

Proofs

PHIL-205-01:Symbolic Logic

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We'll go over the quiz on Wednesday. Homework will be due on Friday.

We'll start with a Fitch style proof.

We need rules that can connect sentences of TFL. We also need rules for removing these connections.

All un-annotated lines are considered to be assumptions or premises. Indented lines represent sub-proofs. We'll use horizontal lines for separation of premises and assumptions, and vertical lines to represent the level of sub-proof we're at. Note that subproofs must be taken as complete chunks, and that attempting to use sub-sub-proofs isn't allowed.

Below are concepts introduced in lecture, using the notation found in the lecture:

1 Conjunction \wedge Rules

1.1 \wedge Elim

	m. $A \wedge B$	
	n. A	\wedge Elim: m

1.2 \wedge Intro

	m. A	
	n. B	
	r. $A \wedge B$	\wedge Intro: m, n

2 Disjunction \vee Rules

2.1 \vee Intro

	m. A	
	n. $A \vee B$	\vee Intro: m

2.2 \vee Elim

	m. $A \vee B$	
	i. A	
	j. C	
	k. B	
	l. C	
	r. C	\vee Elim: m, i-j, k-l

3 Conditional \implies Rules

3.1 \implies Elim (*modus ponens*)

<div style="display: flex; flex-direction: column; align-items: flex-start;"><div>m. $P \rightarrow Q$</div><div>n. P</div><div>r. Q</div></div>	<div style="display: flex; align-items: center;"><div style="flex-grow: 1; border-bottom: 1px solid black; margin-bottom: 5px;"></div>\rightarrow Elim: m, n</div>	Note that order in the reference is important. You must first reference the conditional before referencing the antecedent.
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3.2 \implies Intro

<div style="display: flex; flex-direction: column; align-items: flex-start;"><div style="border-left: 1px solid black; padding-left: 5px; margin-bottom: 5px;">i. A</div><div style="border-left: 1px solid black; padding-left: 5px;">j. B</div></div>	<div style="display: flex; align-items: center;"><div style="flex-grow: 1; border-bottom: 1px solid black; margin-bottom: 5px;"></div>\rightarrow Intro: i-j</div>
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4 Biconditional \leftrightarrow Rules

4.1 \leftrightarrow Intro

<div style="display: flex; flex-direction: column; align-items: flex-start;"><div style="border-left: 1px solid black; padding-left: 5px; margin-bottom: 5px;">i. A</div><div style="border-left: 1px solid black; padding-left: 5px; margin-bottom: 5px;">j. B</div><div style="border-left: 1px solid black; padding-left: 5px; margin-bottom: 5px;">k. B</div><div style="border-left: 1px solid black; padding-left: 5px;">l. A</div></div>	<div style="display: flex; align-items: center;"><div style="flex-grow: 1; border-bottom: 1px solid black; margin-bottom: 5px;"></div>\leftrightarrow Intro: i-j, k-l</div>
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4.2 \leftrightarrow Elim

<div style="display: flex; flex-direction: column; align-items: flex-start;"><div>m. $A \leftrightarrow B$</div><div>n. A</div><div>r. B</div></div>	<div style="display: flex; align-items: center;"><div style="flex-grow: 1; border-bottom: 1px solid black; margin-bottom: 5px;"></div>\leftrightarrow Elim: m, n</div>
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As with \rightarrow 's reference, first list the biconditional, then the condition.

5 Negation \neg Rules

5.1 \neg Elim

Any double negation can be eliminated.

5.2 \perp Intro

This is proven by showing a contradiction.

<div style="display: flex; flex-direction: column; align-items: flex-start;"><div>1. P</div><div>2. $\neg P$</div><div>3. \perp</div></div>
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5.3 \neg Intro

We have to prove this by proof by contradiction. (Shown below)

	1. P	
	2. \perp	
	3. $\neg P$	\perp Intro: 1-2,

5.4 Explosions

	m. \perp	Anything can be proven after a contradiction.
	r. A	X: m

5.5 Tertium non datur

Latin for “no third way”.

	i. A	
	j. B	
	k. $\neg A$	
	l. B	
	r. B	TND: i-j, k-l

6 Example Proofs

6.1 Modus Tollens

In English, If P, then Q. Not Q, therefore not P.

