Curve sketching

Deliverable: The ***last page*** of the worksheet is deliverable.

1. Let $f(x) = \frac{(x-1)^2}{x^2+1}$. We shall go through the steps needed to sketch a graph of $y = f(x)$ by hand.

   (a) What is the domain of $f(x)$?

   (b) What are the x and y intercepts of $y = f(x)$? What is the local behavior of f near its x intercepts?

   (c) Does the graph have any symmetries? (Is it an even function, an odd function, or neither?)

   (d) What are the horizontal and vertical asymptotes of $y = f(x)$ (if any)? [To find the horizontal asymptotes, compute the limit $\lim_{x\to\infty} f(x)$.]

   (e) Compute $f'(x)$, and locate the critical points of f. Make a table showing the intervals of increase and decrease.

(f) Find the local maximum and minimum values of the function.

(g) Compute $f''(x)$, and solve the equation $f''(x) = 0$ for x. Use these points to make a table of signs of f'', showing where the graph of $y = f(x)$ is concave up and where it is concave down. Also find the inflection points.

(h) Sketch the curve, using all of this information. You should label the intercepts, asymptotes, local extrema, and inflection points.

2. Do the same steps (a)-(h), applied to the function $f(x) = \frac{x^3}{3+x^2}$.

3. Do the same for the function $f(x) = \frac{1}{(x-1)(x-3)}$. Pay particular attention to the fact that concavity/monotonicity may change at the asymptotes $x = 1, x = 3$, so these will give extra dividers in your table.