1. Localization of polynomials

Instructional objectives: Given the equation of a polynomial function $p(x)$ in factored form, sketch by hand the graph of the polynomial showing its qualitative behavior near each of the zeros, and the behavior for large x.

1. Sketch the following graphs (separately): $y = x + 1$, $y = (x - 1)^2$, $y = x^3$.

2. Let $f(x) = (x^5 - 4x^4 + 4x^3)(x - 3)$.

   (a) What are the zeros of $f(x)$?

   (b) Take the factored equation of $f(x) = x^3(x - 2)^2(x - 3)$. Substitute $x = 0$ into all but the first factor (which is zero). What is the resulting cubic polynomial? Sketch the graph. This approximates how the graph of $f(x)$ will look for values of x near zero.

   (c) Repeat this for the zero $x = 2$. Substitute $x = 2$ into each term of the factored equation of $f(x)$, except the second factor (which is zero). What is the resulting quadratic polynomial? Graph it. This approximates how the graph of $f(x)$ should look for values of x near two.

(d) Repeat this for the last root $x = 3$.

(e) Now "interpolate". Draw a bunch of little graphs representing the local behavior at each of the roots and connect them in a consistent way.

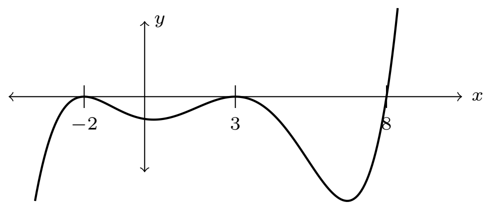
3. Without the aid of a graphing utility, sketch a graph of each of the following. The x-intercepts of each graph should be clearly identified.

   (a) $y = x - 3$, and $y = x + 1$, and $y = (x - 3)(x + 1)$ on the same axes

   (b) $y = (x + 1)^2$ and $y = (x - 3)(x + 1)^2$ on the same axes

   (c) $y = x^3$ and $y = x^3(x - 3)(x + 1)^2$ on the same axes

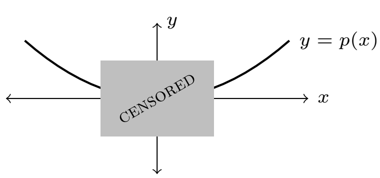
4. Consider the curve shown below.



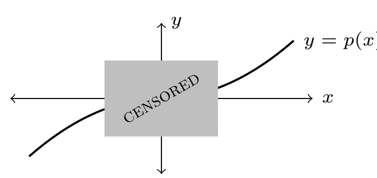
(a) If the curve depicts a polynomial of degree 5, write a factored form of polynomial.

(b) If the curve depicts a polynomial of degree 7, write a factored form of polynomial.

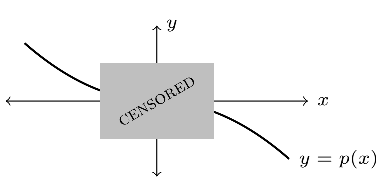
5. For each of the graphs below, determine whether the degree of the corresponding polynomial is even or odd, and whether the leading coefficient is positive or negative.



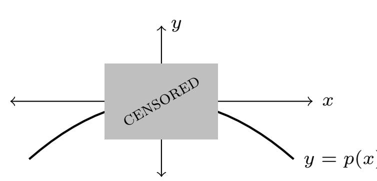
(a)



(c)



(b)



(d)