	1. $P \rightarrow Q$	
	2. $\neg Q$	
	3. P	
	4. Q	\rightarrow Elim: 1, 3
	5. \perp	\neg Elim: 4, 2
	6. $\neg P$	\neg Intro: 3-5

6.2 $A \wedge (B \implies A) \therefore \neg A \implies \neg B$

1. $A \wedge (B \rightarrow A)$	
— 2. $\neg A$	
3. $B \rightarrow A$	
— 4. B	\rightarrow Elim: 3, 4
5. A	\neg Elim: 5, 2
6. \bot	\neg Intro: 4–6
7. $\neg B$	
8. A	
— 9. \bot	\neg Elim: 8, 2
10. $\neg B$	X 9
11. $\neg B$	\vee Elim: 1, 3–7, 8–10
12. $\neg A \rightarrow \neg B$	\rightarrow Intro: 2–11

7 Practice Exercises

A. The following two proofs are *incorrect*. Explain the mistakes made. Proof 1.

1. $\neg L \rightarrow (A \wedge L)$	
2. $\neg L$	
3. A	\rightarrow Elim: 1, 2
4. L	
5. \perp	\neg Elim: 4, 2
6. A	X: 5
7. A	TND: 2–3, 4–6

The problem with this proof falls in line 5. You cannot take only one line out of a sub-proof, nor can you use a sub-proof outside of the proof you are working in. Proof 2.

1. $A \wedge (B \wedge C)$	
2. $(B \vee C) \rightarrow D$	
3. B	\wedge Elim: 1
4. $B \vee C$	\vee Intro: 3
5. D	\rightarrow Elim: 4, 2

The problem with this proof lies in line 3. You cannot expand parentheses out in this model.

B. The following three proofs are missing their citations. Add them. Problem 1.

1. $P \wedge S$	Problem 2.
2. $S \rightarrow R$	
3. P	\wedge Elim: 1
4. S	\wedge Elim: 1
5. R	\rightarrow Elim: 2, 4
6. $R \vee E$	\vee Intro: 5

1. $A \implies D$	Problem 3.
2. $A \wedge B$	
3. A	\wedge Elim: 2
4. D	\rightarrow Elim: 1, 2
5. $D \vee E$	\vee Intro: 4
6. $(A \wedge B) \rightarrow (D \vee E)$	\rightarrow Intro: 2–5

1. $\neg L \rightarrow (F \vee L)$	
2. $\neg L$	
3. $F \vee L$	\rightarrow Intro: 1–2
4. F	
5. $F \wedge F$	\wedge Intro: 4
6. F	\wedge Elim: 5
7. L	
8. \perp	\neg Elim: 2, 7
9. F	X: 8
10. F	TND: 4–6, 7–9

C. Give a proof for each of the following arguments.

1. $F \rightarrow \neg F \therefore \neg F$

1. $F \rightarrow \neg F$	
	2. F
	3. $\neg F$
	4. \perp
	6. $\neg F$

\rightarrow **Elim:** 1, 2

\neg **Elim:** 2, 3

\neg **Intro:** 2–4

2. $Q \rightarrow (Q \wedge \neg Q) \therefore \neg Q$

1. $Q \rightarrow (Q \wedge \neg Q)$	
	2. Q
	3. $Q \wedge \neg Q$
	4. Q
	5. $\neg Q$
	6. \perp
	7. $\neg Q$

\rightarrow **Elim:** 1, 2

\wedge **Elim:** 3

\wedge **Elim:** 3

\neg **Elim:** 4, 5

\neg **Intro:** 3–6

3. $A \rightarrow (B \rightarrow C) \therefore (A \wedge B) \rightarrow C$

1. $A \rightarrow (B \rightarrow C)$	
	2. A
	3. $B \rightarrow C$
	4. $A \wedge B$

\rightarrow **Elim:** 1, 2

4. 4

5. 5

6. 6

7. 7

8. 8

9. 9

10. 10

11. 11

12. 12