MATH-181/181A Practice Core

Print Name: ______________________________

Instructor: ______________________________

- This practice exam is intended to familiarize you with the format and style of the Core exam in calculus. It is not intended to serve as an exhaustive list of topics. The course outline, found at the web site for the School of Mathematical Sciences, includes a list of topics common to all sections of the course. The Core exam will focus on employing the skills and concepts on that list.
- Like this practice test, the actual Core exam will have a cover sheet with a bulleted list of informational items. It will also include the **statement of academic honesty** shown below:

  By signing my name below, I assert that I have neither received nor given assistance in solving the problems contained herein.

  Signature: ______________________________

  you are expected to sign before submitting your bubble sheet and exam packet to your instructor.
- During the actual Core, you will receive a packet like this one, and a separate bubble sheet on which to mark your answers. You **must use a no. 2 pencil** to enter your answers on the bubble sheet.
- **Calculators are prohibited** on the final exam.
- You will have **60 minutes** to complete the Core portion of the final exam, which will consist of about 20 multiple-choice questions.
- Record your answers in the appropriate boxes below.

1	2	3	4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21	22

1. Which of the following is $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$?
   (a) $2h$
   (b) $2x$
   (c) 1
   (d) 4
   (e) The limit is undefined.

2. Determine $\lim_{x\to 0} \frac{e^x-1}{x}$.
   (a) 0
   (b) e
   (c) 1
   (d) ∞
   (e) This limit does not exist.

3. Suppose $f(x) = 2x\cos(x)$. Determine the value of $f'(0)$.
   (a) 0
   (b) 1
   (c) -1
   (d) 2
   (e) -2

4. Determine a general formula for $\frac{dy}{dx}$ when $y = \frac{1}{1-x}$.
   (a) $\frac{1}{1-x}$
   (b) $\frac{1}{x-1}$
   (c) $-\frac{1}{(1-x)^2}$
   (d) $\frac{1}{(1-x)^{-1}}$
   (e) $\frac{1}{(1-x)^2}$

5. Suppose $f(x) = \frac{4x-1}{2x+3}$. Determine a formula for the inverse function, $f^{-1}(x)$.

   (a) $f^{-1}(x) = \frac{3x+1}{4-2x}$
   (b) $f^{-1}(x) = \frac{3x+1}{4+2x}$
   (c) $f^{-1}(x) = \frac{2x+3}{4x-1}$
   (d) $f^{-1}(x) = \frac{3x-1}{4+2x}$
   (e) $f^{-1}(x) = \frac{4x+3}{2x-1}$

6. The table below provides information about the function f and g, and their derivatives. Use it to determine the derivative of $f(g(x))$ with respect to x at $x = 1$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	2	-2
2	-2	3	3	-3
3	4	0	4	0
4	3	1	1	-1
5	2	3	5	2

   (a) 8
   (b) 4
   (c) 2
   (d) -6
   (e) -8

7. Which of the following describes the line that's tangent to the curve $y = e^{2x-4}$ at $x = 2$?

   (a) $y - 1 = e(x - 2)$
   (b) $y + 1 = 2(x + 2)$
   (c) $y - 2 = 2(x - 1)$
   (d) $y - 1 = x - 2$
   (e) $y - 1 = 2(x - 2)$

8. Suppose $f(x) = \frac{x}{x+1}$. Then the second derivative of f with respect to x is...

   (a) $f''(x) = -\frac{2}{(x+1)^3}$
   (b) $f''(x) = \frac{2}{(x+1)^3}$
   (c) $f''(x) = -\frac{1}{x+1}$
   (d) $f''(x) = \frac{1}{x+1}$
   (e) $f''(x) = \frac{1}{(x+1)^2}$

9. Suppose that air is being released from a spherical balloon at a rate of 100 cm^3/sec. Which of the following is the instantaneous rate at which the radius of the balloon is changing when the radius is 5 cm? (The volume and radius of a sphere are related by $V = \frac{4}{3}\pi r^3$.)

