34. Riemann sums and sigma notation

1. Write out the terms for the sum $\sum_{n=1}^{8}(-1)^n 2^{n-2}$. Then evaluate the sum.

2. Compute each of the sums:

   (a) $\sum_{n=1}^{8}(n-(n-1))$

   (b) $\sum_{n=1}^{17}(n-(n-1))$

   (c) Based on your answers to (a) and (b), try to determine the value of the sum $\sum_{n=1}^{2017}(n-(n-1))$.

3. If r_1, $r_2 ... r_{50}$ represent the revenues earned from a company's sales force in each state, interpret the expression $\sum_{k=1}^{50} r_k$.

4. (a) A car's velocity increases, and we measure it at two second intervals:

Time (sec)	0	2	4	6	8	10
Velocity (ft/sec)	20	30	38	44	48	50

   We don't know how fast the car was moving at every instant, so we can't find it exactly. Give an overestimate and underestimate of the distance, using a left/right sampled Riemann sum. How big is the spread?

   (b) Measuring the velocity at one second intervals instead gives:

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (ft/sec)	20	26	30	34	38	41	44	46	48	49	50

   Answer the same questions as (a).

   (c) How much has the spread narrowed?

5. The rate at which a tank leaks depends on how much is in it (more fluid ⇒ more pressure ⇒ faster loss). Suppose the leak in a specific tank is described by the following measured data:

Time (sec)	0	1.3	4.1	6	7.2	8
Rate (L/sec)	20	14	12	9	7	6

   (a) Use right-sampling to estimate the amount of fluid leaked in the first 8 seconds.

   (b) Is your answer from part (a) an overestimate or an underestimate? Explain how you can tell.

6. Suppose a car traveling at 90 kph slows to a full stop in 9 seconds. During that time, its velocity is $v(t) = 90 - 36000t$ kilometers per hour, where t is measured in hours. Estimate the distance the car traveled while stopping by using a left-sampled Riemann sum with 10 intervals.

7. Suppose f is a positive-valued, decreasing function over $[a, b]$. Chris calculates a left-sampled Riemann sum with 10 subintervals, and Pat calculates a right-sampled Riemann sum with 10 subintervals. Who has the larger number, and why?

8. Consider the integral $\int_1^2 (4 - 2x)\, dx$.

   (a) Write down the Riemann sum that approximates this integral using $n = 4$ subintervals, sampled at the right endpoints.

   (b) Write down the Riemann sum that approximates this integral using $n = 4$ subintervals, sampled at the left endpoints.

(c) Which of these is larger?

(d) Decide based on the graph of $f(x) = 4 - 2x$ whether $LEFT(n)$ and $RIGHT(n)$ are greater or smaller than that actual value of the integral $\int_1^2 (4 - 2x)\,dx$.

9. The figure below depicts the heart rate (beats per minute) of a person during a workout.

(a) Use a right-sampled Riemann sum with 4 subintervals to estimate the number of heartbeats that occurred between 10 and 18 minutes into the workout.

(b) Use a left-sampled Riemann sum with 10 subintervals to estimate the number of heartbeats that occurred between 10 and 20 minutes into the workout.