20. Higher Order Derivatives and Curve Sketching

Deliverable. On **separate pages** submit the entire worksheet to be graded.

1. Determine a general formula for $f''(x)$ in each case.

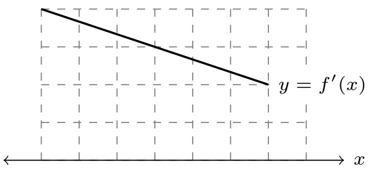
   (a) $f(x) = 19x + 7$

   (b) $f(x) = {}^1\!/_x$

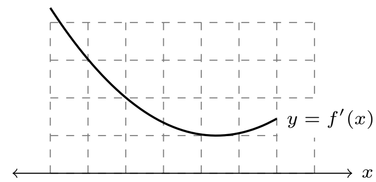
   (c) $f(x) = \sin(8x) + x^6$

   (d) $f(x) = \cos(e^{x^2})$

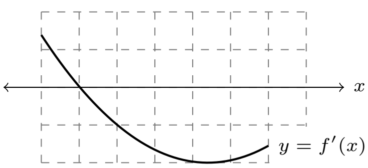
2. The graph of f' is shown below. Draw (i) a possible graph of f, and (ii) a graph of f''



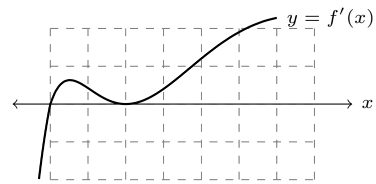
case (a)



case (b)



case (c)

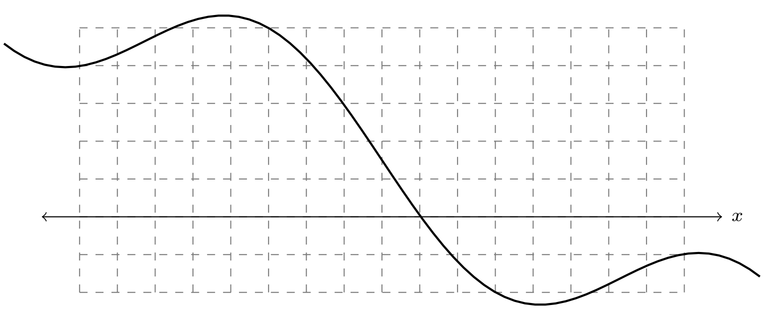


case (d)

3. The graph of $f(x)$ is shown below.

   (a) Locate the second-order critical points of f.

   (b) Determine the interval(s) on which $f''(x)$ remains positive.



4. Determine values of a and b (if any exist) for which the curve $y = ax + \sin(bx)$ is...

   (a) always concave up?

   (b) always increasing?

   (c) always increasing *and* always concave up?

5. When filling a cup with water, the depth of the water in the cup depends on the volume according to

$$h(v) = \left(\frac{27}{\pi}\right)^{1/3} (v + \pi)^{1/3} - 3$$

   (a) Determine a formula for $h''(v)$ and show that it's always negative.

   (b) Explain the practical implication of the fact that $h''(v) < 0$.

   (c) What does the fact that $h''(v)$ tell you about the relationship between a point on the graph of h and the tangent line at that point?

6. Suppose $f''(x)$ is always positive. What must be true about the graph of f as it relates to the tangent line at $x = c$?

7. Suppose $f(x) = \sin(1/x)$. Determine whether the following statements are true or false by computing and analyzing the associated derivative of f.

   (a) $f(x)$ remains bounded as $x \to 0^+$, but $f'(x)$ does not.

   (b) $f''(x)$ grows without bound as $x \to 0^+$.

8. Suppose $f(x) = x\sin(1/x)$. Determine whether the following statements are true or false by computing and analyzing the associated derivative of f.

   (a) $f(x)$ remains bounded as $x \to 0^+$, but $f'(x)$ does not.

   (b) $f''(x)$ grows without bound as $x \to 0^+$.

9. Draw a curve $y = f(x)$ that satisfies all of the following:

   - $f'(x) > 0$ when $x < -1$
   - f has roots only at $x = -2$ and at $x = 1$
   - $f''(x) > 0$ when $x < -2$ and when $x > 2$

10. For each part, design a function with the given characteristic. (You might find it easier to design the function around $x = 0$ and then translate it.)

    (a) The graph of f is concave up at $x = 1$, but $x = 1$ is not a local extremum.

    (b) The value of f is increasing on the left of $x = 1$ and decreasing on the right of $x = 1$, but $x = 1$ is not a local extremum.

    (c) The derivative of f vanishes at $x = 1$ (meaning that $f'(1) = 0$) but $x = 1$ is not a local extremum.

    (d) The second derivative of f vanishes at $x = 1$ (meaning that $f''(1) = 0$) but $x = 1$ is not a local extremum.

    (e) The second derivative of f vanishes at $x = 1$, and $x = 1$ is a local extremum.

11. For each part, determine whether such a function can exist. If so, draw its graph. If not, explain why not.

    (a) $f'(x)$ always negative, but $f(x)$ never 0.

    (b) $f'(x)$ always positive, $f''(x) < 0$ when $2 < x < 5$.

    (c) $f'(x)$ always positive, $f''(x) < 0$ only when $2 < x < 5$.

12. Suppose $f(t)$ is the amount of knowledge acquired after t hours of study and practice.

    (a) What is the practical meaning of the inequality $f'(2) > f'(1)$?

    (b) In practical terms, what does it mean to say that $f'(1) > 0$ and $f''(1) > 0$?

    (c) In practical terms, what does it mean to say that $f'(5) > 0$ and $f''(5) < 0$?

    (d) What is the practical meaning of the inequality $f'(7) < 0$?

    (e) If $f(t)$ is measured in "bits" of information, what are the units associated with $f'(t)$ and $f''(t)$?

13. Suppose $f(t)$ is the total amount of energy provided by a wind turbine, measured in joules, where t is measured in hours after 12:00 noon.

    (a) What is the practical meaning of the inequality $f(2) < f(3)$?

    (b) In practical terms, what does it mean to say that $f'(x) = 0$ when $x \in [0, 1.5]$?

    (c) What is the practical meaning of the inequality $f'(2) > f'(3)$?

    (d) In the context of this problem, what does $f''(4) < f''(5)$ mean?

    (e) What are the units associated with $f'(t)$ and $f''(t)$?