Math 181, fall 2018: Practice Exam 1

Name:____

This is the first exam. You may not use any devices (calculators, smartphones, ipods, etc.) Notes and other textual materials are not permitted. There are 11 questions, with several parts. You have 120 minutes.

1	/12	7	/12
2	/24	8	/12
3	/12	9	/12
4	/12	10	/12
5	/30	11	/6
6	/20		
Total	/110		/54

1. Determine if each of the following equations has a (real) solution x. If there is, find the solution. If not, say why not. If your answer is a whole number, write it as a whole number. Otherwise, you may leave your answer in terms of natural logarithms.

(a) $e^{x+1} = 2^{2x-5}$

(b) $3^x = -3$

(c) $\ln(2x-1) = 2\ln(x)$

2. This problem concerns the function $f(x) = \frac{4x+1}{x-3}$.

(a) Find the *domain of* $f(x)$.

(b) Find any *vertical asymptotes* of $y = f(x)$.

(c) Find any *horizontal asymptotes* of $y = f(x)$.

(d) Compute $\lim\limits_{x \to 3^+} \dfrac{4x+1}{x-3}$.

(e) Compute $\lim_{x \to 3^-} \frac{4x+1}{x-3}$.

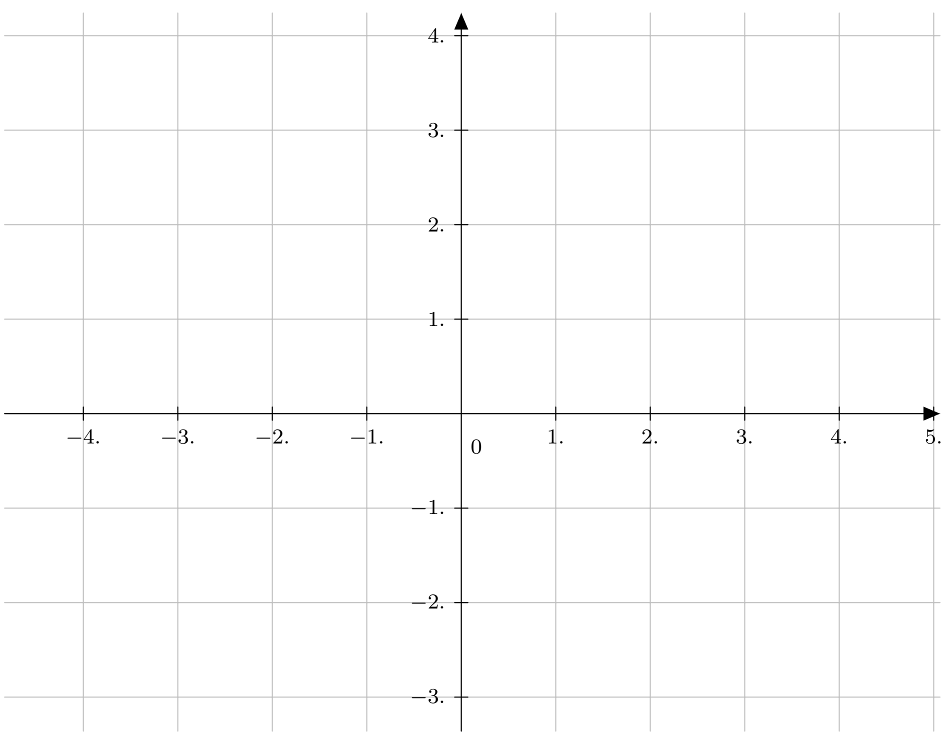
(f) Compute $\lim_{x \to \infty} \frac{4x+1}{x-3}$.

(g) Find the inverse function of $f(x)$.

(h) Find the range of $f(x)$.

3. Consider the polynomial $f(x) = (x+3)(x+1)^2(x-1)^3(x-3)$.

- Determine the local behavior of $f(x)$ near each of the zeros $x = -3, -1, 1, 3$.
- Sketch a graph of the function, showing the correct end behavior.
- Also, determine if this an odd function.



4. Find an exponential function $f(x) = C \cdot b^x$ that satisfies $f(2) = 2$ and $f(4) = 3$.

5. Consider the rational function

$$f(x) = \frac{(x-1)^2}{(x^2-1)(x+2)^2}.$$

(a) Compute the following limits:

(i) $\lim_{x \to 1^-} f(x)$

(ii) $\lim_{x \to 1^+} f(x)$

(iii) $\lim_{x \to 1} f(x)$

(iv) $\lim_{x \to -1^+} f(x)$

(v) $\lim_{x \to -1^-} f(x)$

(vi) $\lim_{x \to -1} f(x)$

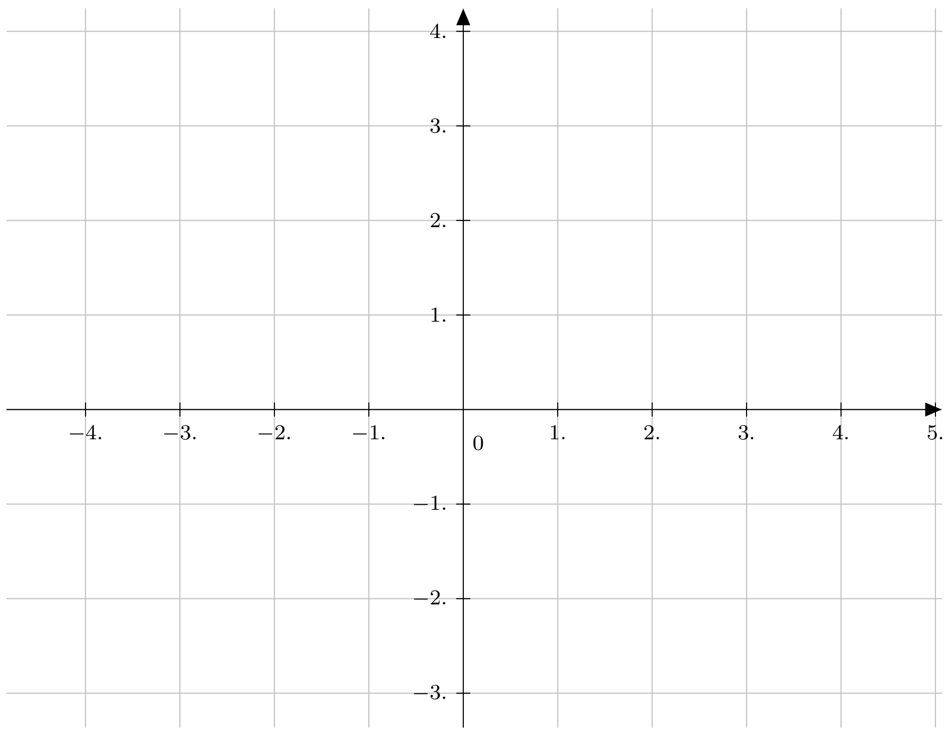
(vii) $\lim_{x \to -2^+} f(x)$

(viii) $\lim_{x \to -2^-} f(x)$

(ix) $\lim_{x \to -2} f(x)$

(b) Find all horizontal asymptotes of $f(x)$.

(c) Sketch the graph of $y = \frac{(x-1)^2}{(x^2-1)(x+2)^2}$, showing the correct behavior at the zeros, poles, removable singularities, and proper end behavior.



6. Evaluate the limits. If they are infinite, write $+\infty, -\infty$ where appropriate.

(a) $\displaystyle \lim_{x \to 2} \frac{\sqrt{x^2+5}-3}{x-2}$

(b) $\displaystyle \lim_{x \to \infty} \sin x$

(c) $\displaystyle \lim_{x \to \infty} \frac{\sqrt{x^2+3}}{2x-1}$

(d) $\displaystyle \lim_{t \to 0} \frac{(2t+1)^{-1}-1}{t}$

(e) $\displaystyle \lim_{x \to -2} \frac{x+2}{x^2+x-2}$

7.

- In complete sentences, state the intermediate value theorem.
- Without using a calculator, find numbers a and b (with justification) so that a solution to the equation $x^2 + \sin(x) = 1$ is guaranteed to exist within the interval $a \leq x \leq b$. (You may assume without proving it that $f(x) = x^2 + \sin(x)$ is a continuous function.)

8. You have the following *limit laws*, assuming that the limits on the right hand side exist, and are finite real numbers:

Law 1: $\lim_{x\to\infty} (f(x) + g(x)) = \lim_{x\to\infty} f(x) + \lim_{x\to\infty} g(x)$

Law 2: $\lim_{x\to\infty} (f(x) - g(x)) = \lim_{x\to\infty} f(x) - \lim_{x\to\infty} g(x)$

Law 3: $\lim_{x\to\infty} cf(x) = c \lim_{x\to\infty} f(x)$ if c is constant

Law 4: $\lim_{x\to\infty} c = c$ if c is constant

Law 5: $\lim_{x\to\infty} [f(x)g(x)] = \lim_{x\to\infty} f(x) \lim_{x\to\infty} g(x)$

Law 6: $\lim_{x\to\infty} [f(x)/g(x)] = \dfrac{\lim_{x\to\infty} f(x)}{\lim_{x\to\infty} g(x)}$ provided $\lim_{x\to\infty} g(x) \neq 0$.

Law 7: $\lim_{x\to\infty} \dfrac{1}{x^n} = 0$ if $n > 0$.

Using only valid algebraic steps, and these laws, evaluate the limit:

$$\lim_{x\to\infty} \frac{2x^2 - x + 1}{-3x^2 + 5x}$$

carefully annotating each step in the derivation of this limit, by citing the appropriate limit laws. (For credit, you *must give* correct justifications.)

9. (a) Explain the squeeze theorem. Support your explanation with graphs and/or examples as needed.

(b) Compute the limit $\lim_{x\to 0^+} \left[\sqrt{x} \cdot \left(\pi + \sin(1/x)\right)\right]$, with justification.

10. Beginning with the graph of $y = x^2$, we perform some geometric operations. For (a)–(f), pick the correct formula from the second column.

(a) The graph is reflected in the line $y = x$.

(b) The graph is moved to the left by one unit, and then stretched horizontally by a factor of 3.

(c) The graph is moved vertically upwards by one unit, and then compressed horizontally by a factor of 1/3.

(d) The graph is moved one unit to the left, and then reflected in the y-axis.

(e) The graph is moved to the left by one unit, and then stretched vertically by a factor of 3.

(f) The graph is moved to the left by one unit, and then reflected in the x-axis.

(1) $y = 3(x-1)^2$

(2) $y = (x-1)^2/3$

(3) $y = 3(x+1)^2$

(4) $y = (x/3+1)^2$

(5) $y = (3x-1)^2$

(6) $y = (x+1)^2/3$

(7) $y = -(x+1)^2$

(8) $y = (-x+1)^2$

(9) $y = -x^2$

(10) The resulting graph is not a function.

(11) None of the above.

11. $\tan(\arcsin(x)) =$ (**Select all that apply.**)

(a) $\sin(\arctan(x)) + \pi$

(b) $\frac{\sqrt{1-x^2}}{x}$

(c) $\sqrt{1-x^2}$

(d) The tangent of an angle whose sine equals x.

(e) $\frac{1}{\sqrt{1-x^2}}$

(f) The sine of an angle whose tangent equals x.

(g) $\frac{x}{\sqrt{1-x^2}}$

(h) None of the above