   (a) $\frac{1}{\pi}$ cm/sec
   (b) $-\frac{1}{\pi}$ cm/sec
   (c) $\frac{1}{25\pi}$ cm/sec
   (d) $-\frac{1}{25\pi}$ cm/sec
   (e) $-\frac{1}{100\pi}$ cm/sec

10. The point $(1, 2)$ is on the curve described by $x^2 + xy + y^2 = 7$. Determine the value of $\frac{dy}{dx}$ at that point.

   (a) $\frac{2}{5}$
   (b) $-\frac{2}{5}$
   (c) $-\frac{3}{5}$
   (d) $\frac{4}{5}$
   (e) $-\frac{4}{5}$

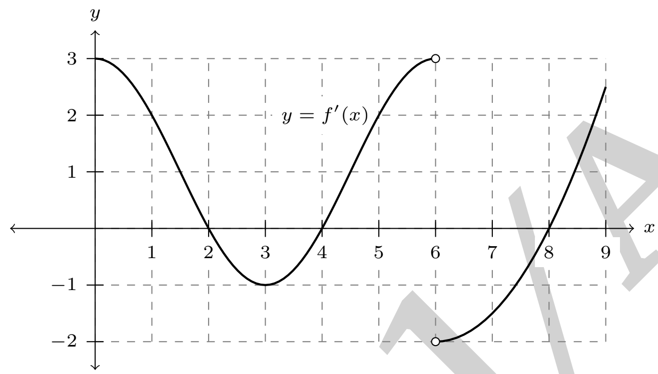
11. Consider the function $f(x) = x^2 + 3x$ on the interval $[1, 3]$. The Mean Value Theorem guarantees that there is a number c in the interval such that $f'(c) = \cdots$

   (a) 14
   (b) 9
   (c) 7
   (d) $\frac{1}{9}$
   (e) $\frac{1}{14}$

12. Suppose $f(x) = \frac{1}{3}x^3 - x^2 - 24x$. Then f is decreasing on which of the following intervals?

   (a) $(-\infty, 5)$
   (b) $(-6, -4)$
   (c) $(-6, 4)$
   (d) $(-4, 6)$
   (e) $(4, \infty)$

13. The **derivative** of f is graphed below, $y = f'(x)$. Which of the following statements about the function f is true?



(a) The function $f(x)$ is increasing on $(0, 2)$.

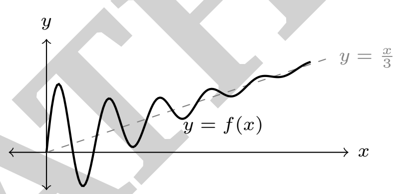
(b) The function $f(x)$ is increasing on $(3, 4)$.

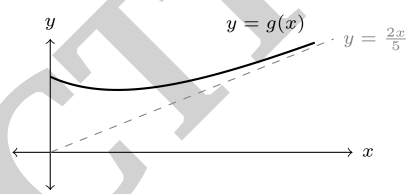
(c) The function $f(x)$ has a local (relative) minimum at $x = 3$.

(d) The function $f(x)$ has a local (relative) maximum at $x = 4$.

(e) The graph of $f(x)$ is concave down on $(3, 4)$.

14. Given the graphs of functions f and g, shown in solid lines below, determine $\lim_{x\to\infty} \frac{f(x)}{g(x)}$.





(a) 0

(b) $\frac{5}{6}$

(c) 1

(d) ∞

(e) None of the above

15. A farmer has 11 feet of fence and wishes to make a rectangular pen, using a barn as one of the sides. What is the maximum area possible for the pen?

    (a) $\frac{363}{8}$ ft^2
    (b) $\frac{121}{8}$ ft^2
    (c) $\frac{61}{4}$ ft^2
    (d) $\frac{31}{2}$ ft^2
    (e) $\frac{121}{9}$ ft^2

16. An object that is moving along a line experiences an acceleration of $a(t) = \cos(t) + \sin(t)$. If it's initial position is $s(0) = 2$ and its initial velocity is $v(0) = -3$, determine its position as a function of time, $s(t)$.

