

Week 8 Activities Problems

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PHYS-111 2pm Lab

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Law of Universal Gravitation

1. At new moon, the earth, moon, and sun are in a straight line.

$$\begin{aligned} \text{Givens :} \\ M_{\text{earth}} &= 5.67 \times 10^{24} \text{ kg} \\ M_{\text{moon}} &= 7.35 \times 10^{22} \text{ kg} \\ M_{\text{sun}} &= 2.00 \times 10^{30} \text{ kg} \\ d_{\text{moon-earth}} &= 3.84 \times 10^5 \text{ km} \\ AU &= 1.50 \times 10^8 \text{ km} \end{aligned} \tag{1}$$

1. What is the net gravitational force exerted on the earth by the moon and the sun?

$$\begin{aligned} F_g &= G \frac{mM}{r^2} \\ F_{\text{net}} &= F_{g1} + F_{g2} \\ F_g &= G \frac{mM}{r^2} + G \frac{mM}{r^2} \\ F_g &= Gm \left(\frac{M}{r^2} + \frac{M}{r^2} \right) \\ F_g &= G(5.67 \times 10^{24} \text{ kg}) \left(\frac{2.00 \times 10^{30} \text{ kg}}{(1.50 \times 10^8 \text{ km})^2} + \frac{7.35 \times 10^{22} \text{ kg}}{(3.84 \times 10^5 \text{ km})^2} \right) \\ F_g &= G(5.67 \times 10^{24} \text{ kg})(8.938734266 \times 10^7 \text{ kg} / \text{m}^2) \\ F_g &= 3.383 \times 10^{22} \text{ N} \\ \underline{F_g = 3.38 \times 10^{22} \text{ N towards the sun}} \end{aligned} \tag{2}$$

2. ... on the moon by the sun and the earth?

$$\begin{aligned} F_g &= Gm \left(\frac{M}{r^2} - \frac{M}{r^2} \right) \\ F_g &= G(7.35 \times 10^{22} \text{ kg}) \left(\frac{2.00 \times 10^{30} \text{ kg}}{(1.50 \times 10^8 \text{ km} - 3.84 \times 10^5 \text{ km})^2} - \frac{5.67 \times 10^{24} \text{ kg}}{(3.84 \times 10^5 \text{ km})^2} \right) \\ F_g &= G(7.35 \times 10^{22} \text{ kg})(5.089360517 \times 10^7 \text{ kg} / \text{m}^2) \\ F_g &= 2.497 \times 10^{20} \text{ N} \\ \underline{F_g = 2.497 \times 10^{20} \text{ N towards the sun}} \end{aligned} \tag{3}$$

3. ... on the sun by the moon and the earth?

$$\begin{aligned}
F_g &= Gm\left(\frac{M}{r^2} + \frac{M}{r^2}\right) \\
F_g &= G(2.00 \times 10^{30} \text{ kg})\left(\frac{7.35 \times 10^{22} \text{ kg}}{(1.50 \times 10^8 \text{ km} - 3.84 \times 10^5 \text{ km})^2} + \frac{5.67 \times 10^{24} \text{ kg}}{(3.84 \times 10^5 \text{ km})^2}\right) \\
F_g &= G(5.67 \times 10^{24} \text{ kg})(3.845215172 \times 10^7 \text{ kg} / \text{m}^2) \\
F_g &= 5.133 \times 10^{27} \text{ N} \\
\boxed{F_g = 5.133 \times 10^{27} \text{ N towards the sun}}
\end{aligned}
\tag{4}$$

2. When the earth, moon, and sun form a right triangle, with the moon located at the right angle, the moon is in its third quarter phase. What is the net gravitational force on the moon?

$$\begin{aligned}
F_g &= G\frac{mM}{r^2} + G\frac{mM}{r^2} \\
F_g &= G\frac{(7.35 \times 10^{22} \text{ kg})(2.00 \times 10^{30} \text{ kg})}{(1.50 \times 10^8 \text{ km})^2} \angle 180^\circ + G\frac{(7.35 \times 10^{22} \text{ kg})(5.67 \times 10^{24} \text{ kg})}{(3.84 \times 10^5 \text{ km})^2} \angle 270^\circ \\
F_g &= 1.910534752 \times 10^{20} \text{ N} \angle 260.8078552^\circ \\
\boxed{F_g = 1.91 \times 10^{20} \text{ N} \angle 261^\circ}
\end{aligned}
\tag{5}$$

3. An asteroid has a mass of $3.35 \times 10^{15} \text{ kg}$ and a radius of 19.0 km . Assume the asteroid is a uniform sphere.

