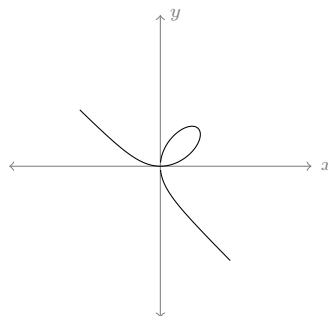# 16. Tangent lines to implicit curves

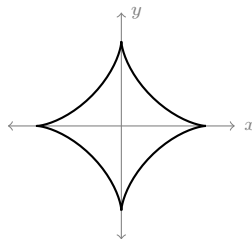**Deliverable.** Submit the *entire worksheet* to be graded. The *last question* is bonus.

1. The folium of Descartes is the curve defined by the equation $x^3 + y^3 = 9xy$.

(a) The point $(x, y) = (2, 4)$ is on the curve. If we increase the value of x from 2 by $dx = 0.1$, estimate the amount dy that we need to increase the y value so that the point $(2 + dx, 4 + dy)$ remains on the curve.

(b) Find the equation of the tangent line through $(2, 4)$.

(c) Find all of the points on the curve with a horizontal tangent line.

(d) Find all of the points on the curve with a vertical tangent line.

2. The equation $x^{2/3} + y^{2/3} = 5$ describes the curve shown below (known as an astroid).

(a) Find the equation of the tangent line to the curve at the point $(8, 1)$.

(b) Take the differential of the equation $x^{2/3} + y^{2/3} = 5$ to find dy/dx in terms of x and y.

(c) If we take the differential of the equation $x^{2/3} + y^{2/3} = 5$, what happens at the x-intercepts? The y-intercepts?

3. The plot shown is for the lemniscate curve, given by the equation $(x^2 + y^2)^2 = x^2 - y^2$. Find the coordinates of the four points along the curve where the slope of the tangent line is zero.