

Lecture 31: Definite integrals by substitution (minimodule)

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*These slides may incorporate material from Hughes-Hallett, et al, "Calculus", Wiley

Skills:

- Evaluate a definite integral $\int_a^b f(x) dx$ by substitution u , first perform the substitution as usual, and then change the limits of integration to the corresponding u values.
- Recognize when a substitution fails to work at first, because the x cannot be eliminated from the integrand (using valid rules for algebra).
- Be able to eliminate x from the integrand by solving the initial substitution $u = \dots$ for x in terms of u .

Definite integrals by substitution

- $\int_0^2 xe^{x^2} dx$
- First find the indefinite integral, then apply the fundamental theorem.
- Alternatively, with the substitution $w = x^2$, $dw = 2x dx$,
 $\int_0^2 xe^{x^2} dx = \int_0^4 e^w \frac{dw}{2}$
- Note that the limits change, because we are integrating with respect to a different variable (w), which covers the range from $w = 0$ to $w = 4$.

Examples

- $\int_0^{\pi/4} \frac{\tan^3 \theta}{\cos^2 \theta} d\theta$
- $\int_1^3 \frac{dx}{5-x}$

More complicated substitutions

- $\int \sqrt{1 + \sqrt{x}} dx$
- $\int (x + 7) \sqrt[3]{3 - 2x} dx$
- Conclusion: Sometimes we have to solve for x in terms of u to eliminate all of the x 's in the integrand.