1. What is the magnitude of gravitational acceleration on the surface of the asteroid?

$$\begin{aligned}
F_g &= G\frac{mM}{r^2} \\
a &= \frac{GM}{r^2} \\
a &= \frac{G(3.35 \times 10^{15} \text{ kg})}{(19.0 \text{ km})^2} \\
\boxed{a = 6.19 \times 10^{-4} \text{ m/s}^2}
\end{aligned}
\tag{6}$$

2. With what speed must you launch a rocket of mass m so that it completely escapes the asteroid's gravitational pull?

$$\begin{aligned}
v_e &= \sqrt{\frac{2GM}{r}} \\
v_e &= \sqrt{\frac{2G(3.35 \times 10^{15} \text{ kg})}{19.0 \text{ km}}} \\
v_e &= 4.851 \text{ m/s} \\
\boxed{v_e = 4.85 \text{ m/s}}
\end{aligned}
\tag{7}$$

4. The largest moon in the solar system is Ganymede, a moon of Jupiter. Ganymede orbits at a distance of $1.07 \times 10^9 \text{ m}$ from the center of Jupiter, with an orbital period of $6.18 \times 10^5 \text{ s}$. Using this information, calculate the mass of Jupiter.

$$\begin{aligned}
v_o &= \sqrt{\frac{GM}{r}} \\
v &= \frac{d}{t} = \frac{2\pi r}{t} = \frac{2\pi(1.07 \times 10^9 m)}{6.18 \times 10^5 s} \\
v_o &= 1.527949235 \times 10^4 m/s \\
\frac{v_o^2 r}{G} &= M \\
M &= \frac{(1.527949235 \times 10^4 m/s)^2 (1.07 \times 10^9 m)}{G} \\
M &= 3.743 \times 10^{27} kg \\
M &= 3.74 \times 10^{27} kg
\end{aligned}
\tag{8}$$

Linear Momentum

1. A puck of mass $m_1 = 0.151 kg$ slides along a frictionless horizontal surface with a velocity of $\vec{v}_{1i} = 0.450 m/s$ directed horizontally to the right. This puck makes a glancing collision with a second puck of mass $m_2 = 0.250 kg$ sliding with a velocity $\vec{v}_{2i} = 0.135 m/s$ directed horizontally to the left. After the collision, m_1 is observed to be moving with a speed $\vec{v}_{1vf} = 0.355 m/s$ at an angle of $\theta_{1f} = 17.1^\circ$.

1. What is the velocity of puck m_2 after the collision?

$$\begin{aligned}
p_i &= p_f \\
m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \\
v_{2f} &= \frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2} \\
v_{2f} &= \frac{(0.151 kg)(0.450 m/s \angle 0^\circ) + (0.250 kg)(0.135 m/s \angle 180^\circ) - (0.151 kg)(0.355 m/s \angle 17.1^\circ)}{0.250 kg} \\
v_{2f} &= 0.009283469707 m/s \angle -137.2232125^\circ \\
v_{2f} &= 0.0928 m/s \angle -137.^\circ \\
v_{2f} &= 0.0928 m/s \angle 137.^\circ \text{ clockwise}
\end{aligned}
\tag{9}$$

2. Is the collision elastic? If not, calculate the fractional change in the kinetic energy ($\frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i}$).

