Lecture 2: New functions from old

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Preview

- Terms: **polynomial**, **power function**, **rational function**, **transformation**, **translation**, **reflection**, **composite function**
- Skills
  - interpret each of the following transformations of a graph $y = f(x)$ geometrically: $y = f(x) + k$, $y = f(x + k)$, $y = cf(x)$, $y = f(cx)$;
  - given two functions f and g, evaluate the composite function $f(g(x))$
  - given a complicated function, like $\sqrt{x^2 + 1}$, write it as the composite of simpler functions

Polynomials

- A polynomial is a function of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
- The terms $a_0, a_1, \ldots, a_n$ are the coefficients.
- The domain of a polynomial is $(-\infty, \infty)$
- Polynomial of degree 1 is linear, degree 2 is quadratic, degree 3 is cubic
- Any quadratic can be obtained from the basic one $y = ax^2$ by shifting. This quadratic opens up if $a > 0$ and opens down if $a < 0$.

Power functions

- A function of the form $f(x) = ax^p$ is called a *power function*.
- Examples: $f(x) = x^2, -5x^2, x^3, \sqrt{x} (= x^{1/2}), 1/x = x^{-1}$ (reciprocal function)
- Compare and contrast power functions, for different powers, graphically

Rational functions

- A *rational function* is a ratio of two polynomials:

$$f(x) = \frac{p(x)}{q(x)}$$

  where p, q are polynomials.
- The domain is all x where $q(x) \neq 0$
- Example: $f(x) = \frac{2x^4+x+1}{x^2-9}$

Other functions we will study

- Trigonometric functions: $\sin x, \cos x, \tan x$
- Exponential functions: 2^x, $P_0 a^t$, e^x
- Logarithms: $\log x, \ln x$

Shifting and stretching

- What happens to the graph of a function $y = f(x)$ when we add k to the value of $f(x)$?
- What happens to the graph of a function $y = f(x)$ when we multiply f by a constant c?
- What happens the the graph of $y = f(x)$ if we replace x by $x - h$?
- What happens to the graph of a function $y = f(x)$ when we multiply f by a constant c ?
- What happens to the graph of a function $y = f(x)$ when we multiply x by a constant c?
- Examples: $f(x) = (x-1)^2$, $f(x) = x^2 - 6x - 5$ (where is the vertex?), $f(x) = 4x^2 + 1$, $f(x) = (x-1)/(x+1)$

Shifting and stretching: summary

- Replacing $f(x)$ by $f(x) + k$ moves a graph up by k (down if k is negative)
- Multiplying a function by a constant c stretches the graph vertically (if $c > 1$) or shrinks the graph vertically (if $0 < c < 1$). A negative sign (if $c < 0$) reflects the graph about the x-axis, in addition to shrinking or stretching.
- Replacing x by $(x - h)$ moves a graph to the right by h (to the left if h is negative)
- Multiplying x by a constant c compresses the graph horizontally (if $c > 1$) or shrinks it (if $0 < c < 1$). A negative sign (if $c < 0$) reflects the graph about the y-axis, in addition to shrinking or stretching.

Combinations of functions

- Two functions can be added, subtracted, or multiplied together to make a new function:

$$(f+g)(x) = f(x)+g(x), \quad (f-g)(x) = f(x)-g(x)$$
$$(fg)(x) = f(x)g(x)$$

- The domain of these is the intersection of the domains of f and g
- Example: If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, then the domain of $(f+g)(x) = \sqrt{x} + \sqrt{2-x}$ is $[0, 2]$
- Two functions can be divided:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

- The domain of f/g is all points of the intersection of the two domains where $g(x) \neq 0$.
- Example: $f(x) = x^2$, $g(x) = x-1$, then the domain of f/g is all $x \neq 1$, i.e., $(-\infty, 1) \cup (1, \infty)$.

Composite functions

- Functions f and g can be *composed*, denoted $f \circ g$, by "feeding" the output of g to f: $(f \circ g)(x) = f(g(x))$
- Example: If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$, then $(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$

Dependency diagrams

- It is often useful to be able to break up a complicated composite function into simpler parts (e.g., for evaluating them in a scientific calculator).
- Example: If $y = (\sqrt{x} + 1)^{10}$, we can break this up into two parts, illustrated by the diagram:

$$\begin{array}{c} y = u^{10} \\ \Big| \\ u = \sqrt{x} + 1 \\ \Big| \\ x \end{array}$$