

# Lecture 31: Definite integrals by substitution (minimodule)

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\*These slides may incorporate material from Hughes-Hallet, et al, "Calculus", Wiley

## Skills:

- Evaluate a definite integral  $\int_a^b f(x) dx$  by substitution  $u$ , first perform the substitution as usual, and then change the limits of integration to the corresponding  $u$  values.
- Recognize when a substitution fails to work at first, because the  $x$  cannot be eliminated from the integrand (using valid rules for algebra).
- Be able to eliminate  $x$  from the integrand by solving the initial substitution  $u = \dots$  for  $x$  in terms of  $u$ .

# Definite integrals by substitution

- $\int_0^2 x e^{x^2} dx$
- First find the indefinite integral, then apply the fundamental theorem.
- Alternatively, with the substitution  $w = x^2$ ,  $dw = 2x dx$ ,  
 $\int_0^2 x e^{x^2} dx = \int_0^4 e^w \frac{dw}{2}$
- Note that the limits change, because we are integrating with respect to a different variable ( $w$ ), which covers the range from  $w = 0$  to  $w = 4$ .

# Examples

- $\int_0^{\pi/4} \frac{\tan^3 \theta}{\cos^2 \theta} d\theta$
- $\int_1^3 \frac{dx}{5-x}$

# More complicated substitutions

- $\int \sqrt{1 + \sqrt{x}} \, dx$
- $\int (x + 7) \sqrt[3]{3 - 2x} \, dx$
- Conclusion: Sometimes we have to solve for  $x$  in terms of  $u$  to eliminate all of the  $x$ 's in the integrand.