Math 181, Fall 2018 Handout: "There is no chain rule."

Here is a selection of problems from section 3.4. You can use the 100 line CAS I wrote:
https://repl.it/@jholland/Derivative-rules

32 $F(t) = \dfrac{t^2}{\sqrt{t^3+1}}$. Find $F'(t)$.

$$\begin{aligned}
dF &= \frac{d(t^2)\sqrt{t^3+1} - t^2 d(\sqrt{t^3+1})}{(\sqrt{t^3+1})^2} && \text{quotient rule}\\
&= \frac{2t\,dt\,\sqrt{t^3+1} - t^2\frac{1}{2\sqrt{t^3+1}}d(t^3+1)}{(\sqrt{t^3+1})^2} && \text{power rule } (\times 2)\\
&= \frac{2t\,dt\,\sqrt{t^3+1} - t^2\frac{1}{2\sqrt{t^3+1}}3t^2\,dt}{(\sqrt{t^3+1})^2} && \text{sum rule+power rule}\\
&= \frac{2t\,\sqrt{t^3+1} - t^2\frac{1}{2\sqrt{t^3+1}}3t^2}{(\sqrt{t^3+1})^2}dt && \text{pulling out a \textit{common factor}}
\end{aligned}$$

so

$$F'(t) = \frac{dF}{dt} = \frac{2t\,\sqrt{t^3+1} - t^2\frac{1}{2\sqrt{t^3+1}}3t^2}{(\sqrt{t^3+1})^2}.$$

36 Find the derivative of $y = x^2e^{-1/x}$.

$$\begin{aligned}
dy &= d\left(x^2e^{-1/x}\right)\\
&= d(x^2)e^{-1/x} + x^2 d(e^{-1/x}) && \text{product rule}\\
&= 2x\,dx\,e^{-1/x} + x^2e^{-1/x}d(-1/x) && \text{power rule+}e^u\text{ rule}\\
&= 2x\,dx\,e^{-1/x} + x^2e^{-1/x}(1/x^2)dx && \text{power rule}\\
&= \left(2x\,e^{-1/x} + x^2e^{-1/x}(1/x^2)\right)dx && \text{pulling out a \textit{common factor}}
\end{aligned}$$

so $dy/dx = 2xe^{-1/x} + e^{-1/x}$

42 Find the derivative of $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$.

$$\begin{aligned}
dy &= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}d\left(x + \sqrt{x+\sqrt{x}}\right) && \text{power rule}\\
&= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(dx + d\sqrt{x+\sqrt{x}}\right) && \text{sum rule}\\
&= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(dx + \frac{1}{2\sqrt{x+\sqrt{x}}}d(x+\sqrt{x})\right) && \text{power rule}\\
&= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(dx + \frac{1}{2\sqrt{x+\sqrt{x}}}\left(dx + \frac{1}{2\sqrt{x}}dx\right)\right) && \text{sum rule + power rule}\\
&= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)\right)dx && \text{collecting } dx
\end{aligned}$$

So,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1 + \frac{1}{2\sqrt{x}}\right)\right).$$