# 27. Optimization, part 2

1. Suppose a natural resource (e.g., gas or oil) is located 1,000 feet below the surface, inside a layer of hard rock. Above the hard rock is a region of "softer" rock, that's easier to drill through. Directly above the resource, the soft rock extends 500 feet down from the surface, and the boundary between the two layers extends linearly from west to east with a slope of $1/2$, as seen in the diagram below. Your company's equipment can drill at a rate of 50 feet per day in the soft rock, and 30 feet per day in the hard rock. The company's plan is to drill straight downward through the soft rock, and once the drill meets the hard rock, to proceed directly toward the resource. Where should the drilling begin in order to minimize the drilling time?
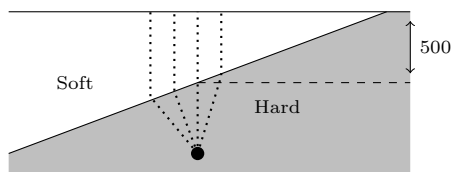
2. Suppose the sun's angle of inclination is $\theta = 30°$ (see diagram below), and a 10-inch book is standing on a table. The book tips over, and its shadow changes length as it falls to the table. Using θ to denote the angle between the book and the table, and x as the length of the book's shadow, determine (a) a formulation for $\frac{dx}{d\theta}$, and (b) the maximum length of the shadow.
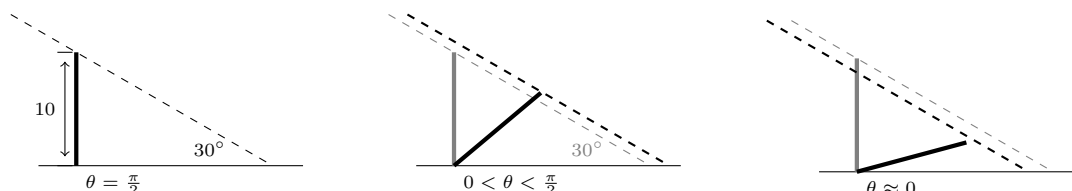
3. Suppose light has to travel through two different mediums (e.g., air and glass, or air and water) in order to arrive at a detector. The source is a distance of A from the interface, the detector is a distance of B, and they are separated by a distance of L along the (straight) interface of the media, as depicted in the diagram below.
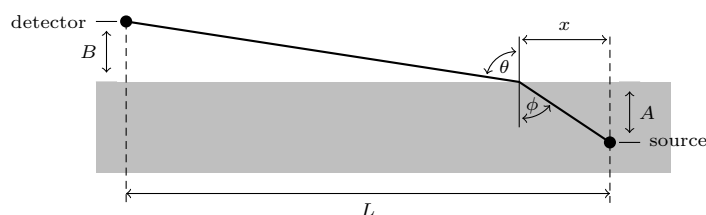
   (a) Suppose the speed of light is v_a in the lower medium, and is v_b the upper medium. If x is the horizontal distance that the light travels in the lower medium, write the length of time it takes light to travel from the source to the detector as a function of x.

   (b) Fermat's Principle says that *light always travels along the path of least time*. Use this principle to show that $(\sin\theta)/v_b = (\sin\phi)/v_b$, which is known as *Snell's Law*.