Handout: Related rates review

Problem: A boat sails parallel to a straight beach at a constant speed of 12 miles per hour, staying 4 miles offshort. How fast is it approaching a lighthouse on the shoreline at the instant it is exactly 5 miles from the lighthouse?

A related rates problem has three parts: a **modeling** part, a **calculus** part, and a **solve for the unknown rate** part. It is very important to keep these parts separate. We do the modeling part first. Anything with a rate (i.e., units of time in the denominator) should play no role in the modeling part. If you use the differential or derivative (i.e., the letter d) anywhere in the modeling part, then it's wrong.

Modeling: no calculus allowed!

Step 1: Draw a picture. This is the *most important step* in the problem. To begin with, your picture should not have any symbols in it. Think of the problem as telling a story. You want to illustrate that story. Your picture should be a good model of what is described in the problem. You may need to look at the numbers (and units!) in the problem to do this. For instance, the problem says that the boat is exactly 5 miles from the lighthouse, so we shouldn't draw the boat right next to the lighthouse because this would not be compatible with the story!

Step 2: Now, you have the picture. Set the problem aside for a moment and look at your picture. Try to identify all of the quantities that depend on time. Pick variables for all of these quantities.

Step 3. Write equations relating the quantities you found in step 2.

Calculus

Step 4: Compute the *differential* of the equation(s) you found in step 3. (Yes ***the differential***. Do not attempt to "take the derivative", because students typically take the derivative with respect to the wrong variable, or do not fully understand what "take the derivative with respect to time" means when none of the variables explicitly depend on time.)

Step 5: If you've taken the differential correctly, every term should have a bird-with-food in it. Divide both sides of the equations by dt to get the related rates equation.

Solve for the unknown rate

Now, we should be able to substitute numbers in and solve for the unknown rate(s). *This is the only part of the problem where one should think in terms of specific values of the quantities involved.*