17. Introduction to the derivative and rates of change

Deliverable. While everyone is expected to complete *the entire worksheet*, the last two questions are deliverable.

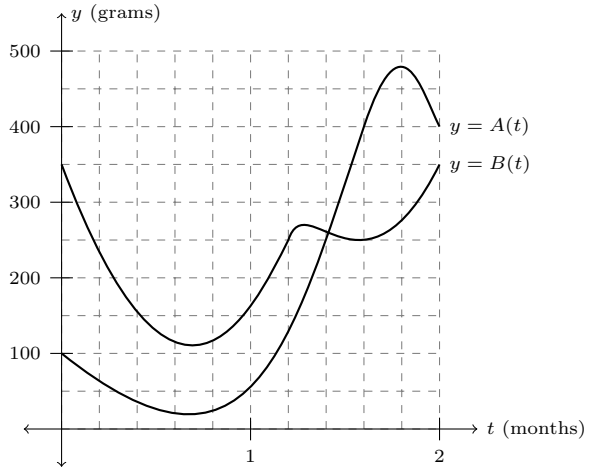
1. The figure depicts the biomass of two different species of algae that have been introduced into a new climate.

   (a) What are the units of $A'(t)$ and $B'(t)$?

   (b) What practical interpretation do we ascribe to the fact that $B'(0.5) < 0$?

   (c) Which plant species appears better suited to the new climate at $t = 0.8$?

   (d) Estimate $B'(1)$ with correct units.



2. Alex and Taylor are running a shuttle race, 25 meters out and then back to the start line. Their distances from the start line are given by $A(t)$ and $T(t)$ respectively, as shown in the graph below.

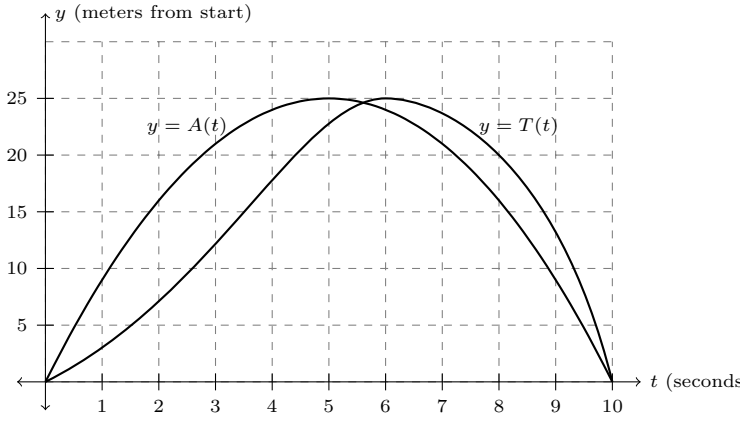
   (a) Who wins the race?

   (b) At what time(s) is Alex farthest ahead of Taylor?

   (c) What are the units of $A'(t)$ and $T'(t)$?

   (d) Estimate the runners' speeds when $t = 1$ and $t = 7$.

   (e) Is Taylor ever running at a rate of 3 meters per second toward the finish line? If so, determine T' at that time. If not, how can you tell?



3. In each case below, determine the units associated with f'.

   (a) $f(t)$ = the altitude (in meters) of a model rocket after t seconds.

   (b) $f(t)$ = the temperature of a chemical solution (in kelvins) after t minutes.

   (c) $f(x)$ = the cost (in dollars) of making a carbon nanotube that is x centimeters long.

   (d) $f(z)$ = the average number of nerf darts fired per day on RIT's campus when there are z zombies.

4. In each case below, explain the practical (real-world) interpretation of f'.

   (a) $f(k)$ = the energy stored in a substance (measured in joules) when its temperature in k degrees kelvin.

   (b) $f(s)$ = the resistance (measured in newtons) offered by a particular fluid when an object moves through it at a speed of s meters per second.

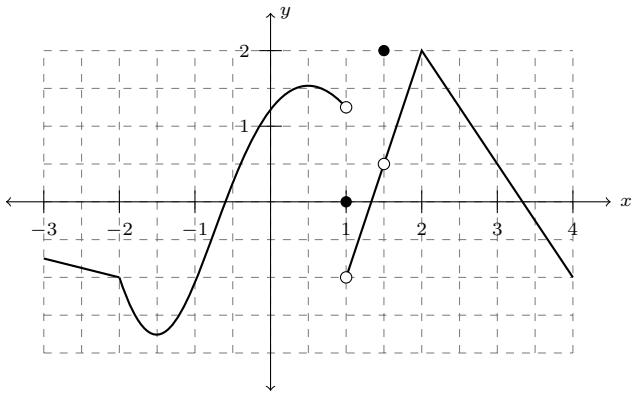
5. Suppose $f(t)$ is the temperature (in degrees Fahrenheit) of a cold yam that's been placed in a hot oven, where t is the time in minutes since the yam entered the oven.

   (a) What is the sign of $f'(t)$? Why?

   (b) What are the units of $f'(10)$?

   (c) What is the practical meaning of the statement $f'(10) = 2$?

6. Determine all values of x where function f, graphed below, fails to be differentiable:



7. In the classic arcade game of *asteroids*, you need to pilot a ship around and blow up asteroids with your ship's blaster (or perish in the attempt). The rules of the game are such that you may only fire in your direction of motion. Suppose that you follow a trajectory that can be represented by the function

$$y = 1 + \frac{1}{x}.$$

A row of four asteroids are positioned at the (x, y) coordinates (0,1), (0, 2), (0,3), and (0,4), and although the asteroids are not in your path, you wish to clear them for the safety of future travelers. You can shoot lasers capable of destroying the asteroids, but only in the direction tangent to your trajectory. Which asteroid(s) can you hit when your spaceship is positioned at the coordinate (1,2)?

8. Suppose that a single line of people is moving to the right. A common element of mathematical models that describe the flow of people is a characterization of the relationship between crowd density and speed; typically we begin with the idea that higher densities correspond to slower speeds. An example of such a relationship is

$$f(p) = \begin{cases} 1.35 - 0.45p & \text{if } 0 \le p \le 3 \\ 0 & \text{if } p > 3 \end{cases}$$

where p is the density of pedestrians (the average number of people per meter in a given region), and $f(p)$ is the associated speed in meters per second.

The rate at which pedestrian traffic is crossing over a particular point (perhaps the entrance to a tunnel) depends on both the density of the pedestrians and the speed at which they are moving.

(a) What does $f(0)$ tell us?

(b) In practical terms, what does it mean that $f(3) = 0$?

(c) Determine the units associated with $F(p) = p\,f(p)$.

(d) Distribute p onto $f(p)$ to write $F(p)$ as a quadratic polynomial.

(e) Graph the parabola $y = F(p)$.

(f) Determine the interval(s) on which $F'(p) > 0$, and the interval(s) on which $F'(p) < 0$.

(g) What is the practical meaning of $F'(p) > 0$ in the context of this scenario?

(h) Where is $F'(p) = 0$? What does this point have to do with the graph of F?

(i) If we want to maximize the flow of pedestrians, what speed should they walk at?