Quiz # 9

PHIL–205–01:Symbolic Logic

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1 Section 1

Annotate the following proof :

1. $\neg\forall x(Fx \to Gx)$	
2. $\exists x\neg(Fx \to Gx)$	**CQ:** 1
3. $\neg(Fa \to Ga)$	
4. $\neg\exists x(Fx \wedge \neg Gx)$	
5. $\forall x\neg(Fx \wedge \neg Gx)$	**CQ:** 4
6. $\neg(Fa \wedge \neg Ga)$	$\forall$**Elim:** 5
7. $\neg Fa \vee \neg\neg Ga$	**DeM:** 6
8. Fa	
9. $\neg\neg Ga$	**DS:** 7, 8
10. Ga	**DNE:** 9
11. $Fa \to Ga$	$\to$ **Intro:** 8–10
12. $\perp$	$\perp$ **Intro:** 3, 11
13. $\neg\neg\exists x(Fx \wedge \neg Gx)$	$\neg$ **Intro:** 4–12
14. $\exists x(Fx \wedge \neg Gx)$	**DNE:** 13
15. $\exists x(Fx \wedge \neg Gx)$	$\exists$**Elim:** $2, 3-14$
16. $\exists x(Fx \wedge \neg Gx)$	
17. $Fa \wedge \neg Ga$	
18. $Fa \to Ga$	
19. Fa	$\wedge$ **Elim:** 17
20. Ga	$\to$ **Elim:** 18, 19
21. $\neg Ga$	$\wedge$ **Elim:** 17
22. $\perp$	$\perp$ **Intro:** 20, 21
23. $\neg(Fa \to Ga)$	$\perp$ **Intro:** 18-22,
24. $\exists x\neg(Fx \to Gx)$	$\exists$**Intro:** 23
25. $\exists x\neg(Fx \to Gx)$	$\exists$**Elim:** 24
26. $\neg\forall x(Fx \to Gx)$	**CQ:** 25
27. $\neg\forall x(Fx \to Gx) \leftrightarrow \exists x(Fx \wedge \neg Gx)$	$\leftrightarrow$ **Intro:** 1–15, 16–26

2 Section 2

Construct a formal proof of one (or both) of the following argument(s).

A. $(\forall x(Fx \vee Gx);\ \neg\forall xGx) \vdash \exists xFx$

1. $\forall$x(Fx $\vee$ Gx)	
2. $\neg\forall$xGx	
3. $\exists$x$\neg$Gx	**CQ:** 2
4. $\neg$Ga	
5. Fa $\vee$ Ga	$\forall$**Elim:** 1
6. Fa	**DS:** 5, 4
7. $\exists$xFx	$\exists$**Intro:** 6
8. $\exists$xFx	$\exists$**Elim:** $3, 4-7$

B. $\vdash [\forall x(Fx \to Gx) \wedge \exists x(Fx \wedge Hx)] \to \exists x(Gx \to Hx)$

1. $\forall$x(Fx $\to$ Gx) $\wedge$ $\exists$x(Fx $\wedge$ Hx)	
2. $\forall$x(Fx $\to$ Gx)	$\wedge$ **Elim:** 1
3. $\exists$x(Fx $\wedge$ Hx)	$\wedge$ **Elim:** 1
4. Fa $\wedge$ Ha	
5. Ga	
6. Ha	$\wedge$ **Elim:** 4
7. Ga $\to$ Ha	$\to$ **Intro:** 5–6
8. $\exists$x(Gx $\to$ Hx)	$\exists$**Intro:** 7
9. $\exists$x(Gx $\to$ Hx)	$\exists$**Elim:** $4-8$
10. [$\forall$x(Fx $\to$ Gx) $\wedge$ $\exists$x(Fx $\wedge$ Hx)] $\to$ $\exists$x(Gx $\to$ Hx)	$\to$ **Intro:** 1–9