy &= d\left(e^p(p + p\sqrt{p})\right) \\ &= d(e^p)(p + p\sqrt{p}) + e^p d(p + p\sqrt{p}) \\ &= e^p\,dp(p + p\sqrt{p}) + e^p\left(dp + \frac{3}{2}p^{1/2}dp\right) \\ &= \left(p + p\sqrt{p} + 1 + \frac{3}{2}p^{1/2}\right)e^p\,dp. \end{aligned}$$

23:

$$\begin{aligned}
df &= d\left(\frac{x^2e^x}{x^2+e^x}\right) \\
&= \frac{d(x^2e^x)(x^2+e^x) - x^2e^x d(x^2+e^x)}{(x^2+e^x)^2} \\
&= \frac{(d(x^2)e^x + x^2 d(e^x))(x^2+e^x) - x^2e^x(2x\,dx + e^x\,dx)}{(x^2+e^x)^2} \\
&= \frac{(2x\,dx\,e^x + x^2e^x dx)(x^2+e^x) - x^2e^x(2x\,dx + e^x\,dx)}{(x^2+e^x)^2} \\
&= \frac{(2x\,e^x + x^2e^x)(x^2+e^x) - x^2e^x(2x + e^x)}{(x^2+e^x)^2}dx
\end{aligned}$$

(further simplification is possible)

33: We have $dy = d(2xe^x) = 2dx\,e^x + 2xe^x\,dx = (2+2x)e^x\,dx$ Localizing at $(x,y) = (0,0)$, this gives

$$dy = (2 + 2\cdot 0)e^0\,dx = 2dx$$

Finally, substituting the displacements $dy = y - 0$ and $dx = x - 0$ gives

$$y = 2x$$

for the tangent line.

35: We have $dy = d((1+x^2)^{-1}) = -(1+x^2)^{-2}d(1+x^2) = -(1+x^2)^{-2}2x\,dx$. We obtain the tangent line by localizing at $(x,y) = (-1, 1/2)$. At that point, we have

$$dy = -(1+(-1)^2)^{-2}2(-1)\,dx = \frac{1}{2}dx$$

. Finally, the tangent line is found by substituting the displacements $dy = y - 1/2$ and $dx = x + 1$:

$$y - 1/2 = \frac{1}{2}(x+1).$$

43: (a) By the product rule for *derivatives*, $(fg)'(5) = f'(5)g(5) + f(5)g'(5) =$ .... (b) By the quotient rule for derivatives,

$$\begin{aligned}
(f/g)'(5) &= \frac{f'(5)g(5) - f(5)g'(5)}{g(5)^2} = ... \\
(g/f)'(5) &= \frac{g'(5)f(5) - g(5)f'(5)}{f(5)^2} = ...
\end{aligned}$$