Math 181: Practice final exam

1. Sketch the graph of a function $y = f(x)$ on the interval $[0, 3]$ satisfying *all* three conditions: $\int_0^3 f(x)\,dx = 5$, $\int_1^3 f(x)\,dx = 1$, and $\int_0^2 f(x)\,dx = 1$.

2. Using geometry (or any other method), calculate

$$\int_{-3}^{0} \left(x + \sqrt{9 - x^2} \right)\, dx$$

3. Calculate the integral $\int_0^2 |x^2 - x| \, dx$.

4. Evaluate the definite integral

$$\int_0^2 \left(\sin\left[(x-1)^2\right] + \cos\left[(x-1)^4\right]\right)(x-1)\,dx$$

using an appropriate substitution. Show your steps.

5. You are given a function such that $\int_0^2 f(x)\,dx = \sqrt{\pi}$. You take the graph of $f(x)$, and subject it to the following transformations: first, you stretch the graph vertically by a factor of 3. Next, you squash the graph horizontally by a factor of $1/2$. Finally, you translate the graph upwards by 2 units. Let $y = g(x)$ be the resulting graph. What is the value of the integral

$$\int_0^1 g(x)\,dx?$$

6. Calculate the integrals

(a) $\displaystyle\int \left(\sqrt{x} - \frac{2}{x} + 1\right) dx$

(b) $\displaystyle\int\limits_0^{\pi} \frac{2x\,dx}{x^2+1}$

(c) $\displaystyle\int_0^1 (x^2+1)(x^3+3x)^{1/2}\,dx$

(d) $\displaystyle\int_0^1 \frac{1+x+x^2}{\sqrt{x}}\,dx$

7. Compute the derivative

$$\frac{d}{dx} \int_{x}^{x^2} \cos(t^2)\, dt.$$

8. Let $F(x) = \displaystyle\int_0^x \frac{4 - t^2}{t^4 + 1}\, dt$. Find the largest interval on which $F(x)$ is an increasing function.

9. A black body in a medium held at absolute zero cools according to Newton's law of cooling, which is that the temperature $T(t)$ decays exponentially in time t. It also radiates a small amount of its heat energy in the form of black body radiation, according to the Stefan–Boltzmann law, which states that the amount of heat radiated as electromagnetic energy is

$$E(t) = \sigma A \int_0^t T(s)^4 \, ds$$

where $\sigma \approx 5.7 \times 10^{-8} J s^{-1} m^{-2} K^{-4}$ is the Stefan–Boltzmann constant, and A is the surface area of the body in m^2. Assume that $A = 2m^2$. Find the rate at which electromagnetic energy radiates (i.e., the luminosity, in Js^{-1}) when the temperature is $100°K$.