FOL and TFL Proof Rules

PHIL-205-01:Symbolic Logic

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This is a combination of the notes from 03/11, 08/11, and 17/11. Note that all rules for TFL will also work in FOL.

1 Conjunction $\wedge$ Rules

1.1 $\wedge$ Elim

m.	$A \wedge B$	
n.	A	$\wedge$ **Elim:** m

1.2 $\wedge$ Intro

m.	A	
n.	B	
r.	$A \wedge B$	$\wedge$ **Intro:** m, n

2 Disjunction $\vee$ Rules

2.1 $\vee$ Intro

m.	A	
n.	$A \vee B$	$\vee$ **Intro:** m

2.2 $\vee$ Elim

m.	$A \vee B$	
i.	A (assumption)	
j.	C	
k.	B (assumption)	
l.	C	
r.	C	$\vee$ **Elim:** m, i–j, k–l

3 Conditional → Rules

3.1 → Elim (*Modus Ponens*)

m. $P \to Q$
n. P
r. Q $\to$ **Elim:** m, n

Note that order in the reference is important. You must first reference the conditional before referencing the anticedent.

3.2 → Intro

i. A
j. B
r. $A \to B$ $\to$ **Intro:** i–j

4 Biconditional ↔ Rules

4.1 ↔ Intro

i. A
j. B
k. B
l. A
$A \leftrightarrow B$ $\leftrightarrow$ **Intro:** i–j, k–l

4.2 ↔ Elim

m. $A \leftrightarrow B$
n. A
r. B $\leftrightarrow$ **Elim:** m, n

As with →'s reference, first list the biconditional, then the condition.

5 Negation ¬ Rules

5.1 ¬ Elim

Any double negation can be eliminated.

5.2 ⊥ Intro

This is proven by showing a contradiction.

1. P
2. $\neg P$
3. $\bot$

5.3 ¬ Intro

We have to prove this by proof by contradiction. (Shown below)

1. P
2. ⊥

3. ¬P ⊥ **Intro:** 1–2,

5.4 Explosions

m. ⊥

r. A **X:** m

Anything can be proven after a contradiction.

5.5 Tertium non datur

Latin for "no third way".

i. A
j. B

k. ¬A
l. B

r. B **TND:** i–j, k–l

6 Misc. TFL

6.1 Disjunctive Syllogism

m. P ∨ Q
n. ¬Q
r. P **DS:** 1, 2

6.2 Modus Tollens

m. P → Q
n. ¬Q
r. ¬P **MT:** 1, 2

7 Conversion of Quantifiers

This allows us to move a negation into or out of a sentence:

m. ∀x¬Dx
n. ¬∃xDx **CQ:** m

This can be done in either direction, and with either quantifier. Note that if a sentence has multiple quantifiers, this rule must be done for each quantifier.

8 Universal Elimination

m. ∀xFx
n. Fa ∀**Elim:**1

9 Existential Introduction

m. Fa
n. $\exists$xFx **$\exists$Intro:**m

10 Universal Introduction

This is incredibly hard, because we have to prove that *any* value is true. That being said, it will basically never be used, because its not a tool we need very often.

m. $\forall$xFx
n. Fb **$\forall$Elim:**m
o. $\forall$yFy **$\forall$Intro:**n

11 Existential Elimination

The goal of this rule isn't actually to remove the existential quantifier. Its as part of a subproof, generally.

m. $\exists$xAx
i. Ac
j. $\exists$xBx
r. $\exists$xBx **$\exists$Elim:**$m, i - j$

12 Equivalence Identity

Example identity:

I see a President and no one else.
I see Joe Biden.
Therefore, Biden is President.

1. $\forall$x[(Px $\wedge$ Six) $\wedge$ $\forall$y(Siy $\rightarrow$ y = x)]
2. Sib
3. Pa $\wedge$ Sia $\wedge$ $\forall$y(Siy $\rightarrow$ y = a)
4. $\forall$y(Siy $\rightarrow$ y = a) **$\wedge$ Elim:** 3
5. Sib $\rightarrow$ b = a **$\forall$Elim:**4
6. b = a **$\rightarrow$ Elim:** 5, 2
7. Pa **$\wedge$ Elim:** 3
8. Pb **=Elim:** 6, 7
9. **$\exists$Elim:**1, 3 − 8

The F is a G. a is not a G. Therefore a is not an F.

1. $\exists x \forall y(Fx \wedge (Fy \rightarrow x = y) \wedge Gx)$	
2. $\neg Ga$	
3. $\forall y(Fb \wedge (Fy \rightarrow b = y) \wedge Gb)$	
4. $Fb \wedge (Fa \rightarrow b = a) \wedge Gb$	$\forall$**Elim:**3
5. Fa	
6. $Fa \rightarrow b = a$	$\wedge$ **Elim:** 4
7. $b = a$	$\rightarrow$ **Elim:** 6, 5
8. Gb	$\wedge$ **Elim:** 4
9. Ga	=**Elim:**7,8
10. $\bot$	$\bot$ **Intro:** 9, 2
11. $\neg Fa$	$\neg$ **Intro:** 5-10
12. $\neg Fa$	$\exists$**Elim:**$1, 3 - 11$