    (a) $s(t) = 3 - \cos(t) - \sin(t)$
    (b) $s(t) = -4 + 3t + \cos(t) + \sin(t)$
    (c) $s(t) = 3 + 4t - \cos(t) + \sin(t)$
    (d) $s(t) = 3 - 2t - \cos(t) - \sin(t)$
    (e) $s(t) = 3 + 2t - \cos(t) - \sin(t)$

17. Which of the following is a correct Riemann sum for the integral $\int_0^3 x^2 \, dx$ using 500 subintervals?

   (a) $\displaystyle\sum_{i=1}^{500} \left(\frac{i}{500}\right)^2 \left(\frac{3}{500}\right)$
   (b) $\displaystyle\sum_{i=1}^{500} 3\left(\frac{i}{500}\right)^2 \left(\frac{3}{500}\right)$
   (c) $\displaystyle\sum_{i=1}^{500} i^2 \left(\frac{3}{500}\right)$
   (d) $\displaystyle\sum_{i=1}^{500} \frac{(3i)^2}{500} \left(\frac{3}{500}\right)$
   (e) $\displaystyle\sum_{i=1}^{500} \left(\frac{3i}{500}\right)^2 \left(\frac{3}{500}\right)$

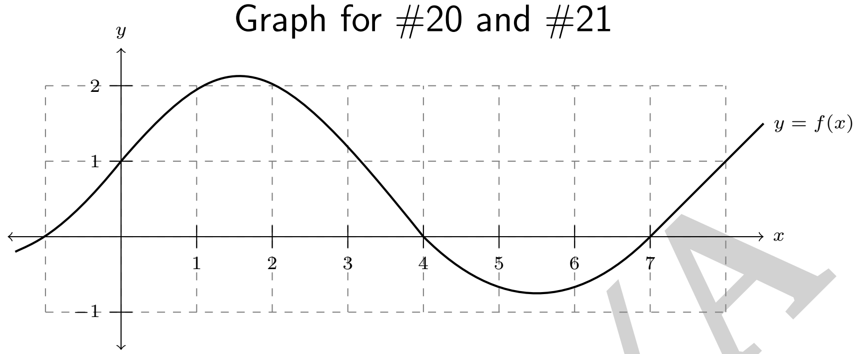
18. The derivative of $F(t) = \int_5^t \sin(x^2) \, dx$ is…

   (a) $F'(t) = \cos(t^2)$
   (b) $F'(t) = \sin(t^2)$
   (c) $F'(t) = \sin(2t)$
   (d) $F'(t) = 2t\cos(t^2)$
   (e) $F'(t) = 2t\sin(t^2)$

19. Suppose $f(x) = \frac{1}{x}$. Which of the following is the linear approximation of f constructed at $x = 5$?

   (a) $\ell(x) = \frac{1}{25} - \frac{1}{5}(x-5)$
   (b) $\ell(x) = -\frac{1}{25} + \frac{1}{5}(x-5)$
   (c) $\ell(x) = \frac{1}{5} + \frac{1}{x^2}(x-5)$
   (d) $\ell(x) = \frac{1}{5} + \frac{1}{25}(x-5)$
   (e) $\ell(x) = \frac{1}{5} - \frac{1}{25}(x-5)$

Graph for #20 and #21



20. Based on the graph above, which of the following is the best estimate for $\int_0^1 f(x)\,dx$?

 (a) 0
 (b) 1
 (c) 1.5
 (d) 2
 (e) 2.5

21. Use the graph above to determine which statement is true about $F(t) = \int_0^t f(x)\,dx$.

 (a) The function F is decreasing on $4 < t < 7$
 (b) The function F is decreasing on $2 < t < 5$
 (c) The function F is decreasing on $0 < t < 2$
 (d) The function F is decreasing on $-1 < t < 4$
 (e) The function F is decreasing on $-1 < t < 7$

22. Suppose R is the finite region bounded by the curve $y = \frac{1}{x}$ and the lines $y = 0$, $x = 1$, and $x = 5$. Determine the area of R.

 (a) $\frac{12}{25}$
 (b) e^5
 (c) $e^5 - 1$
 (d) $\ln(5)$
 (e) $\ln|x| + C$