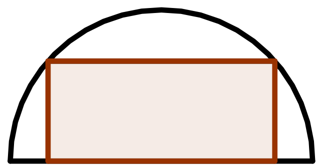
Final exam practice workshop

1. Find the largest rectangle that can be incribed, as shown, in a semicircle of radius 1.



2. Suppose that $\int_0^3 f(x)\,dx = 3$, $\int_1^3 f(x)\,dx = -2$, and $\int_0^2 f(x)\,dx = 1$.

   Find $\int_1^2 f(x)\,dx$.

3. Using geometry, find

$$\int_{-4}^{4} \left(1 + \sqrt{16 - x^2}\right)\,dx$$

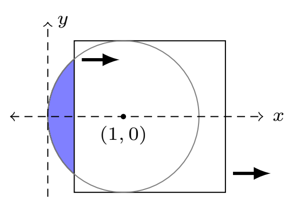
4. Evaluate the definite integral

$$\int_{-1}^{1} \left(1 + \frac{x}{\pi + x^2 + x^4}\right)\,dx$$

   (Hint: $u = x^2$.)

5. Find $\frac{d}{dx}\int_0^x \sin(t^3)\,dt$.

6. A circular pipe has a radius of 1 inch, but flow through the pipe is blocked by a valve. When the valve is open (either fully or partially, as shown below) the flow is proportional to the opened area.



   When the left-edge of the valve is at x, the flow is

$$f(x) = 2k\int_0^x \sqrt{1-(w-1)^2}\,dw \quad \frac{\text{in}^3}{\text{sec}}$$

   where k is a constant of proportionality. Suppose the valve is sliding to the right at $0.25\ \frac{\text{in}}{\text{sec}}$. What's the rate of change in the flow through the pipe when $x = 1.4$in?

7. Find the integrals

(a) $\int_0^4 |2x - x^2| \, dx$

(b) $\int (x^2 - 2x + 1) \, dx$

(c) $\int_0^1 \frac{dx}{x+1}$

(d) $\int_{-1}^1 (x+1)\sqrt{x^2 + 2x + 1} \, dx$

(e) $\int \frac{dx}{1+4x^2}$ (hint: $u = 2x$)