$$\text{Elastic : } \Delta K = 0 \quad (10)$$

$$\Delta K = K_f - K_i$$

$$\Delta K = \frac{1}{2}(m_1 v_{1i}^2 + m_2 v_{2i}^2 - (m_1 v_{1f}^2 + m_2 v_{2f}^2))$$

$$\Delta K = \frac{1}{2}((0.151 \text{ kg})(0.450 \text{ m/s} \angle 0^\circ)^2 + (0.250 \text{ kg})(0.135 \text{ m/s} \angle 180^\circ)^2 - \dots$$

$$((0.151 \text{ kg})(0.355 \text{ m/s} \angle 17.1^\circ)^2 + (0.250 \text{ kg})(0.0928 \text{ m/s} \angle -137^\circ)^2))$$

$$\Delta K = 0.01156154894 \text{ J}$$

$$\Delta K \neq 0 \text{ J}$$

$$\therefore$$

inelastic

$$\frac{\Delta K}{K_i} = \frac{\frac{1}{2}(m_1 v_{1i}^2 + m_2 v_{2i}^2 - (m_1 v_{1f}^2 + m_2 v_{2f}^2))}{\frac{1}{2}(m_1 v_{1i}^2 + m_2 v_{2i}^2)}$$

$$\frac{\Delta K}{K_i} = \frac{0.01156154894 \text{ J}}{((0.151 \text{ kg})(0.355 \text{ m/s} \angle 17.1^\circ)^2 + (0.250 \text{ kg})(0.0928 \text{ m/s} \angle -137^\circ)^2)}$$

$$\frac{\Delta K}{K_i} = \frac{0.01156154894 \text{ J}}{0.01022235025 \text{ J}}$$

$$\frac{\Delta K}{K_i} = 1.131006927$$

$$\boxed{\left| \frac{\Delta K}{K_i} = 1.13 \right|}$$

2. A block of mass m_1 is attached to a horizontal spring that is at equilibrium. The block rests on a frictionless horizontal surface. A wad of putty mass m_2 is thrown horizontally at the block, hitting it with a speed of v_{2i} and sticking to the block.

1. Derive an expression for the maximum amount that the putty-block system compresses the spring after the collision.

$$U_{\text{spring}} = \frac{1}{2} kx^2; K = \frac{1}{2} mv^2; p_i = p_f \quad (11)$$

$$m_2 v_{2i} = m_{1+2} v_f$$

$$v_f = \frac{m_2 v_{2i}}{m_{1+2}}$$

$$kx^2 = mv^2$$

$$x^2 = \frac{mv^2}{k}$$

$$x = \sqrt{\frac{m_{1+2} \left(\frac{m_2 v_{2i}}{m_{1+2}} \right)^2}{k}}$$

2. Determine the compression distance given the following:

$$\begin{aligned}
 m_1 &= 0.430\text{kg} \\
 m_2 &= 0.0500\text{kg} \\
 k &= 20.0\text{N/m} \\
 v_{2i} &= 2.330\text{m/s}
 \end{aligned}
 \tag{12}$$

$$\begin{aligned}
 x &= \sqrt{\frac{m_1 + 2\left(\frac{m_2 v_{2i}}{m_1 + 2}\right)^2}{k}} \\
 x &= \sqrt{\frac{(0.430\text{kg} + 0.500\text{kg})\left(\frac{(0.500\text{kg})(2.330\text{m/s})}{0.430\text{kg} + 0.500\text{kg}}\right)^2}{20.0\text{N/m}}} \\
 x &= 0.2701279068\text{m} \\
 \boxed{x = 0.270\text{m}}
 \end{aligned}
 \tag{13}$$

3. Block A in the figure below has a mass $m_A = 1.50\text{kg}$, and block B has a mass $m_B = 3.00\text{kg}$. The blocks are forced together, compressing a spring between them. The system is released from rest on a level, frictionless surface. The spring which has negligible mass, is not attached to either block, and it drops to the surface after it has expanded and pushed on both blocks. Block B acquires a speed of 1.25m/s .

