Math 181, Fall 2018 Handout: Limit laws

Guess the limit

Here are some sample values of a function:

x	$f(x)$
2.	0.25
1.1	0.460829
1.01	0.49586
1.001	0.499584
1.0001	0.499958
1.00001	0.499996
0.99999	1.99999
0.9999	1.9999
0.999	1.999
0.99	1.99
0.9	1.9
0.	1.

Determine

$$\lim_{x\to 1^-} f(x)$$

and

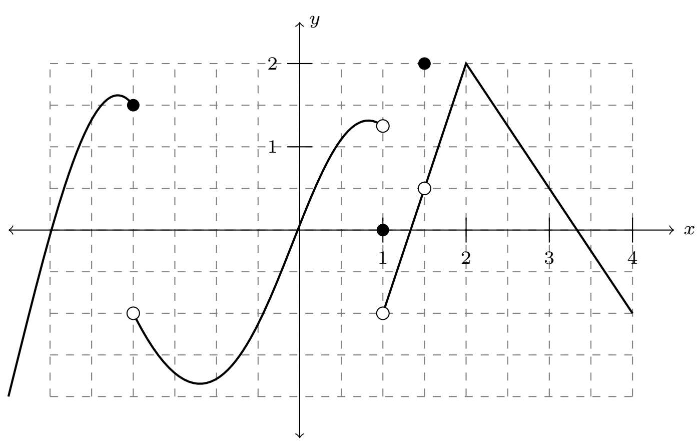
$$\lim_{x\to 1^+} f(x).$$

Also decide if the limit $\lim\limits_{x\to 1} f(x)$ exists.

Guess the limits

The figure below depicts the graph of f. Use it to calculate the following values.

1. $\lim_{x\to -2^-} f(x)$
2. $\lim_{x\to -2^+} f(x)$
3. $\lim_{x\to 1^-} f(x)$
4. $\lim_{x\to 1^+} f(x)$
5. $\lim_{x\to 3^-} f(x)$
6. $\lim_{x\to 3^+} f(x)$
7. $\lim_{x\to 1.5^+} f(x)$
8. $\lim_{x\to 1.5^-} f(x)$



Limit laws

Assuming all the limits on the right hand side exist:

1. If b is a constant, then $\lim_{x\to c}(bf(x)) = b\lim_{x\to c} f(x)$ (contant multiple law)
2. $\lim_{x\to c}(f(x)g(x)) = \lim_{x\to c} f(x)\lim_{x\to c} g(x)$ (product law)
3. $\lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$ (sum law)
4. $\lim_{x\to c}(f(x)/g(x)) = \lim_{x\to c} f(x)/\lim_{x\to c} g(x)$, provided $\lim_{x\to c} g(x) \neq 0$ (quotient law)
5. $\lim_{x\to c}[f(x)]^n = [\lim_{x\to c} f(x)]^n$ if n is a positive integer (power law)
6. $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)}$ provided $f(x) > 0$ for n even (root law)
7. $\lim_{x\to c} x = c$, $\lim_{x\to c} k = k$ for any constant k.
8. $\lim_{x\to\infty} \frac{1}{x^n} = 0$ for $n > 0$

Theorem 1. *If $f(x)$ is a polynomial or rational function and $x = c$ is in the domain of f, then*

$$\lim_{x\to c} f(x) = f(c).$$

Theorem 2. *A limit exists if and only if the left and right hand limits are equal:*

- $\lim_{x\to c} f(x) = L$ *means that* $\lim_{x\to c^+} f(x) = L = \lim_{x\to c^-} f(x)$

Theorem 3 (Squeeze theorem)**.** *Suppose that $f(x), g(x), h(x)$ are three functions, and the following two conditions hold:*

- $f(x) \leq g(x) \leq h(x)$ *for all x on the domain*
- $\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$.

Then $\lim_{x\to c} g(x) = L$ as well.

(The squeeze theorem is also true for one-sided limits.)