Handout: Implicit curves review

- Chain rule: If $v = f(u)$, then $dv = f'(u)du$
- Power rule: $d(u^a) = au^{a-1}du$
- Constant multiple rule: $d(au) = a\,du$
- Sum rule: $d(u + v) = du + dv$
- Constant rule: $da = 0$
- Product rule: $d(uv) = v\,du + u\,dv$
- Quotient rule: $d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$
- $d(e^u) = e^u\,du, \qquad d(a^u) = (\ln a)a^u\,du$
- $d(\sin u) = \cos u\,du, \qquad d(\cos u) = -\sin u\,du$
- $d(\tan u) = \sec^2 u\,du$
- $d(\ln u) = \frac{du}{u}$
- $d(\arctan u) = \dfrac{du}{1+u^2}, \qquad d(\arcsin u) = \dfrac{du}{\sqrt{1-u^2}}$

1. If $x^4 + x^2y^2 + y^3 = 5$, find dy/dx.

Step 1. Apply the differential d to both sides of the equation.

Step 2. Use the rules (above) until all appearances of the differential d are with an atomic variable (bird-with-food).

Step 3. Solve for dy.

Step 4. Divide by dx.

2. Find an equation of the tangent line to $x^2 - xy - y^2 = 1$ through the point $(2, 1)$.

Step 1. Apply the differential d to both sides of the equation.

Step 2. Use the rules (above) until all appearances of the differential d are with an atomic variable (bird-with-food).

Step 3. Replace x by 2 and y and by 1, and then dx by $x - 2$ and dy by $y - 1$.

3. The Lambert W-function $W(x)$ is defined implicitly by the equation

$$x = We^W.$$

Show that $\dfrac{dW}{dx} = \dfrac{W}{x(1+W)}$