26. Optimization

Refer to the handout for a guide to the process below.

1. Find the dimensions of a rectangle of given (constant) perimeter P that encloses the largest area.

   Draw a picture and choose variables.

   What is the objective function?

   What is the constraint?

   Solve the constraint and substitute back into the objective function.

   Find the stationary points of the objective function.

   Analyze the stationary points.

2. Find the point on the line $y = 2x + 3$ that is closest to the origin. (The square of the distance to the origin is $x^2 + y^2$.)

   Draw a picture and choose variables.

   What is the objective function?

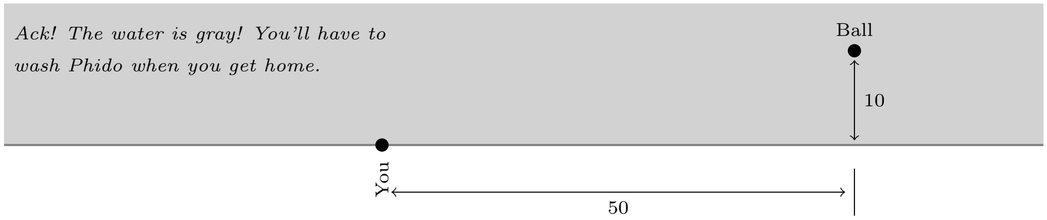
   What is the constraint?

   Solve the constraint and substitute back into the objective function.

   Find the stationary points of the objective function.

   Analyze the stationary points.

3. Suppose you're walking along the straight edge of a lake with your enthusiastic dog, Phido. When you throw a ball 50 feet forward, and 10 feet out into the water, Phido wants to get to it in the least amount of time. He can run at 4 feet per second on land, and can swim at 1/2 foot per second. How far along the shore should Phido run before diving into the lake?



What is the objective function?

What is the constraint?

Solve the constraint and substitute back into the objective function. (Note: This will involve square roots.)

Find the stationary points of the objective function. (The derivative will require the use of the chain rule. Also, the algebra is tricky: to solve an equation like $2 - \frac{x}{\sqrt{1-x^2}} = 0$, subtract the constant from both sides and then square both sides of the equation, clear denominators and rearrange to make it a quadratic to which you can apply the quadratic formula.)