

Homework #5

PHIL-205-01:Symbolic Logic

Blizzard MacDougall

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1 Section I

Construct an interpretation to show that the following is invalid:

$$\begin{array}{l} \exists x(Px \wedge \neg Qx) \\ \forall x(Qx \rightarrow Rx) \\ \therefore \exists x\neg Rx \end{array} \tag{1}$$

English translation: There is someone who both is P and is not Q. For everyone, if they are Q, then they are R. Therefore, there is someone who is not R.

Domain:all right-handed individuals

Px : x pays taxes.

Qx : x has lost their left hand.

Rx : x is right-handed.

2 Section II

Add annotations to the following proofs:

1. Proof 1

1. $\neg P \vee \neg Q$	
2. $P \wedge Q$	
3. P	\wedge Elim: 2
4. Q	\wedge Elim: 2
5. $\neg P$	
6. \perp	\neg Elim: 3, 5
7. $\neg Q$	
8. \perp	\neg Elim: 4, 7
9. \perp	\vee Elim: 1, 5–6, 7–8
10. $\neg(P \wedge Q)$	\neg Intro: 1, 2–9

2. Proof 2

1. $P \rightarrow Q$	
2. P	
3. Q	\rightarrow Elim: 1, 2
4. $\neg P \vee Q$	\vee Intro: 3
5. $\neg P$	
6. $\neg P \vee Q$	\vee Intro: 5
7. $\neg P \vee Q$	TND: 2–4, 5–6

3. Proof 3

1. $\neg P \vee Q$	
2. P	
3. Q	
4. $Q \wedge Q$	\wedge Intro: 3
5. Q	\wedge Elim: 4
6. $\neg P$	
7. \perp	\neg Elim: 2, 6
8. Q	X: 7
9. Q	\vee Elim: 1, 3–5, 6–8
10. $P \rightarrow Q$	\rightarrow Intro: 2–9