11. Review for exam 1

1. Find all solutions x to the equation $2^{x^2} = 3^{x+1}$. You may leave your answer in terms of natural logarithms.

2. Compute the limits:

   (a) $\displaystyle \lim_{x \to \infty} \frac{x - 2}{x^2 - 1}$

   (b) $\displaystyle \lim_{x \to 1^+} \frac{x - 2}{x^2 - 1}$

   (c) $\displaystyle \lim_{x \to \infty} \frac{3x^2 - 12}{x^2 - 3x + 2}$

   (d) $\displaystyle \lim_{x \to 2} \frac{3x^2 - 12}{x^2 - 3x + 2}$

   (e) $\displaystyle \lim_{x \to \sqrt{2}} \frac{3 - \sqrt{5x^2 - 1}}{x^2 - 2}$

(f) $\displaystyle\lim_{x\to\infty} \frac{1 - \sqrt{4x^2 + 1}}{3x - 2}$

3. This problem concerns the function $f(x) = \frac{x+1}{x+2}$.

(a) Find the *domain of* $f(x)$.

(b) Find any *vertical asymptotes* of $y = f(x)$.

(c) Find any *horizontal asymptotes* of $y = f(x)$.

(d) Compute $\displaystyle\lim_{x\to-2^+} \frac{x+1}{x+2}$.

(e) Compute $\displaystyle\lim_{x\to-2^-} \frac{x+1}{x+2}$.

(f) Compute $\displaystyle\lim_{x\to\infty} \frac{x+1}{x+2}$.

(g) Find the inverse function of $f(x)$.

(h) Find the range of $f(x)$.

4. Consider the polynomial $f(x) = x^2(x-2)^3(x-1)(x+1)^2$. Determine the local behavior of $f(x)$ near each of the zeros $x = -1, 0, 1, 2$. Sketch a graph of the function, showing the correct end behavior.

5. In 2006, the population of Iceland was 300,000. In 2016, the population is 330,000. Assume that the population grows exponentially.

   (a) Write the exponential model $P = P_0 a^t$ that best fits this data. (The variable t can be in years from 2006.)

   (b) Use your model to estimate the population of Iceland in 2026.

   (c) Find how long it will take the population to double.

6. Suppose we start with the graph of the function $y = f(x)$. How would we sketch the graph of $y = 2f(x - 3) + 1$? Give an example.

7. (a) State the intermediate value theorem.

   (b) The function $f(x) = x^3 + \sin x$ is known to be continuous. Prove that a solution of the equation $f(x) = 1$ exists.

8. Give an example of a rational function $f(x)$ satisfying *all* of the following conditions:

   - $\lim\limits_{x \to \infty} f(x) = 1$
   - $\lim\limits_{x \to 1} f(x) = -\infty$
   - $f(0)$ is undefined, but $\lim\limits_{x \to 0} f(x) = 2$.