1. What is the final speed of block A?

$$\begin{aligned}
 p_i &= p_f \\
 p_i &= 0 \\
 m_A v_A &= -m_B v_B \\
 v_A &= \frac{m_B v_B}{m_A} \\
 \frac{m_B}{m_A} &= 2 \\
 \boxed{v_A = 2.50\text{m/s}}
 \end{aligned}
 \tag{14}$$

2. How much potential energy was stored in the compressed spring?

$$\begin{aligned}
 P &= K \\
 K &= \frac{1}{2}(m_A v_A^2 + m_B v_B^2) \\
 P &= \frac{1}{2}((1.50\text{kg})(2.50\text{m/s})^2 + (3.00\text{kg})(1.25\text{m/s})^2) \\
 P &= 63.28125\text{J} \\
 \boxed{P = 63.3\text{J}}
 \end{aligned}
 \tag{15}$$

4. A chunk of ice of mass m_1 is sliding with a horizontal velocity of magnitude v_{1i} on the floor of an ice-covered valley when it collides with and sticks to another chunk of ice mass m_2 that is at rest at the base of a hill. The two blocks stick together and move up the hill together. Since the valley and the hill are icy, there is no friction between the chunks and the ground.

1. Which conservation laws may be applied to understand the motion of the ice chunks in this problem?

Conservation of energy and conservation of momentum.

2. What is the maximum vertical distance H that the combined ice chunks will go up the hill after the collision? [^1]

$$\begin{aligned}
 p_f &= p_i \\
 m_f v_f &= m_i v_{1i} \\
 v_f &= \frac{m_i v_{1i}}{m_f} \\
 v_f &= \frac{m_1 v_{1i}}{m_1 + m_2} \\
 U &= K \\
 mgh &= \frac{1}{2} m v^2 \\
 H &= \frac{\left(\frac{m_1 v_{1i}}{m_1 + m_2} \right)^2}{2g}
 \end{aligned} \tag{16}$$

3. Calculate the numerical value H given:

$$\begin{aligned}
 m_1 &= 5.00 \text{ kg} \\
 m_2 &= 7.50 \text{ kg} \\
 v_{1i} &= 12.0 \text{ m/s}
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 H &= \frac{\left(\frac{m_1 v_{1i}}{m_1 + m_2} \right)^2}{2g} \\
 H &= \frac{\left(\frac{5.00 \text{ kg}(12.0 \text{ m/s})}{(5.00 \text{ kg}) + (7.50 \text{ kg})} \right)^2}{2(9.81 \text{ m/s}^2)} \\
 H &= 1.174311927 \text{ m} \\
 \underline{H} &= \underline{1.17 \text{ m}}
 \end{aligned} \tag{18}$$

4. How much kinetic energy is lost due to the collision?

$$\begin{aligned}
\Delta K &= K_f - K_i; \quad K_f = U_f \\
\Delta K &= \frac{1}{2}mv^2 - mgh \\
\Delta K &= \frac{1}{2}m_1v_{1i}^2 - m_{1+2}gH \\
\Delta K &= \frac{1}{2}(5.00kg)(12.0m/s)^2 - (5.00kg + 7.50kg)(9.81m/s^2)(1.17m) \\
\Delta K &= 216J \\
\boxed{\Delta K = 216.J}
\end{aligned} \tag{19}$$

5. A Prius of mass $m_1 = 1.61 \times 10^3 kg$ is traveling north with an unknown initial speed v_{1i} . A RAV4 of mass $m_2 = 1.61 \times 10^3 kg$ is travelling with an initial velocity of $\vec{v}_{2i} = 17.8m/s \angle 160^\circ$. The two vehicles collide and stick together. Immediately after the collision, the combined vehicles are moving with a velocity of $\vec{v}_f = 13.1m/s \angle 133.4^\circ$. What is the initial speed of the prius before the collision?

$$\begin{aligned}
p_i &= p_f \\
m_1v_{1i} + m_2v_{2i} &= m_{1+2}v_f \\
v_{1i} &= \frac{m_{1+2}v_f - m_2v_{2i}}{m_1} \\
v_{1i} &= \frac{((1.38 + 1.61) \times 10^3 kg)(13.1m/s \angle 133.4^\circ) - (1.61 \times 10^3 kg)(17.8m/s \angle 160^\circ)}{1.38 \times 10^3 kg} \\
v_{1i} &= 13.51999850m/s \angle 89.94724050^\circ \\
\boxed{v_{1i} = 13.5m/s}
\end{aligned} \tag{20}$$