Lecture 19: Derivatives of implicit functions, logarithms, and inverse trig functions

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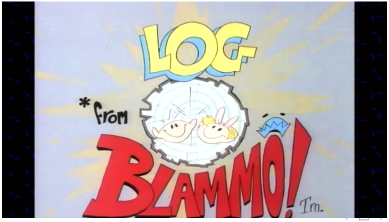

Preview

- Terms: **Implicit function**, **relative differential**, *logarithmic derivative*
- Concepts:
  - The differential of the natural logarithm is $d(\ln u) = \frac{du}{u}$ (the *relative differential of u*).
  - The equation $du = u\, d\ln u$ is useful for calculations where u has lots of products or exponentials in it (*logarithmic differentiation*)
  - The differential of the arctangent is $d(\arctan u) = \frac{du}{1+u^2}$
  - The differential of the arcsine is $d(\arcsin u) = \frac{du}{\sqrt{1-u^2}}$
- Skills:
  - To find the equation of the tangent line to a curve given implicitly, like $x^2 + xy = 1$, at a point (x_0, y_0), take the d of both sides, then set $x = x_0$, $y = y_0$, and $dx = x - x_0$, $dy = y - y_0$.
  - To find the slope dy/dx to the tangent line of an implicit curve like $x^2 + xy = 1$: (1) take the differential of both sides, (2) solve for dy, (3) divide through by dx.

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- An *explicit function* is one of the form $y = f(x)$, where y is solved for in terms of x.
- For example, $y = 1/x - x^2$ is an explicit function.
- An *implicit function* is a relation between the x and y variables in which y is not isolated on one side.
- For example, $x^2 + xy = 1$ is an implicit function.
- Sometimes it is possible to solve an implicit function for y, thus converting it to an explicit function.
- But this is not always desirable. E.g.: The unit circle $x^2 + y^2 = 1$. If we try to solve, we get $y = \pm\sqrt{1 - x^2}$, which is rather awkward.
- It may not even be possible. For example $ye^{-y^2} = x$ cannot be solved for y explicitly.

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Example

Find dy/dx if $x^2 + xy = 1$.

Note: Stewart says "use implicit differentiation". Equivalent to the class way, but the class way is better.

- Take the differential of both sides:

$$d(x^2 + xy) = d(1) \implies 2x\,dx + x\,dy + y\,dx = 0$$

- This is a linear equation for dy, which we solve and simplify (factor):

$$x\,dy = -2x\,dx - y\,dx \implies dy = \frac{-2x - y}{x}\,dx$$

- Finally, dividing by dx gives

$$\frac{dy}{dx} = \frac{-2x - y}{x}$$

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Theorem

$d \ln x = \frac{dx}{x}$

Proof.

Let $y = \ln x$. We want to find dy/dx. Remember that $y = \ln x$ means that $x = e^y$. Taking differentials of both sides gives $dx = e^y\, dy = x\, dy$. So $\frac{dx}{x} = dy$ as required. □

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Derivatives of mixed exponentials

- The power rule says $d(x^a) = ax^{a-1}dx$ for constant a
- The rule for exponentials says $d(a^x) = (\ln a)a^x dx$ for constant a
- What about $d(x^x)$?
- Note that it's not $x^{x-1}dx$ because the exponent isn't constant. Also, it's not $(\ln x)x^x\,dx$ because the base isn't constant.
- Instead a trick is to write it as $x^x = (e^{\ln x})^x = e^{x\ln x}$
- Now we can use the chain rule:

$$d(x^x) = d(e^{x\ln x}) = e^{x\ln x}d(x\ln x)$$

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Logarithmic differentiation (handout)

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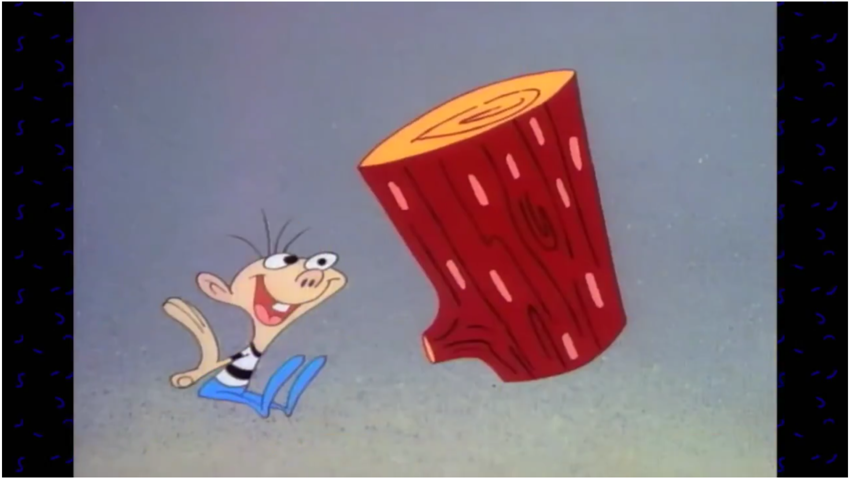

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Derivative of the arctangent

Theorem

$d \arctan x = \frac{dx}{1+x^2}$

Proof.

Remember that $y = \arctan x$ means $x = \tan y$. So $dx = d \tan y = \sec^2 y \, dy$. Solving for dy gives $dy = \frac{dx}{\sec^2 y}$. From the triangle $\sec^2 y = 1 + x^2$. So $dy = \frac{dx}{1+x^2}$. $\square$

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