1. Review workshop

1. Consider the parabola described by $y = x^2 + bx + c$. Under which of the following conditions (if any) can we conclude that the curve has two distinct x-intercepts. Support your answers by sketching appropriate graphs.

   (a) $y = 0$ when $x = -1$

   (b) $y < 0$ when $x = 0$.

   (c) $y > 0$ when $x = 1$

   (d) $y = 0$ when $x = 2$

2. Here we review how to complete the square.

   (a) Here's an example: Suppose we have a quadratic polynomial

   $$x^2 + 6x + 1$$

   We look at the first two terms $x^2 + 6x$. These can be made to participate in a perfect square, since

   $$(x + 3)^2 = x^2 + 6x + 9.$$

   This is called the *associated square*. So $x^2 + 6x = (x + 3)^2 - 9$. Substitute this back into the original polynomial:

   $$x^2 + 6x + 1 = (x^2 + 6x) + 1 = ((x + 3)^2 - 9) + 1 = (x + 3)^2 - 8.$$

   (b) Now consider $x^2 + 8x + 19$.

   (c) Now consider $x^2 + 8x + 19$.
   The associated square is: $\underline{(x + 4)^2 = x^2 + 8x + 16}$
   So

   $$x^2 + 8x + 19 = (x^2 + 8x) + 19 = \underline{(x + 4)^2 + 3}$$

3. The polynomial $x^2 - 2x - 3$ is negative for x values in the interval $(-1, 3)$. We can prove this by completing the square; that is, by writing $x^2 - 2x - 3 = (x - 1)^2 - 4$. Find the largest interval on which each of the following polynomials is negative. Justify your answer by completing the square.

   (a) $x^2 + 4x - 5$

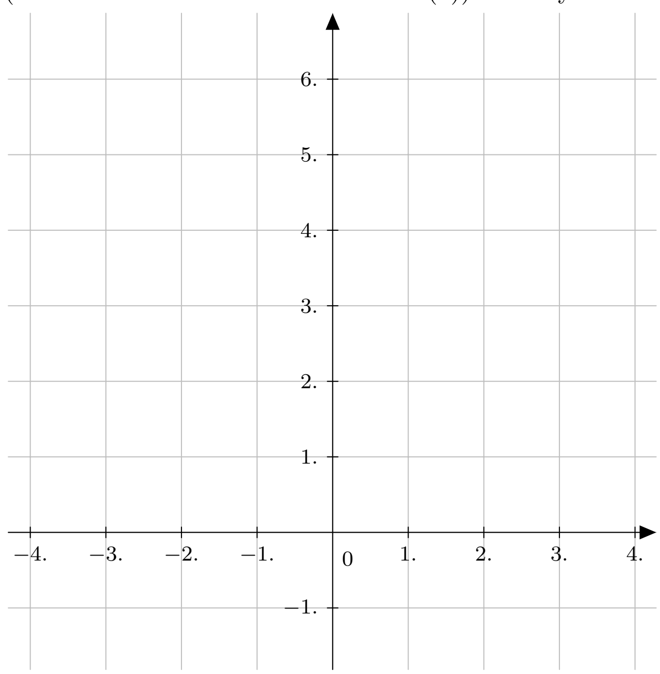
   (b) $x^2 + 2x + 3$

   Answer: (a) $x^2 + 4x - 5 = (x + 2)^2 - 9$, which is negative only if $x + 2$ is between -3 and 3; that is, if $x \in (-5, 1)$. (b) $x^2 + 2x + 3 = (x + 1)^2 + 2$ is the sum of two non-negative quantities, so is never negative.

4. Let $f(x) = x^2 - 4x + 5$. In this exercise, we will sketch the graph of $y = f(x)$.

(a) Complete the square to write $f(x)$ as the sum of a perfect square and a number: $y = (x + A)^2 + B$.

(b) In the space below, sketch the graph of $y = x^2$. Then translate the graph up B units (where B is the constant term from (a)). Finally translate to the left A units.



5. Let $f(x) = x^2 - 4x + 5$. In this exercise, we will sketch the graph of $y = f(x)$.

   (a) Complete the square to write $f(x)$ as the sum of a perfect square and a number: $y = (x + A)^2 + B$.

$$y = (x - 2)^2 + 1$$

   (b) In the space below, sketch the graph of $y = x^2$. Then translate the graph up B units (where B is the constant term from (a)). Finally translate to the left A units.
   This is the graph $y = x^2$ with the vertex shifted to the point $(2, 1)$.

6. Explain what it means for a function to be *decreasing*. Then draw the graphs of the functions described below when $-5 \leq x \leq 5$.

   (a) f is always decreasing, never zero, and $f(-2) = 1$.

   (b) f is always decreasing, never zero, and $f(1) = -2$.

   (c) f is always decreasing, never zero, $f(-2) = 1$ and $f(1) = -2$.

   Answer: A function is decreasing if the value of the function at a later x is always smaller than the value of the function at an earlier x. Symbolically, if $x_1 < x_2$, then $f(x_1) > f(x_2)$. (Hint for part c) your graph will have to jump over the x-axis

7. A function is said to be *even* if $f(x)$ and $f(-x)$ are the same, and is said to be *odd* if $f(x)$ and $f(-x)$ are opposites.

   (a) Draw the graph of a non-constant even function.

   (b) Draw the graph of a non-constant odd function.

   (c) Draw the graph of a function that's neither even nor odd.

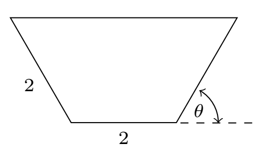
   (d) Draw the graph of a function that's both even and odd.

   Answer: (d) The only such function is $f(x) = 0$!

8. Complete the following table with exact values (no decimal approximation)

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos(\theta)$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\sin(\theta)$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1

9. The cross section of an irrigation ditch is an isosceles trapezoid that is wider at the top than at the bottom. The bottom and the sides of the trapezoid are each 2 meters long. If θ is the acute angle between the horizontal and the side of the ditch, find the cross-sectional area as a function of θ.



   Answer: $A = 4\sin(\theta) + 4\sin(\theta)\cos(\theta)$

10. Determine the equation of the line that is inclined from the x-axis by $\pi/6$ radians and passes through the point $(5, 11)$. **Answer:** $y = 11 + \frac{\sqrt{3}}{3}(x - 5)$

11. Suppose the line ℓ is inclined from the x-axis by θ radians, where $\theta \in (0, \pi/2)$. What is the slope of a perpendicular line? **Answer:** $-\cot(\theta)$

12. Determine the wavelength (period) and amplitude of the curves (a) $y = 15\cos(3x)$ and (b) $y = 7\sin(0.2x)$. **Answer:** (a) wavelength $\frac{2\pi}{3}$ and amplitude 15; (b) wavelength 10π and amplitude 7