

Expanded TFL Proofs

PHIL-205-01:Symbolic Logic

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Below is a complete list of available rules for TFL Proofs.

1 Conjunction \wedge Rules

1.1 \wedge Elim

m. $A \wedge B$	
n. A	\wedge Elim: m

1.2 \wedge Intro

m. A	
n. B	
r. $A \wedge B$	\wedge Intro: m, n

2 Disjunction \vee Rules

2.1 \vee Intro

m. A	
n. $A \vee B$	\vee Intro: m

2.2 \vee Elim

m. $A \vee B$	
i. A	
j. C	
k. B	
l. C	
r. C	\vee Elim: m, i-j, k-l

3 Conditional \rightarrow Rules

3.1 \rightarrow Elim (*Modus Ponens*)

m. $P \rightarrow Q$	Note that order in the reference is important. You must first reference
n. P	
r. Q	\rightarrow Elim: m, n

the conditional before referencing the antecedent.

3.2 \rightarrow Intro

	i. A	
	├	
	j. B	
	r. A \rightarrow B	\rightarrow Intro: i-j

4 Biconditional \leftrightarrow Rules

4.1 \leftrightarrow Intro

	i. A	
	├	
	j. B	
	k. B	
	├	
	l. A	
	A \leftrightarrow B	\leftrightarrow Intro: i-j, k-l

4.2 \leftrightarrow Elim

	m. A \leftrightarrow B	
	n. A	
	r. B	\leftrightarrow Elim: m, n

As with \rightarrow 's reference, first list the biconditional, then the condition.

5 Negation \neg Rules

5.1 \neg Elim

Any double negation can be eliminated.

5.2 \perp Intro

This is proven by showing a contradiction.

	1. P	
	2. \neg P	
	3. \perp	

5.3 \neg Intro

We have to prove this by proof by contradiction. (Shown below)

	1. P	
	├	
	2. \perp	
	3. \neg P	\perp Intro: 1-2,

5.4 Explosions

	m. \perp	Anything can be proven after a contradiction.
	r. A	X: m

5.5 Tertium non datur

Latin for “no third way”.

	i. A	
	j. B	
	k. $\neg A$	
	l. B	
r. B	TND: i–j, k–l	

6 New Additions

6.1 Disjunctive Syllogism

	m. $P \vee Q$	
	n. $\neg Q$	
	r. P	DS: 1, 2

6.2 Modus Tollens

	m. $P \rightarrow Q$	
	n. $\neg Q$	
	r. $\neg P$	MT: 1, 2

7 Example Theorem Proofs

7.1 Frege’s Theorem

		1. $P \rightarrow (Q \rightarrow R)$		
			2. $P \rightarrow Q$	
			3. P	
			4. $Q \rightarrow R$	\rightarrow Elim: 1, 3
			5. Q	\rightarrow Elim: 2, 3
			6. R	\rightarrow Elim: 4, 5
			7. $P \rightarrow R$	\rightarrow Intro: 3–6
		8. $(P \rightarrow Q) \rightarrow (P \rightarrow R)$	\rightarrow Intro: 2–7	
	9. $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$	\rightarrow Intro: 1–8		